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Damage Modelling in Cross-Ply Composite Laminates with Double-Edge-Semicircular Notches

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Abstract

In order to study the failure mechanisms of the matrix-dominated damage in composite laminates, a local-global progressive failure finite element (FEM) approach was created in combination with the local application of a group of strength criteria. Constitutively, it is assumed that the laminate is made up of homogeneous orthotropic layers before the matrix-dominated failure occurs. Matrix damage reduces the stiffness. Consequently, the average reduced layer stiffness as a function of the applied strain is separated from the reduced laminate stiffness, and it is used as the input of the FEM model. A group of strength criteria is separately applied to the layers at the integration points of elements to induce multi-mode of local matrix failure into the material, e.g. transverse cracks and notch-induced splits. Two types of intralaminar matrix failure, transverse cracking and notch-induced splitting (NIS), as well as their interactions are then simulated. The FEM simulation predicts a thickness-dependent transverse crack growth and a constant onset strain for the first transverse crack (FTC). The initiation and propagation of the notch-induced splits are simulated incorporating the effects of the earlier transverse cracking. The influence of local delamination was also briefly discussed. One of the most important conclusions is that the effects of the earlier matrix damage have to be taken into account for a correct prediction of the occurrence of the notch-induced split, its initiation and growth.

1. Introduction

Before the final catastrophic collapse, a number of different forms of matrix-dominated failure have already occurred in a fibre reinforced polymer composite laminate with or without notches. These matrix cracks (displacement discontinuity in a single layer or between laminae) cause local composite decomposition and hence local stress redistribution on a laminar scale or between laminae (three-dimensionally). A geometric notch may speed up the global structural failure by inducing new forms of matrix failure, or by radically quickening the failure process through the notch intensified stress field. Therefore, various forms of matrix dominated subcritical failure in polymer composites have been a subject of extensive investigation in the field of composite materials.

Various matrix damage modes were observed by Kortschot *et al.* [1-4], Liu *et al.* [5,6], and Reifsnider *et al.* [8] in notched laminates. These typical damage modes in a notched cross-ply laminate under tensile load are shown in Figure 1, which includes transverse matrix cracks, longitudinal matrix cracks and interlaminar delamination. Fibre breaking occurs at later stage of loading.

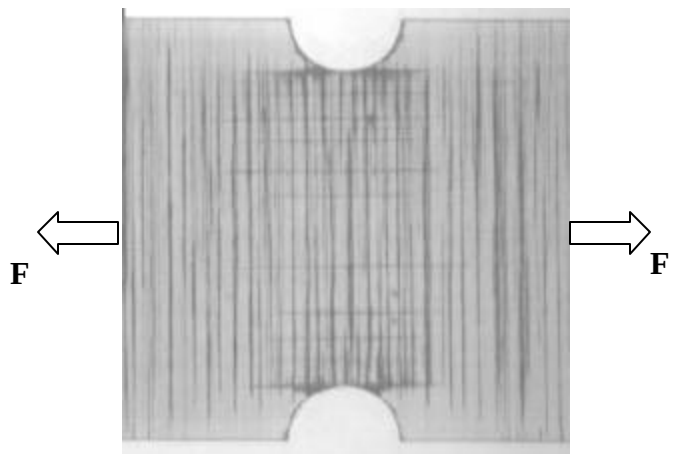


Figure 1. Typical matrix failure modes in a cross-ply laminate with double-edge-semicircular notches (DEN) at s_{net}=174 MPa.

The first type of the matrix dominated failure, observed in a laminate under tensile load, tension-tension fatigue, or even residual load from manufacture, is the matrix microdamage along the fibre direction. Traditionally, they are named as transverse cracks in cross-ply laminates. As a special case of this type intralaminar cracking, the mismatch of the Poisson's ratios, υ_{LT}-υ_{TL}, between the longitudinal layers and transverse layers in a cross-ply laminate may induce longitudinal cracking in longitudinal layers. Nevertheless, this type of intralaminar matrix cracks may never attain its full saturation as it does in transverse layers. Transverse matrix cracking takes place in a very early stage of loading, e.g. at 0.2% to 0.5% average strain in a monotonously increasing tensile test in glass-fibre reinforced unsaturated polyester and epoxy observed by Garrett *et al.* [15] and by Parvizi *et al.* [16], and in unsaturated polyester recently by Liu *et al.* [5,6]. This strain value will noticeably be influenced by the residual stresses/strains induced during manufacture. In fact, initial matrix cracks were observed in glass-fibre reinforced polyester laminates [10] due to the residual stresses/strains induced by chemical and thermal shrinkage. It is very difficult to treat the behaviour of

such cracks in a highly anisotropic material, with detailed micro mechanics, due to their large number and more or less irregular appearance. However, the damage state of a laminate caused by the transverse cracks, from a point of average macroscale effects, is traditionally represented by the transverse cracks density, which is defined as the transverse crack number in a unite length. As the applied load increases, the transverse crack density tends to a saturation value [5-7].

