THE APPLICATION OF A GENERAL FIBRE ARCHITECTURE MODEL FOR COMPOSITE MECHANICAL PROPERTY PREDICTION

J.J. Crookston, B.J. Souter, A.C. Long and I.A. Jones
School of Mechanical, Materials, Manufacturing Engineering and Management,
University of Nottingham, University Park, Nottingham, NG7 2RD, UK

SUMMARY: An approach to composite component modelling is described, incorporating drape modelling to determine deformation of the reinforcement due to preform manufacture and element-wise property prediction for subsequent macroscopic (component) mechanical performance prediction. A modular method is employed such that various mechanical property models can be incorporated as appropriate. Results of finite element (FE) analyses are included showing the change in mechanical response over the geometry of a component. An important section of the work is development of a three-dimensional description of a textile unit-cell to enable FE analysis to be used to determine properties by mesomechanical methods. This textile description is general, enabling accurate modelling of both woven and warp-knitted (non-crimp) reinforcements.

KEYWORDS: Textile Preforms, Fabric Shear, Fabric Microstructure, Composite Lamina Properties, Unit Cell FE Analysis, Mesomechanics.

INTRODUCTION

For liquid composite moulding (LCM) processes such as resin transfer moulding (RTM), reinforcement materials need to be deformed to conform to the component shape, altering both the fibre orientation and the angle between warp and weft fibres. Many studies have been conducted in the field of drape analysis of textile composite preforms [1-4] and in some cases this work has been performed to facilitate permeability or mould filling simulations [5]. Considerably less work has been published relating to the mechanical properties of the resultant composite structure after draping.

It is generally considered that the only significant mode of deformation during draping is fabric shear [1]. It is clear that when a fabric is sheared the mechanical characteristics of the material will be changed, as the reinforcement orientation has a significant effect on stiffness and strength; if the reinforcement is sheared by up to 40°, so the principal stiffness and strength directions will also change.

Figure 1. Conical component mesh (left); Drape simulation output for plain-weave material (middle) and warp-knitted (non-crimp) material (right).
The nature of the work in hand is to interface a drape model developed previously [6] with macroscopic FE analysis techniques. This involves determination of component mechanical performance based on properties obtained by mesomechanics, determined at element level according to reinforcement shear during draping. This is implemented in a modular fashion such that properties predicted by any model can be easily incorporated enabling rapid evaluation of models using simple or complex components. Results from the drape model for a conical component for two fabrics, a plain-weave and a warp-knitted textile, are shown in Figure 1. These results will be used to demonstrate the effect of reinforcement deformation on mechanical performance.

**PREDICTION OF MECHANICAL PROPERTIES**

When conducting structural analyses of composite components, it is important to consider the effects of reorientation of reinforcement to maximise the advantages obtainable from such materials. Many approaches exist to predict mechanical properties of composite materials with respect to their orientation and, in some cases, with respect to shear deformation, e.g. due to preform manufacture or draping. Basic analytical methods begin with the Rule of Mixtures (ROM). Rudd et al [7] tested samples manufactured with sheared fabric reinforcement and compared the tensile moduli with those found using the modified ROM. Agreement was noted, with predictions suggesting higher moduli than those determined experimentally. Smith et al [8] also conducted experimental tests using flat plaque samples manufactured from sheared fabrics with a range of ply angles. Comparisons were made with predictions from classical laminate theory (CLT) which appeared to give good agreement. CLT has been adopted by the authors as an acceptable first approximation, particularly when used to simulate stitched (non-crimp) fabrics, since the approach neglects the effects of crimp, or waviness, present in woven reinforcement materials. This issue was addressed by Hofstee and van Keulen [9] who propose a method incorporating a repeating element for woven fabrics, in which reinforcement tows are assumed to follow curved paths. This method also allows for nesting of multilayered reinforcements by incorporating a geometric shift such that the warp/weft crossover of one layer lies above the gap between tows of the layer below; this improves fibre volume fraction predictions for laminates. Subsequently isostress or isostrain assumptions are made, i.e. constant stress or strain respectively are assumed throughout the tow volume, and CLT is performed using these properties. The latter two methods calculate laminate properties assuming two layers of fibre and a thin layer of (isotropic) material to represent the matrix properties. Hofstee and van Keulen consider this to be a major cause of the predicted stiffness in bias tensile tests being considerably lower than that determined experimentally for angles included between warp and weft fibres of less than 90°. Conversely it was noted that excellent agreement was observed for angles between fibres greater than 90°. To overcome the limitations of such simplified approaches, the three dimensional unit cell model have been proposed by a number of authors, notably Lomov et al [10] and Tan et al [11]. A similar method was also suggested by Bigaud and Hamelin [12], whose predicted results appear to show excellent correlation to experimental data for plain- and satin-weave fabrics. The fundamental shortcomings of the approaches detailed in the literature are the ability to describe: (i) general fabric architecture, (ii) effects of shear deformation on mechanical properties due to reorientation and subsequent change in fibre volume fraction, and (iii) failure envelopes for composite materials.

