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An Application of Finite Element Method for laminated Composite Plate

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SUMMARY: This research is to investigate the effective tensile modulus for composite laminates, made from plain-weave fabrics of E-glass type and polyester resin, of which family is [(0/90)°/±45°]. Making variation in percentage of ±45°ply yields chosen specimen configurations as follow :

$(0/90)_{10}°$, $[(0/90)_4°/\pm45_2°/(0/90)_4°]$, $[(0/90)_3°/\pm45_4°/(0/90)_3°]$, $[(0/90)_2°/\pm45_6°/(0/90)_2°]$, $[(0/90)°/\pm45_8°/(0/90)°]$, . $\pm45_{10}°$.

For experimental data, effective modulus acquired from a tensile test are coMPared with those obtained by using Ansys FEA package version 5.4 and by direct calculation to solve for the engineering property. For such problem solving, when dealing with woven cloth ,single layer of plain-weave glass cloth is assumed as if the fiber in the warp (0°) direction and weft(0°) orientation were separated from each other and lay in two separate plies and its properties of constituent should follow basic lamination theory and the same micromechanic expressions used for a lamina with unidirectional fiber, however this assumption should be accoMPanied with some factors of difference. One of the important aspects of this research is to find the proper way of getting four of mechanical lamina constants : longitudinal modulus (E_1) , transverse modulus (E_2) ,major Poisson's ratio(υ_{12})and shear modulus(G_{12}), which will be used as inputs for further calculation to determine effective laminate constants. When E_2 are set equivalent to the tensile modulus of matrix and υ_{12} is approximated to 0.3, E_1 can be derived from modeled $(0/90)_{10}°$ and G_{12} , with the corrected E_1 ,can be done from ±45°. Using both of the calibrated modulus, the FEA results of the rest configurations .are almost the same as those obtained from the experiment

KEYWORDS : Composite Laminate, Plain-Weave, Effective Tensile Modulus,Lamina

INTRODUCTION

Woven fiber has lost its stiffness and it strength due to its fiber crimping. The composite laminate with such fiber can exhibits a low modulus and strength. Mechanical properties of a single layer is presumably the properties of half fiber contents soaked in half of its matrix. Without a concern in loss of stiffness and strength, the alleged comment may still be far from the actual values.

The equations from macromechanic study being useful for mechanical properties determination for single ply of composite containing unidirectional fiber reinforcement are as follows :

$$E_1 = E_f V_f + E_m (1 - V_f) \quad , \quad \upsilon_{12} = \upsilon_f V_f + \upsilon_m (1 - V_f)$$

$$E_2 = E_f E_m /(E_m V_f + E_f (1 - V_f) \quad , \quad G_{12} = E_1 E_2 /(E_1 + E_2 + 2E_1 \upsilon_{12})$$

where subscript 1 defines the direction parallel to fibers'direction

2 defines the direction transverse to fibers'direction

and E_f = fiber Young's modulus V_f = volume faction of fibers

E_m = matrix Young's modulus E_1 = lamina longitudinal Young 's modulus

E_2 = lamina transverse Young 's modulus υ_{12} = major Poisson's ratio

υ_f = fiber Poisson's ratio υ_m = matrix Poisson's ratio

G_f = fiber shear modulus G_m = matrix shear modulus

For a bi-directional reinforcement , the longitudinal modulus is found from

$E_1 = \alpha E_f V_f + E_m (1 - V_f)$ where α is the fiber efficiency

However this research does not aim at using all those expressions but to get the data out from the produce which is reasonable because some loss factor from manufacturing process will automatically included.

GENERAL CONSTITUTIVE EQUATION

Generalized Hook's law for a state of plane stress is given by

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & O \\ S_{21} & S_{22} & O \\ O & O & S_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}$$

where

$$S_{11} = \frac{E_1}{(1 - \upsilon_{12}\upsilon_{21})} \qquad S_{12} = \upsilon_{12}\upsilon_{21} \qquad S_{22} = \frac{E_2}{(1 - \upsilon_{12}\upsilon_{21})} \qquad S_{66} = G_{12}$$

When dealing with laminate , x and y are used for an axis notation and the plane stress matrix is then changed to