In addition to the conventional longitudinal cracks, mainly caused by the mismatch of the Poisson's ratios, notch-induced longitudinal cracks (splits) were observed by Kortschot [1-4], Liu [5,6] and Reifsnider *et al*. [8] in the longitudinal plies of notched cross-ply laminates, and by Bazhenov [9] in notched unidirectional composites. The propagation of this type matrix cracks along the fibre direction is stable. A function associating the split growth with the applied stress in the cross-ply laminate was proposed by Kortschot *et al*. [1-4]. One of their conclusions is that the matrix dominated subcritical damage must be considered in developing models to predict the notched strength of composites laminates. Reddy et al. [12] and Liu *et al*. indicated that the notch-induced splits are initiated by the high inplane shear stress and the normal stress perpendicular to the fibre direction in the 0° plies. It is observed that the growth of the notch-induced split is about linearly proportional to the far-field applied strain. Bazhenov found that the fracture toughness of this failure mode increases with the split length growth in notched unidirectional composites, which he attributed to the fibre bridging effect. Reifsnider *et al*. studied the micro-damage development in notched composite laminates under fully reversed cyclic loads. The materials used were carbon fibre epoxy systems (T300/Narmco 5208 and AS4/Cycom 1808). Notch-induced splits and notch-induced delamination were observed in laminate $[0/45/90/-45]_{s4}$ and $[0/45/0/-45]_{s4}$ with a central circular cutout. They found that the matrix cracking provided the paths for the development of delamination and fibre fracture, and regulate the regions of delamination initiation. Schulte [11] observed microscale interlaminar delamination at the intersection of a transverse crack and a longitudinal crack in fatigued carbon-epoxy cross-ply composites. Summarizing, it can be stated in general that a matrix failure event occurred at an earlier stage may affect the failure evolution in later stages. In other words: damage events interact with each other to induce or quicken (may be even delay) another mode of damage. The failure mechanisms and the failure evolution become very complicated due to this kind of interaction or intersection. Vaidya *et al*. [13, 14] inspected the effects of the ply thickness on the fracture of cracked cross-ply laminates and on quasi-isotropic laminates with a man-made crack. They concluded that the layer thickness or the layer cluster numbers in a cross-ply laminate have significant effects on the fracture behaviour.

As mentioned above, the notch-induced split is an often-observed matrix-dominated failure mode in continuous fibre reinforced composite plates with notches. This paper presents a numerically based progressive failure approach to study the formation and propagation of the notch-induced split and its interaction with transverse cracks and local delamination in cross-ply laminates with double-edge-semicircular notches (DEN). A group of strength criteria was implemented in the three-dimensional finite element model to induce the local matrix-dominated failure into the laminae. The characteristic stress components were first inspected both globally and locally. Then, the formation of the notch-induced split, and its interaction with transverse cracks and delamination, was simulated.

2. Material Modelling

2.1. The initial elastic constants [5,6]

A highly transparent material system of E-glass fibre reinforced unsaturated polyester resin (Synolite 0593 -N-2) was used in the study. The initial elastic properties of the unidirectional (UD) laminae, with an average fibre volume of 55%, are

$$E_L=42 \text{ GPa}, E_T=14 \text{ GPa}, G_{LT}=5.6 \text{ GPa}, \nu_{LT}=0.26, \nu_{TT}=0.45$$

where L and T represent the longitudinal direction and the transverse direction of the laminae, respectively.

2.2. The residual transverse stiffness

Transverse cracking in both the longitudinal layers and the transverse layers has already taken place at a low applied load and leads to stiffness degradation of the laminate in both directions. The equivalent (reduced) laminae stiffness in the direction perpendicular to the fibres can be estimated from the reduced laminate stiffness, by assuming that the tensile

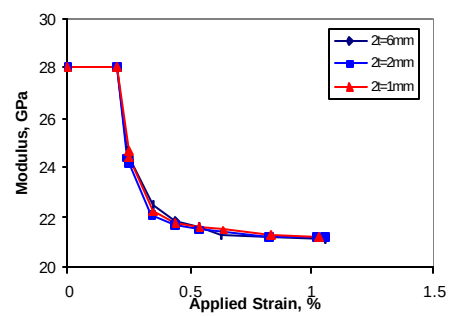


Figure 2a. The stiffness reduction of laminate

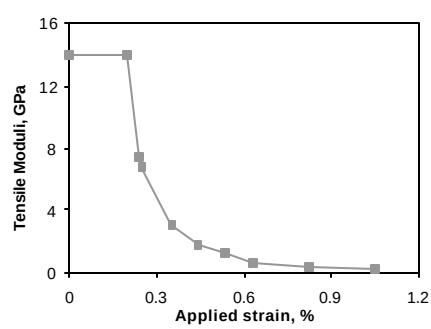


Figure 2b. The reduced laminae stiffness in the transverse direction.