The approach taken by the present authors is to use a general geometric description of a fabric such that any reinforcement can be easily modelled once certain simple geometric parameters are known. The foundations of this model were developed by Robitaille et al [13,14] and the purpose of the work in hand is to extend the description to enable the solid modelling of a unit cell of material, taking into consideration shear deformation due to preform manufacture. This will be further extended to incorporate the generation of a three dimensional mesh with each element characterised and assigned appropriate properties, such that FE simulations can be carried out on the micromechanical model to obtain mesomechanical (unit-cell) properties. A schematic illustration of the modelling process under development is shown in Figure 2.
The basis of the model is a general description of reinforcement tow paths in vector form. Each path is constructed from a series of vectors, the start and end points of which define where possible interactions may occur with crossing tows (or threads in stitched materials).

Geometric parameters of the fabric are measured, and the linear density of the tow (Tex) is taken from manufacturer’s data. Using Equation 1, the tow height, $H_{ti}$, can be calculated from the fabric height or thickness, $H_f$, tow width, $W_t$, and linear density, Tex.

$$H_{ti} = \frac{H_f}{\sum_{i=0}^{i=n} \left( \frac{Tex_i}{W_{ti}} \right)}$$

where the subscript $i$ represents the $i^{th}$ tow at a crossover where there are $n_t$ tows.

Equation 1 assumes that the packing fraction of all tows is equal, and that all tows are in contact, i.e. no gaps exist between tows at crossover points. To enable an accurate three dimensional description of a fabric based on the geometric parameters available, other assumptions are required, specifically: (i) crimped tows follow lenticular paths, (ii) tow width varies during fabric shearing, while the crimp angle (i.e. the maximum angle between the tangent to the tow path and the nominal fabric plane) remains constant, (iii) contacting tow surfaces are consistent due to an even pressure distribution at the crossover, and (iv) free tow surfaces at crossovers have an elliptical boundary.

The basis of the description of a woven fabric is the definition of the lenticular tow paths. For fabrics with multilayered tows, i.e. one tow on top of another forming a double thickness in the weave, a common lenticular path is defined, with each tow following this path with a suitable shift, as shown in Figure 3. If the tow height, $H_{n,i}$, and distance from common path, $D_{n,i}$, (subscript $n$ depicts start or end value for vector) are assumed to change linearly with angle subtended, $\alpha$, then the expression for radius of tow path, $r_i(\alpha)$, in Equation 2, can be derived.

$$r_i(\alpha) = R \pm \left( D_{start} \left( 1 - \frac{\alpha}{2\psi_c} \right) + D_{end} \left( \frac{\alpha}{2\psi_c} \right) \right)$$

where $R$ is the radius of the common lenticular path, $\psi_c$ is the crimp angle (Figure 3), and $0 \leq \alpha \leq \psi_c$. Similar expressions can be developed to describe the radius change for the top and bottom surfaces of the tows by adding a tow height term to the equation. This is illustrated in Figure 3.