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\end{bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & Q_{16}\\ Q_{21} & Q_{22} & Q_{26}\\ Q_{61} & Q_{62} & Q_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix}$$

$$Q_{11}=S_{11}m^4+2(S_{12}+2S_{66})n^2m^2+C_{22}n^4$$

$$Q_{22}=S_{11}n^4+2(S_{12}+2S_{66})n^2m^2+C_{22}m^4$$

$$Q_{12}=(S_{11}+S_{22}-4S_{66})n^2m^2+C_{12}(n^4+m^4)$$

$$Q_{66}=(S_{11}+S_{22}-2S_{12}-2C_{66})n^2m^2+C_{66}(n^4+m^4)$$

$$Q_{16}=(S_{11}+S_{12}-2C_{66})nm^3+(C_{12}-C_{22}+2C_{66})n^3m$$

$$Q_{26}=(S_{11}+S_{12}-2C_{66})n^3m+(C_{12}-C_{22}+2C_{66})nm^3$$

where $n=\sin\theta$, $m=\cos\theta$ and θ is the angle between the x-y and principle material directions.

The general constitutive equation without thermal term for plates and shell which shows relationship between the shell resultants(force: N and moment : M) and plane strains (ε) and curvatures(κ) on the mid-plane of shell is :

$$\left[\begin{array}{c}N_X\\ N_Y\\ N_{XY}\\ \hline M_X\\ M_Y\\ M_{XY}\end{array}\right]=\left[\begin{array}{ccc}A_{11} & A_{12} & A_{16}\\ A_{21} & A_{22} & A_{26}\\ A_{61} & A_{62} & A_{66}\\ \hline B_{11} & B_{12} & B_{16}\\ B_{21} & B_{22} & B_{26}\\ B_{61} & B_{62} & B_{66}\end{array}\right]\begin{bmatrix}\varepsilon_X\\ \varepsilon_Y\\ \varepsilon_{XY}\end{bmatrix}+\left[\begin{array}{ccc}B_{11} & B_{12} & B_{16}\\ B_{21} & B_{22} & B_{26}\\ B_{61} & B_{62} & B_{66}\\ \hline D_{11} & D_{12} & D_{16}\\ D_{21} & D_{22} & D_{26}\\ D_{61} & D_{62} & D_{66}\end{array}\right]\begin{bmatrix}\kappa_X\\ \kappa_Y\\ \kappa_{XY}\end{bmatrix}$$

where

$$A_{ij}=\sum_{k=1}^{n}(Q_{ij})_k(Z_k-Z_{k-1})$$

$$B_{ij}=\sum_{k=1}^{n}(Q_{ij})_k(Z_k{}^2-Z_{k-1}{}^2)/2\quad D_{ij}=\sum_{k=1}^{n}(Q_{ij})_k(Z_k{}^3-Z_{k-1}{}^3)/3$$

A_{ij} is extensional stiffness matrix , B_{ij} is extensional-bending coupling stiffness matrix

D_{ij} is bending stiffness matrix

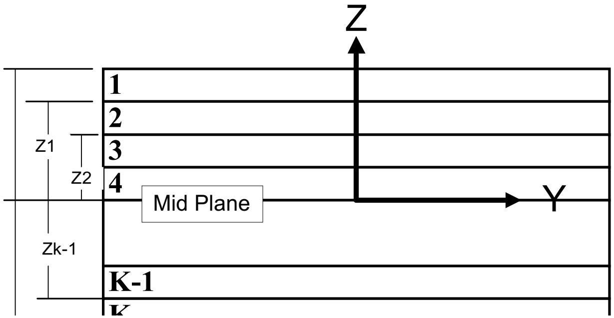

Figure 1 Laminate ply notation

Substituting coefficients from the extensional stiffness matrix into expressions below, effective elastic constants of laminate, when each layer has the same thickness , are given by

$$E_x = \frac{A_{11}A_{22} - A_{12}^{\ 2}}{A_{22}t} \qquad E_y = \frac{A_{11}A_{22} - A_{12}^{\ 2}}{A_{11}t}$$

$$\upsilon_{xy} = -\frac{A_{12}}{A_{22}} \qquad \upsilon_{yx} = -\frac{A_{12}}{A_{11}} \qquad G_{xy} = \frac{A_{66}}{t}$$

These are effective properties for finite element analysis to be carried out.