modulus of the laminae in the fibre direction remains constant, using simplified classical laminate theory (CLT):

$$E_T^* = \frac{tE_C - t_L E_L}{t_T} \qquad (1)$$

where E^*_T, E_C and E_L stand for the equivalent modulus in the direction perpendicular to the fibre direction, young's modulus of the laminate in the loading direction and longitudinal modulus of the lamina. t, t_L and t_T stand for the thickness of the laminate, the longitudinal layers, and the transverse layers, respectively. The laminate stiffness degradation is shown in Figure 2*a*, and the calculated stiffness degradation in the counterpart 90° layer is presented in Figure 2*b*.

2.3. The degradation of the inplane shear modulus

As a result of the matrix cracking along the fibre direction, the in-plane shear stiffness of the laminate decays in a manner as shown in Figure 2, which was measured using a two-rail shear tests. It is assumed that the inplane shear moduli of both the longitudinal layers and the transverse layers degenerate in the same manner. The theoretical initial in-plane shear modulus is about 5.6 GPa (the measured average value is about 5.4 GPa). The residual in-plane shear modulus is about 10% of the original value at a stress level about 35 MPa.

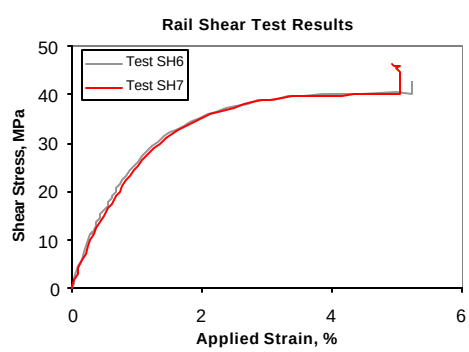


Figure 3. The degeneration of in-plane shear modulus

The local stiffness degradation in other directions can be obtained by the transversely isotropic assumption in each layer.

2.4. Assumptions - material modelling

Two essential aspects have to be included when a matrix failure is modelled: (1). The matrix-dominated failure behaviour, including its initiation and its growth, has to be modelled; (2). There must be a new constitutive law to represent the stress-strain relationships of the damaged materials. For modelling the damage behaviour of the materials, it is assumed that:

(1) Traditional composite modelling assumes laminates to be made of homogeneous orthotropic elastic layers. The laminate remains to be orthotropic on a single layer scale, but it becomes considerably less homogeneous after the matrix dominated failure occurs. Based on the traditional assumption, the classical laminate theory (CLT) is applicable to separate the reduced stiffness of a laminate into layer scale stiffness degradation, as that stated in section 2.2. The global stress-strain relationship of the laminate is then written as an incremental function of the global strain under homogenous assumption:

$$\sigma_{ij} = \sum_{k=1}^{n} [\Delta C_{ij}]^k \, \Delta\varepsilon_{ij}^k \qquad (2)$$

where $[\Delta C_{ij}]$ is the stiffness incremental matrix as a function of the applied load, σ_{ij} and $\Delta\varepsilon_{ij}$ stand for the stress and strain change, respectively, and k refers to the kth step of loading.

(2) The locally reduced stiffness of the damaged materials can be represented by the product of the initial constitutive constants and a group of scalar parameters as a function of the applied average load, in terms of stress or strain:

$$[C_{ij}]^L = [\alpha_{ij} C_{ij}^0] \qquad (3)$$

where L and 0 refer to the local and original stiffness, respectively. α_{ij} is the global load dependent damage parameters. In this way, the local matrix failure is constitutively represented by the local stiffness changes. Consequently, the assumption of homogeneity is not necessary anymore on a small scale where local failure is present.

(3) A group of strength criteria is used to induce the multi-sites of matrix failure in multiple layers. The global non-linearity of the composite laminate is only caused by the matrix-dominated failure.