The co-ordinates of the tow path can be expressed using this radius, $r_i(\alpha)$, and the start or end co-ordinate of the path, $P_{n,i}$, using the expressions in Equations 3a and 3b:
where, $\phi_i$ is the angle subtended by the vector to the global $x$ axis, $n = \text{start}$ and sign is positive for the first half of the path; $n = \text{end}$ and sign is negative for second half of the path. For the $z$ co-ordinate of the path, Equation 3c is used.

$$z_i = |P_{n,i}|_z \pm (r_i(0) - r_i(\alpha) \cos \alpha)$$  

where the sign changes are as follows:
1. If $|P_{\text{start},i}|_z > |P_{\text{end},i}|_z$ then: $n = \text{start}$, sign = negative for the first half of the path; $n = \text{end}$, sign = positive for second half of path.
2. If $|P_{\text{start},i}|_z < |P_{\text{end},i}|_z$ then: $n = \text{start}$, sign = positive and for the first half of the path; $n = \text{end}$, sign = negative for second half of path.

Figure 3. Tow path description with multiple levels of tows, where the tow paths are defined from a single lenticular path, preventing interference between tows following the same path.

To complete the geometric description, the cross-section shape and volume occupied by each tow must be defined along its length. Many descriptions impose a uniform shape to define the tow cross-sections within a unit cell [15-17], but these models were created to analyse specific weave styles. In the general case, the assumption is used that the tow perimeter in contact with a crossing tow is dictated by the crossing tow path, enabling full three-dimensional descriptions for all weave styles to be defined, where the cross-section shape is able to change along the tow length. To demonstrate the cross-sectional shapes that are produced, a simple model is presented for three common types of crossover found in biaxial woven reinforcements, shown in Figure 4.
By a method similar to that proposed by McBride [17] the tow width parameter, $W_t$, is recalculated when the fabric is sheared through an angle $\theta_s$ according to the relationship in Equation 4.

\[
\begin{align*}
W_t &= W_{t0} & \theta_s \leq \theta_{\text{Limit}} \\
W_t &= S_t \cos \theta_s & \theta_s > \theta_{\text{Limit}}
\end{align*}
\]

where $S_t$ is the tow spacing, $W_{t0}$ is the initial (measured) tow width, and the angle at which adjacent tows come into contact, $\theta_{\text{Limit}}$, is defined in Equation 5.

\[
\theta_{\text{Limit}} = \cos^{-1}\left(\frac{W_{t0}}{S_t}\right)
\]

This tow width variation model was compared with experimental measurements obtained using a video camera during shearing experiments [6] and an excellent correlation was observed.

![Figure 4. Three different types of tow crossover used to describe woven fabrics, before shearing, top, and during shearing, bottom. ($L_c = $ contact length, $T_s = $ tow spacing and $T_w = $ tow width)](image)

The final step in generating the solid model is to produce a mathematical description of the tow surfaces. This is implemented using Ferguson (bicubic polynomial) patches, which are defined using 16 points on the surface. These are used to allow continuity of curvature between areas on the tow surface. This is required as contacting tow surfaces, i.e. the area between crossing tows, are defined using the same surface description, hence the various adjacent surfaces must have very different, but continuous, curvature.

In order to generate a mesh for the unit cell of composite, the rhombohedral volume containing the repeat unit must be determined, and within this the tow boundary points must be identified to give mesh co-ordinates for each axis. These are points coincident with the
edges of tows, found by scanning along each axis and finding the points where tows cross one another. This procedure is repeated for each axis, and forms a basis for dividing the volume into prismatic elements. A more complete explanation of the mesh generation routine will be presented orally, as coding is still in progress at present and hence further details are not yet available.

Once a mesh is generated for the rhombohedral cell volume, a test must be performed to determine whether each element lies within the matrix or the reinforcement to enable determination of the material type. For the elements in the reinforcement, their proximity to the matrix/tow interface must be established as infiltration of resin into the fibre bundle may change with distance from the tow surface. For these elements, the tow direction must also be specified as the principal material direction for the element. Appropriate mechanical properties must subsequently be assigned to each element type.

On completing this procedure for a number of unit cells with different shear angles, each one is to be analysed using appropriate boundary conditions and load cases to enable the determination of the elastic constants for the bulk material in each case. Similarly, by the application of appropriate failure criteria within the analysis, the failure envelope may be determined. Results from these analyses can then be used within the procedure described in the following section as a model for mechanical properties to be incorporated in the analysis of components on an element-wise basis.