FINITE ELEMENT MODELLING

Ansys 5.4 ,multiphysic code is used here.The analysis is set up following the pattern of specimen tested in the actual environment. Only half length of model is drown. At the left end -side of the model is constrained with symmetrical displacement which means there is a mirrored half of this model to its left side. Force per width length is applied as a load on the right end. The bottom left node is fixed (no node translation is allowed in any direction.) Element type defined for the meshing is Shell99L.

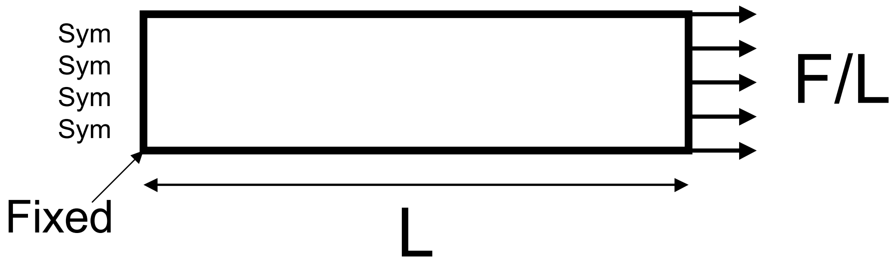

Figure 2 Constraints used on the FEA model

$$E_x = \frac{F}{A} \times \frac{L}{\Delta L}$$

where

F = force
L = length of specimen
A = area
ΔL = effective length

TEST RESULT

Specimen Configuration	Averaged Longitudinal Tensile Modulus(MPa)
$(0/90)_{10}°$	**3872**
$[(0/90)_4°/\pm45_2°/(0/90)_4°]$	**3550**
$[(0/90)_3°/\pm45_4°/(0/90)_3°]$	**3370**
$[(0/90)_2°/\pm45_6°/(0/90)_2°]$	**3201**
$[(0/90)°/\pm45_8°/(0/90)°]$	**2931**
$.\pm45_{10}°$	**2541**

Figure 3 Test results of specimens after being performed on a tensile test.

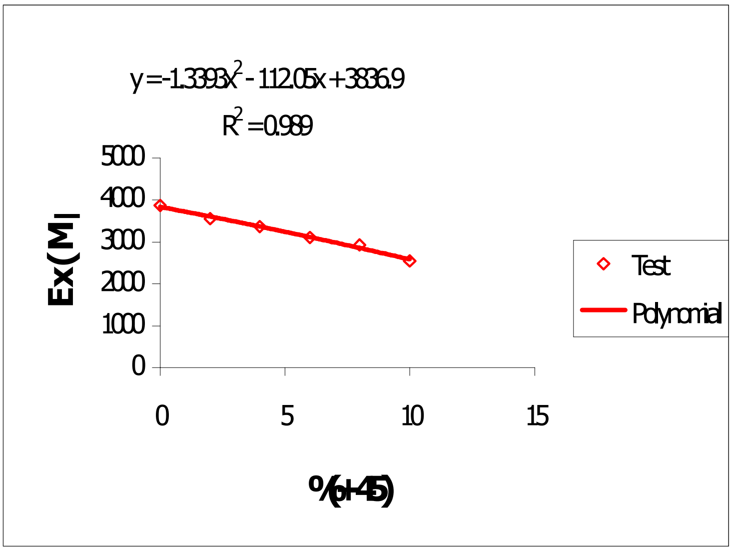

Figure 4 Plot of %±45° versus Averaged Longitudinal tensile modulus ,*E*x (MPa)

From the test results it is found that when the percentage of [±45°] increases, the tensile modulus decreases in a polynomial trend, with a small rate of change in slope. The modulus for [±45°] specimen hit the minimum value of 2541 MPa. Although $(0/90)_{10}°$ has reach the maximum value but such data is so small when compared to the tensile modulus of fiber of 77000 MPa.