3. Construction of 3D FEM Models

3.1. 3D FE Model Construction

Figure 4 shows one eighth of a double edge notched (DEN) cross-ply laminate and the co-ordinate system used in the simulation. The load direction, (x, 1), is the same as the fibre direction in the 0° plies; the (y, 2) direction is defined as the width direction of the plate, which is perpendicular to the fibre direction in 0° layers; and the (z, 3) direction is the thickness direction. The planes defined by x=0, y=0, and z=0 are the symmetric surfaces perpendicular to the x, y and z, respectively. For a more accurate representation of the

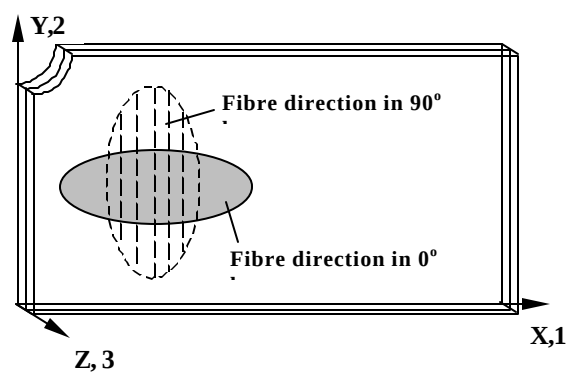


Figure 4. One eighth of the laminate with coordinate system.

stresses near the free edges and in the damaged areas, 4 to 6 layers of 20-node brick elements in the thickness (z, 3) direction were used to model half of the laminate. The mesh is first generated in the mid-plane (z=0) then expanded in z direction according to the thickness of the laminae. Local notch-induced split in 0° plies and Notch-Induced

Delamination (NID) at the interface between 0° and 90° plies were constructed through merging separate mesh blocks and sweeping nodes. A commercial FEM package MARC 7.1, running on a Silicon Graphics Origin 200 workstation, was used to perform the simulations. A group of prescribed global displacement was implemented through one of the boundary conditions in the progressive failure model.

3.2. Strength criteria and failure definition

3.2.1. General forms of strength criteria

The matrix-dominated failure in a laminate takes place progressively according to specific failure mechanisms prior to the ultimate failure. The general form of a strength criterion in terms of the local stress components can be written as

$$f_s(s_{ij}) - R_s(S_{ij}) = 0 \qquad (4)$$

or in terms of local strains:

$$f_e(e_{ij}) - R_e(e_{ij}) = 0 \qquad (4')$$

where $\sigma_{ij}/\varepsilon_{ij}$ represent the local relevant stress/strain components with a subscript *ij*, S_{ij}/e_{ij} represent the critical strength parameters in terms of stresses and strains, respectively. It is worthy to note that the critical values for a specific failure mode depends on the constituent materials of the composite and the lay-up of the laminate, as well as the failure mode. For each case, these critical values should be determined by a group of tests.

3.2.2. Definition of failure

The transverse layers have failed when the maximum local strain in the layers reaches its critical value, 0.2%, in the direction perpendicular to the fibres and a transverse crack starts. This also means that the maximum strain criterion is applied for the prediction of transverse cracking.

Interlaminar delamination between 0/90 layers occurs when Hill's criterion is satisfied. This is due to the fact that the interlaminar delamination is dominated by the interlaminar normal stress, σ_{33}, and the interlaminar shear stress, τ_{13}.

The notch-induced split is predicted by the local application of Hill's criterion, including the normall stress, σ_{22}, and the inplane shear stress, τ_{12}, at the split tip.

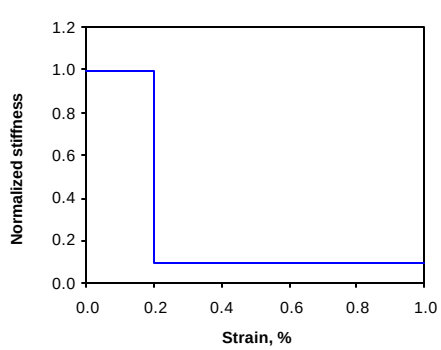


Figure 5. Uniform stiffness degradation model.

3.2.3. Uniform stiffness degradation model

As a special case of equation (3), figure 5 shows a uniform stiffness degradation assumption: upon the strength criteria are satisfied, the local stiffnesses are reduced to 10% of the original values in all of the directions. Our previous work has showed that this approximation gives good prediction of the transverse cracking in cross-ply laminates with notches at lower load levels. As the first attempt, this model is again applied to study the formation of the notch-induced split.

4. Stress distribution and damage patterns

4.1. Global 3D stress distribution

As mentioned in [5], the occurrence of the notch-induced split in 0° plies results from two driving forces: the notch-enhanced in-plane shear component and the normal stress component perpendicular to the fibre direction induced by the mismatch of the Poisson's ratios, (ν_{LT} - ν_{TL}), and during manufacturing.