**PREDICTION OF COMPONENT PERFORMANCE**

Some work has been carried out to interface the University of Nottingham Drape Model with the Abaqus™ Standard FE package. The methodology used to produce input files for Abaqus, using the draped output, is described below.

The basis for this analysis is an iterative model for draping of bi-directional textiles, which is able to predict the variation in fibre orientations and volume fraction \( V_f \) over a three dimensional component. This is described in detail elsewhere [6]. A geometric model is loaded into the drape model, and a draping simulation is performed. At this stage, material shear data are incorporated to give an accurate representation of fabric behaviour during draping. The drape model is used to provide node co-ordinates, element definitions (node numbers used as vertices) and material directions (consisting of direction vectors for the warp and weft fibres). These data are then used by the property prediction module. This module is designed to be easily modified, and as a first approach calculates properties, using the ROM (Equations 6a and 6b) for properties in the fibre direction:

\[
E_1 = V_f E_f + (1 - V_f) E_m \quad (6a) \\
v_{12} = V_f v_f + (1 - V_f) v_m \quad (6b)
\]

where subscripts \( f \) and \( m \) represent fibre and matrix properties respectively. The Halpin-Tsai relationships (Equations 7a and 7b) for transverse and shear properties:

\[
E_2 = \frac{E_m (1 + \xi \eta V_f)}{1 - \eta V_f} \quad \text{where } \eta = \frac{E_f - E_m}{E_f + \xi E_m} \quad (7a) \\
G_{12} = \frac{G_m (1 + \xi \eta V_f)}{1 - \eta V_f} \quad \text{where } \eta = \frac{G_f - G_m}{G_f + \xi G_m} \quad (7b)
\]

where \( \xi = 2 \) for \( E_2 \) and \( \xi = 1 \) for \( G_{12} \) [18].
This provides data for Abaqus to calculate laminate material properties for each element. CLT calculations will be incorporated into the module in due course, producing a complete stiffness modelling module, which will be suitable for incorporating new stiffness models as well as including failure criteria. As results become available from the unit-cell FE analyses described above, these may also be utilised using suitable semi-empirical property models. The approach adopted is described schematically in Figure 5, while the current implementation of the intermediate model is shown in Figure 6.

The results presented in Figure 7 show the effect of draping on displacement under a fixed load in two simulations of a conical component subjected to a radial pinch loading. Material data used are given in Table 1. Multiple load cases are used, each applying the pinch load at a different position around the draped component geometry. The trend is toward quarter-cone symmetry for plain-weave reinforced component, while the peak present at 90° for this simulation is observed to shift in correlation with the shift of the 0° shear path for the warp-knitted fabric, illustrated in Figure 1. Correlation with the output from the drape model is evident, the results highlighting the validity of the incorporation of drape data into structural analyses.

![Figure 5](image-url) Overall approach to component mechanical performance modelling.

![Figure 6](image-url) Current implementation of intermediate stiffness model, incorporating internal routines within Abaqus Standard to calculate laminate properties.
DISCUSSION
The results shown using the initial implementation of the described methodology illustrate the potential application for such a technique in composite component design and analysis. The importance of more accurate descriptions of material behaviour is demonstrated, particularly for highly draped components.

The intention of the authors is to develop a generic approach to mechanical analysis of components via a mesoscopic model for mechanical properties at element level. Ultimately the intermediate stiffness model will be provided by FE analysis of solid models for a general textile composite unit cell, incorporating changes in properties due to shear deformation during preform manufacture.

Models will be validated using mechanical test data for a range of engineering components in addition to simple test specimens. Failure criteria will also be incorporated to enable complete component performance modelling with a modular approach, allowing the incorporation of new models as they become available.

ACKNOWLEDGEMENTS
The work is funded by the EPSRC, and the authors wish to express their gratitude to The Ford Motor Company, Dowty Aerospace Propellors and ESI for their support. The authors also wish to thank Tim Baillie for his contribution to the implementation of the stiffness model.

REFERENCES