MODIFICATION OF PLY PROPERTIES

To find the effective elastic constants of laminate, it is necessary to acquire those mechanical parameters of a unidirectional lamina where standard tests should also be carried out. The following section in this paper is the alternative to extract two of four basic properties, E_1 and G_{12} ,of the assumed unidirectional ply of woven glass in a lamina by retrieving the data directly from experimental tensile testing result of $(0/90)_{10}°$ and $\pm 45_{10}°$ specimens and then to use them to predict the tensile modulus of the rest having the different combination of $\pm 45_{10}°$.

Based on the assumption that E_2 equals to the tensile modulus of resin and Poisson's ratio is 0.3, the exact longitudinal tensile modulus of a lamina(E_1) that builds up the tensile modulus of $(0/90)_{10}°$ model is to be calibrated. For the exact lamina shear modulus(G_{12}), the model that correlates with this parameter is $\pm 45_{10}°$.

STEP 1 E_1 calibration from $(0/90)_{10}°$ model

Input properties E_2 = Tensile Modulus of resin = 2100 MPa , $\upsilon_{12} = 0.3$

$$G_{12} = E_1 E_2 / (E_1 + E_2 + 2E_1 \upsilon_{12}) = 1039.604 \text{ MPa}$$

Concept in calibration is that E_1 is varied to find E_x , using ansys/multiphysics code and direct calculation, so that the range of the E_x value is to cover result of E_x obtained from the real experiment. The known parameter here is the one from the test(E_x = 3872) is used in interpolation to get E_1 in a backward manner.(It is noted here that E_x from FEA and direct calculation give the same result of E_x , so in figure 5 only one column of E_x is shown.

E_1 (MPa)	E_x (MPa)
5500	3827
6000	4081

Figure 5 Interpolation table between E_1 and E_x

To get E_x at 3872 MPa , E_1 has to be 5589 MPa.

E_1 (MPa)	E_2 (MPa)	υ_{12}	G_{12} (MPa)
5589	2100	03	1039.604

Figure 6 Modified E_1 and other principle properties

Now properties, from figure 6 ,are ready to be used as a new input to find E_x of the other models as shown below:

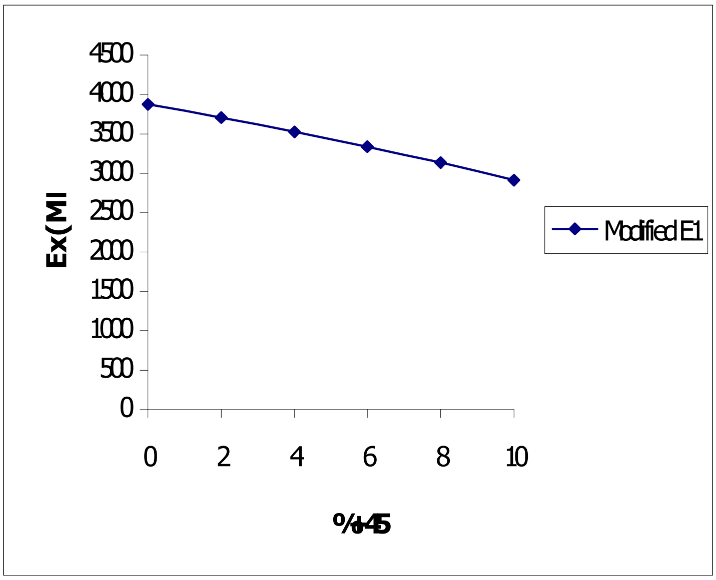

Figure 7 Tensile modulus prediction after having E_x calibrated.

Section 2 G_{12} calibration from $\pm 45_{10}°$ model

Input properties E_1 = 5589 MPa

E_2 = Tensile Modulus of resin = 2100 MPa , $\upsilon_{12} = 0.3$

Like section 1, the value of G_{12} that give E_x of $\pm 45_{10}°$ of 2541 is 875.99 MPa

G_{12} (MPa)	E_x (MPa)
900	2592.3975
800	2378.3162

Figure 8 Interpolation table between G_{12} and E_x

Now new ply- properties are found.

E_1 (MPa)	E_2 (MPa)	υ_{12}	G_{12} (MPa)
5589	2100	03	875.99

Figure 9 Modified E_1, moified G_{12} and other principle properties

Properties from figure 9 is, again ,used to predict E_x of the others.