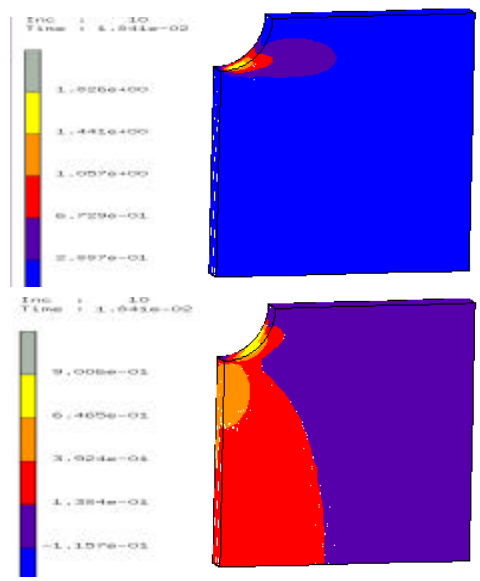


Figure 6. Contour bands of t_{12} & s_{22} at $e_{app} = 7.364 \times 10^{-5}$

Globally, Figure 6 illustrates the contour bands of τ_{12} and σ_{22} at an applied global strain of 7.364×10^{-5} in a 1 mm laminate with a notch to width ratio 0.2. It can be seen that these stresses distribute three-dimensionally around the notch, and their maximum values appear at different positions in the notch. The maximum stress of the notch-induced inplane shear stress (τ_{12}) in the 0° plies of the laminate is at a position, $\theta=18^o$, from the notch root. This angle coincides with the notch-induced split initiation position observed in the experiments [5]. Along with the highly intensified local inplane shear component (τ_{12}) in the notch, its far-field distribution is rather uniform and minute. Additionally, the stress normal to the fibres (σ_{22}) in 0° layers distributes evenly in the far field with a relatively low value with respect to the maximum in the notch. However, this stress at the region around the notch is rather high, as seen in Figure 7.

The local distribution of the two characteristic stress components, τ_{12} and σ_{22}, is again shown in Figure7. Figure 7a

presents the inplane shear stress and the normal stress around the notch, and 7*b* represents the in-plane shear stress and the normal stress along the line of the notch-induced split, both are the stress components in 0° layers.

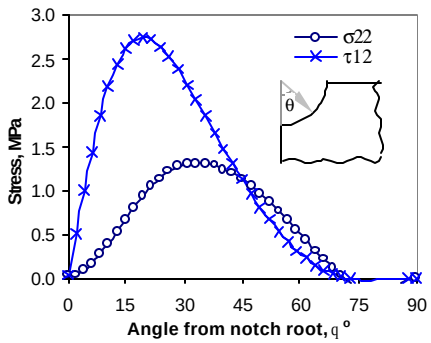


Fig. 7a. Stresses along the notch boundary.

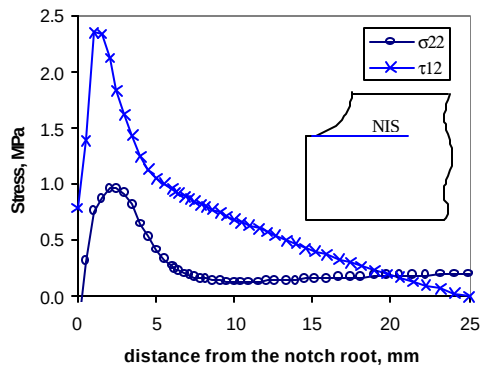


Fig. 7b. Stresses along the NIS growth line.

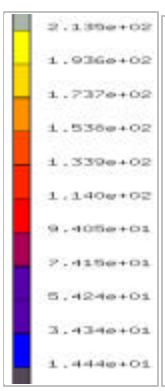
4.2. Stresses at notch-induced split tip

In order to identify the real driving forces for the notch-induced split growth, the local stresses at the notch-induced split tip were separately examined in three cases:

Case 1**,** without the First Transverse Cracking (FTC) and NID;

Case 2, with FTC and without NID;

Case 3, with FTC and NID round the NIS.



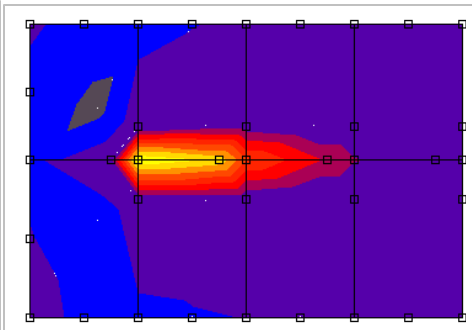


Figure 8. Local stress at the NIS tip.

Quarter-point elements were constructed at the crack-tip for a better approximation of the stress singularity at the notch-induced split tip. For the case with FTC and local delamination, Figure 8 presents the contour bands of the inplane shear stress at the notch-induced split tip in a very local view, with the local mesh and the nodes, at a global strain of 1%.