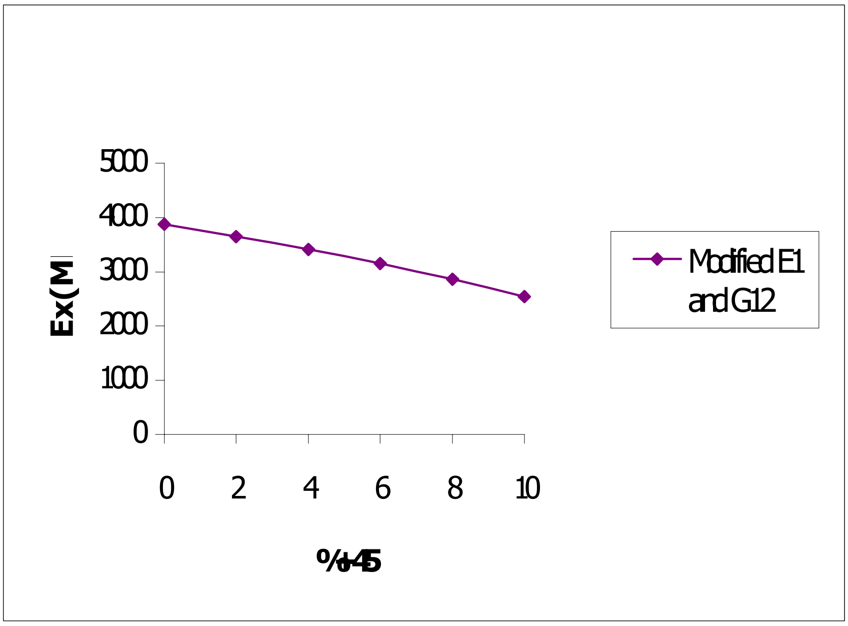

Figure 10 Tensile modulus prediction after having E_x and G_{12} calibrated.

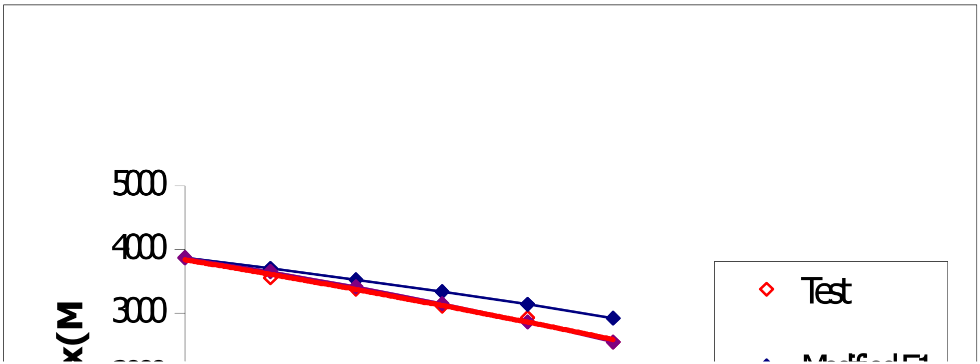

Figure 11 CoMParision among tensile modulus prediction with modified E_x ,

with modified E_x and G_{12} to the real experiment

It is obviously seen that if only E_1 is modified, the predicted results do not much come close to that of which are tested when there is a high percentage of %±45° ; however the rate of reduction in longitudinal tensile modulus when %±45° increases can be added up by reducing the value of G_{12} . When both of the values, E_1 and G_{12} , are corrected, the predicted results are almost the same as what are experimented, which prove that a laminate theory for unidirectional-fiber laminate can also be suitable for the bi-directional fiber-reinforce laminate.

CONCLUSION

This paper describes the method to back out the two main properties of the bi-directional fiber-reinforce lamina with equal strength in both directions which are longitudinal modulus of elasticity(E_1) and shear modulus(G_{12}) of a ply. The experimented result of E_x for $(0/90)_{10}$° is a target value used in interpolation to find E_1 in a backward manner and in the same way the experimented result of E_x for %±45°specimen are used to get the proper value of G_{12} .

These two developed properties with two other constants are used in the calculation to find E_x of other configuration having the percentage of %±45° in between 0-100 %. It is satisfactory to have only E_1 modified but when both E_1 and G_{12} are corrected , the predictions are close to the exact solution.

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