Figure 9 illustrates the inplane shear stress at the middle of the 0° layers of the three cases at same applied global load, about 1% global strain. It can be seen that this stress component is enhanced by the first transverse crack and by the local delamination, although the effect of the FTC seems not significant (the maximum value of τ_{12} changes from 128 MPa to 129 MPa). This is because only the FTC is taken into account in this case and it is far from the split tip. Therefore, its influence on the local stresses is limited. However, the multiplication of transverse cracks will also multiply this effect much more significantly. This will be dis cussed in next section. The effect of the local delamination is obvious: the maximum value of τ_{12} increase from 128 MPa to about 210 MPa.

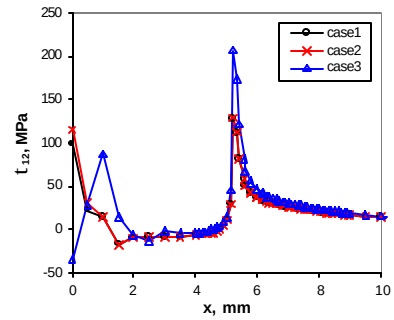


Figure 9. Local shear stress in three cases.

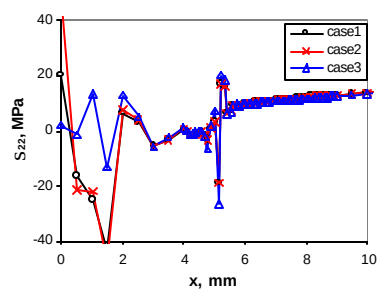


Figure 10. Local stress, s_{22}, in three cases.

Figure 10 presents the local normal stress perpendicular to the fibre in the 0° layers, particularly at the split tip. Compared with the inplane shear stress component, this normal stress is low and it is not much influenced by other forms of matrix failure. First of all, the absolute value of the normal stress, σ_{22}, is low at the split tip, which is about an order of magnitude lower than the maximum shear stress at the notch-induced split tip. Secondly, its maximum value is not much influenced by other forms of matrix failure, neither transverse cracks nor local interlaminar delamination. It is known that the strain energy release rate is quadratically proportional to the effective stresses at the crack tip [17,18], so the difference in energy is even about two orders of magnitude, and the critical strain

energy release rates of this material system for mode I and mode II are of the same magnitude (unpublished results from a EUCLID project). One can, from this local stress calculation, conclude that the growth of the notch-induced split in the area far from the notch is mode II overwhelmingly dominated.

Briefly, the notch-induced split growth is predominated by the inplane shear stress component, the influence from other stress components, particularly from the normal stress σ_{22}, is not important in the area far from the notch. Consequently, the growth of the notch-induced split is mode II dominated in this area. Based on this conclusion, the maximum shear stress/strain strength criterion can then be used to predict the split growth in the area. Only if the situation very close to the notch has to be considered, e.g. the onset of the notch-induced split is studied, the relatively high normal stress, σ_{22}, near the notch must be taken into account.

5. Transverse cracking and notch-induced splitting

It was observed that a few transverse cracks in the transverse layers in the area agaist the notches are always expected to occur before the initiation of the notch-induced split [6]. More transverse cracks occur with the split growth as the sample is continuously loaded. First of all, the formation of these transverse cracks will be discussed. Then, their influence on the formation of the notch-induced split will be inspected. This was performed in two ways:

(1). A single transverse crack, the FTC, was induced by applying the fixed boundary condition only on the longitudinal layers;

(2). More transverse cracks were induced by a strength criterion, e.g. the maximum strain criterion, applying only upon the transverse layers in the area against the notch.

5.1. Transverse cracking

5.1. 1. Local and global stresses

Transverse cracking causes local and global stress redistribution between the laminae in the laminate. Due to the local material failure of the transverse layer, the local stress in

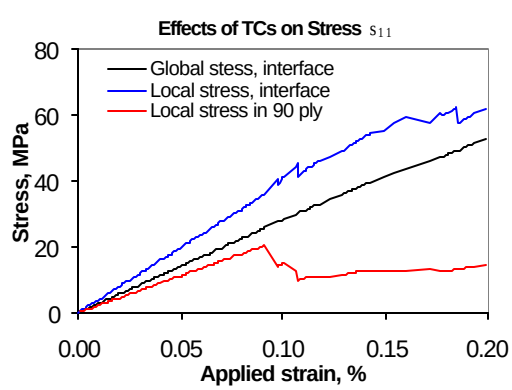


Figure 11. Simulated local & global stresses.

the transverse layer decreases in the loading direction, while the local stress in the longitudinal layer increases in the loading direction. The local stress drop in the transverse ply is compensated by the stress enhancement in the longitudinal ply, and local force balance is achieved in this way. Figure 11 presents the simulated local stresses at different sites in the laminate in the loading direction. The global response of the laminate shows somewhat non-linearity due to the local failure of the transverse plies, although it is not very significant.

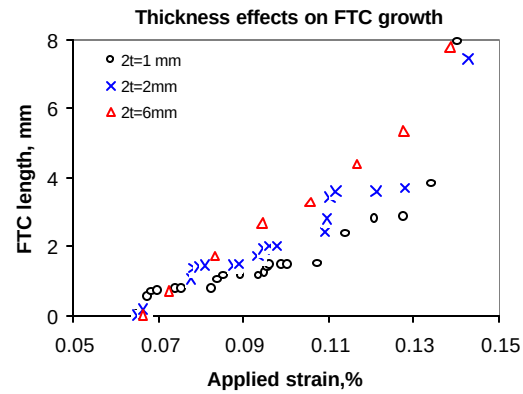


Figure 12. Thickness effect on FTC growth.

5.1.2. Growth of transverse cracking

Most of the shear-lag analysis focused on the scale effect of the thickness on the average transverse crack density [7,15,16]. However, another thickness scale effect was more or less ignored. As shown in Figure 12, a bigger transverse-layer thickness quickens the growth of the first transverse crack. This effect may be the results of a three-dimensional redistribution of the local stresses. On the other hand, the interaction of the transverse cracks in thinner laminates makes the FTC growth in a step-by-step manner.

5.1.3. Transverse cracking pattern

Figure 13 shows the simulated pattern of transverse cracking for 2 mm laminate with a notch-width aspect ratio 0.2 at about 40 MPa. Physically, the darker strips against the notch should be treated as sites with a higher local strain, and the stiffness degradation occurs at the sites.

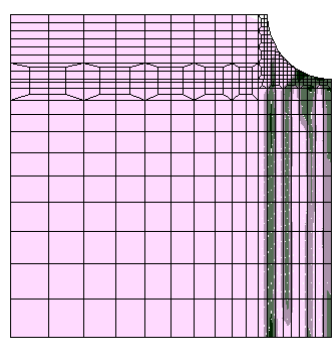


Figure 13. Simulated transverse crack pattern.

5.2. The initiation and growth of the notch-induced split

5.2.1. The onset of the notch-induced split

As discussed above, the growth of the notch-induced split in the area far from the notch is dominated by the inplane-

shear stress (τ_{12}). A parameter study was performed for two main purposes. The first is to check the fisibility of the approach in modelling the notch-induced split without involving other forms of matrix failure. The second is to exam the influence of the critical inplane-shear strength on the growth of the notch-induced split in case without other forms of matrix failure. First of all, it is found that the model predicts a multi-site notch-induced split onset without taking other forms of matrix failure into account. Figure 14 is the simulated notch-induced split pattern without taking other forms of matrix failure into consideration. In this case, the local damage in the 0° layers is overestimated due to the ignorance of the transverse cracks in the 0° layers and the interlaminar delamination between 0/90 layers.

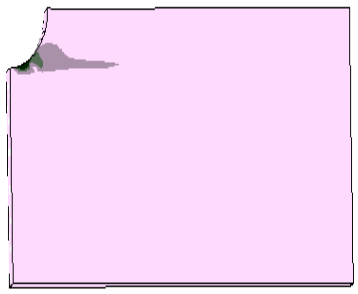


Figure 14. local NIS pattern

Figure 15 presents the influence of the critical inplane-shear strength on the growth of the notch-induced split in the area relatively far from the notch root. However, the calculated strain value at which the notch-induced split starts is noticeably lower than the observed value because of the ignorance of the transverse cracks. The earlier transverse cracking will increase the global average strain, and hence the strain at which the notch-induced split starts will be increased.

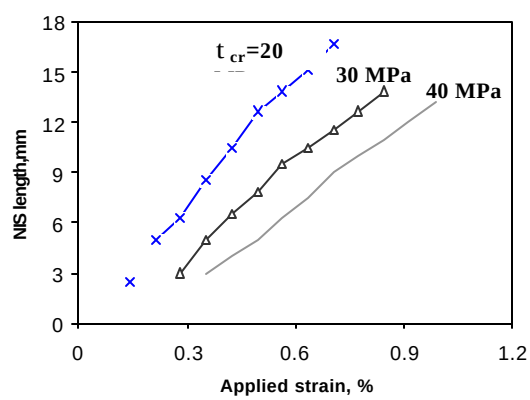


Figure 15. In-plane shear strength (t_{cr}) effects on the notch-induced split growth.

5.2.2. FTC effects

As illustrated in Figure 16, the FTC delays the onset of the notch-induced split by as much as 0.06% in term of the global strain, compared with the case without FTC. On the other hand, the influence of a single transverse crack on the notch-induced split growth is not obvious and the growth rate (the slope in Figure 16) keeps almost a constant. This agrees with the result of the local stress analysis as discussed above. However, the notch-induced split starts after local transverse cracking fully developed. Therefore, all of the influences from the transverse cracks occurred before the notch-induced split should be included in estimating the growth of the notch-induced split.

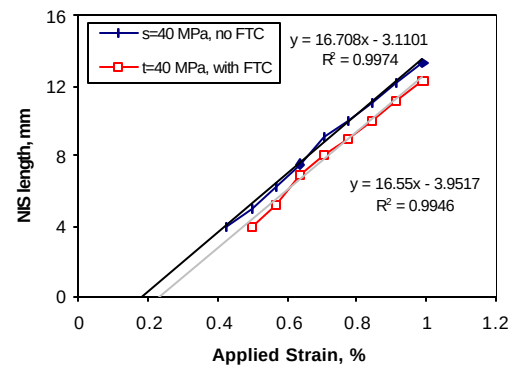


Figure 16. FTC effect on the NIS onset strain.

5.2.3. Effects of more transverse cracks

The transverse cracks soften the transverse plies and intensify the local stresses at the notch-induced split tip. Figure 17 present the simulated notch-induced split growth with the global applied strain with the influence of transverse cracks. The transverse cracks are induced by applying the maximum strain criterion to the transverse plies, as that stated in section 5.1. Figure 17 indicates that the growth of the notch-induced split is considerably accelerated by these transverse cracks. Figure 18 illustrates a part of the global view of the simulated notch-induced split including the effects of transverse cracks, at about 1% global strain.

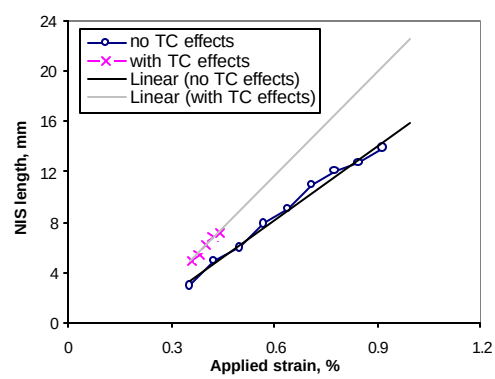


Figure 17. TC influence on NIS growth

5.3. Influence of the notch-induced delamination

As one of the main damage forms, the notch-induced delamination (NID) will influence the growth of the notch-

induced split. In fact, this influence has already been mentioned in the case study: the shear stress at the split tip is significantly enhanced. Further inspection of the influence of the NID on the notch-induced split growth is still in progress.

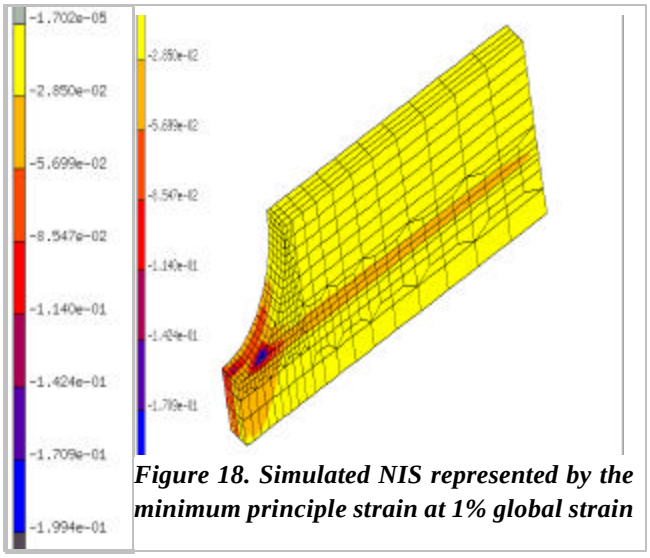


Figure 18. Simulated NIS represented by the minimum principle strain at 1% global strain

6. Discussions and Conclusions

A progressive failure approach was created and applied to study the matrix-dominated failure in terms of transverse cracking and notch-induced splitting in cross-ply laminates with double-edge-semicircular notches, till a rather high load level, about 1% applied strain. In the outline of the approach, the authors attempted to construct a model in which does not contain the usual assumed material homogeneity assumption. Consequently, simulation of local damage (extreme inhomogeneity) becomes within reach. The local matrix-dominated failure was introduced by the application of a group of strength criteria on the relevant plies. The constitutive relationships of the damage materials were modelled by a uniform stiffness reduction approximation. Particularly, the simulation here gives a sense of the interaction effect between the transverse cracking and the notch-induced splitting around the notch. The onset of the notch-induced split is delayed by the earlier occurred transverse cracks. On the other hand, the growth is enhanced by the occurrence of transverse cracks. For a correct prediction of the notch-induced split initiation and its growth, the transverse cracks in the 90° layers has to be taken into account. No arguments are given on the ultimate failure of a notched laminate till now. The attention is preliminarily focused on both the transverse cracks and the splits.

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