STRENGTH AND ITS RELIABILITY OF CERAMIC FIBER BUNDLE

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KEYWORDS: SiC Fiber, Fiber Bundle, Tensile Test, Monte-Carlo Simulation, Coleman Theory, Strength, Reliability

INTRODUCTION

It is very important to clarify strength and its reliability of reinforcement fiber and fiber bundle when discussing mechanical properties of fiber-reinforced composite. It is widely known that Weibull distribution [1] is often used to evaluate strength distribution of brittle fiber. On the other hand, Daniels [2] has discussed the relationship between strength of fiber bundle and strength of the constituent fibers, and established the theory about strength and the distribution of fiber bundle. Thereafter, Coleman [3] has applied Weibull distribution to Daniels's theory and got equation (1) to build the relationship between strength of fiber bundle and strength of the constituent fibers.

$$\frac{\sigma_b}{\sigma_f} = \left(\frac{1}{m_f e}\right)^{\frac{1}{m_f}} \frac{1}{\Gamma(1+1/m_f)} \quad \ldots\ldots (1)$$

Where σ_b is bundle strength, σ_f is the fiber strength, m_f is Weibull shape parameter of the constituent fiber strength, e is the base of the natural logarithms, and Γ is gamma function. According to equation (1), the relationship between strength of fiber bundle and strength of the constituent fibers is only related to Weibull shape parameter, m_f of the constituent fiber strength. Presently, Coleman's theory is often used to discuss bundle strength or as the base of building mechanical models about fiber-reinforced composite [4-8]. However, it should be paid attention to about the above two theories that both Daniels and Coleman assumed the number of the constituent fibers to be very large in their theories.

In this study, a tensile test and a Monte-Carlo simulation were carried out for SiC fiber bundle. Bundle strength and its reliability were examined and discussed with increasing the number of the constituent fiber and comparing with Coleman's theory.

EXPERIMENT AND THE SIMULATION METHOD

Tensile test of fiber bundle

SiC_{CVD} fiber, produced by Textron Specialty Materials (type SCS-9, diameter: 70μm), was used as the constituent fibers of fiber bundle. A tensile test was done at room temperature in a screw driven constant crosshead tensile testing machine (Shimadzu, AG-5000ES) under a constant tensile speed of 0.5mm/min and a gauge length of 30mm for the single fiber and the fiber bundles, which included 13, 50, 100 and 200 constituent fibers. The tabs of aluminum pieces about 30mm length were adhered to the

both ends of all tensile specimens. The constituent fibers were arranged on the tabs in parallel along the tensile direction and made to form one layer. Figure 1 gives tensile specimens of the single fiber and the fiber bundle. To prevent making damage to the fibers till tensile test, support paper sheet was applied to one side of each tensile specimen, and was burnt and cut by burning stick just before tensile test. The fibers used as the specimens were selected at random from a lot of SiC fibers, which were cut into length of 70mm from obtained continuous SiC fiber. The number of tensile specimen for each fiber bundle was over 40.

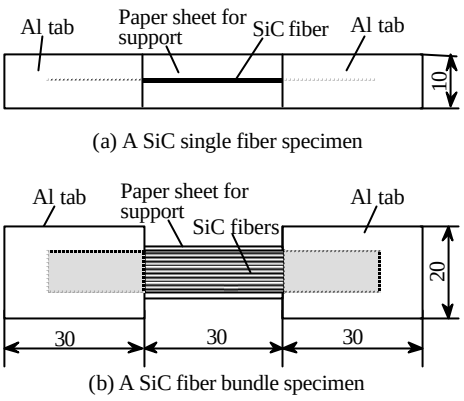


Fig.1 Tensile specimens of single fiber and bundle.

Monte-Carlo simulation method

A Monte-Carlo simulation was carried out for tensile test of the bundles. In the present simulation, it was assumed that the fiber strength agrees to two-parameter Weibull distribution shown as equation (2). Therefore, the strengths of the constituent fibers can be obtained in computer program from the inverse function of Weibull distribution shown as equation (3).

Table 1 The parameters of SiC single fiber for the present simulation.

Sample number	E_f (GPa)	σ_f (GPa)	m_f	σ_0 (GPa)
114	421	4.221	9.095	4.455

$$F(\sigma_f) = 1 - \exp\{-(\frac{\sigma_f}{\sigma_0})^{m_f}\} \quad \ldots\ldots (2)$$

and

$$\sigma_f = \sigma_0 \{ln\,(\frac{1}{1-Z})^{\frac{1}{m_f}}\} \quad \ldots\ldots (3)$$

Where σ_f shows fiber strength, m_f and σ_0 represent Weibull shape and scale parameters of fiber strength, respectively. Z is uniformly random numbers in (0,1). Also, it was assumed that the diameter and gauge length of the constituent fiber are constant, respectively. It was presumed that the given load applies each the constituent fiber uniformly and the load, which was applying to a broken fiber, distributes the other fibers uniformly. The parameters obtained from tensile test of SiC single fiber and used in the present simulation are listed in Table 1. In the present simulation, Weibull scale parameter, σ_0 was as a constant and substituted by the experimental value, 4.455GPa. Weibull shape parameter, m_f was given some values, 1, 3, 6, 9.095 and 15 based on the experimental one.

Figure 2 shows flow chart of the present simulation. In this figure, σ, ε, E, P and A mean strength, strain, Young's modulus, load

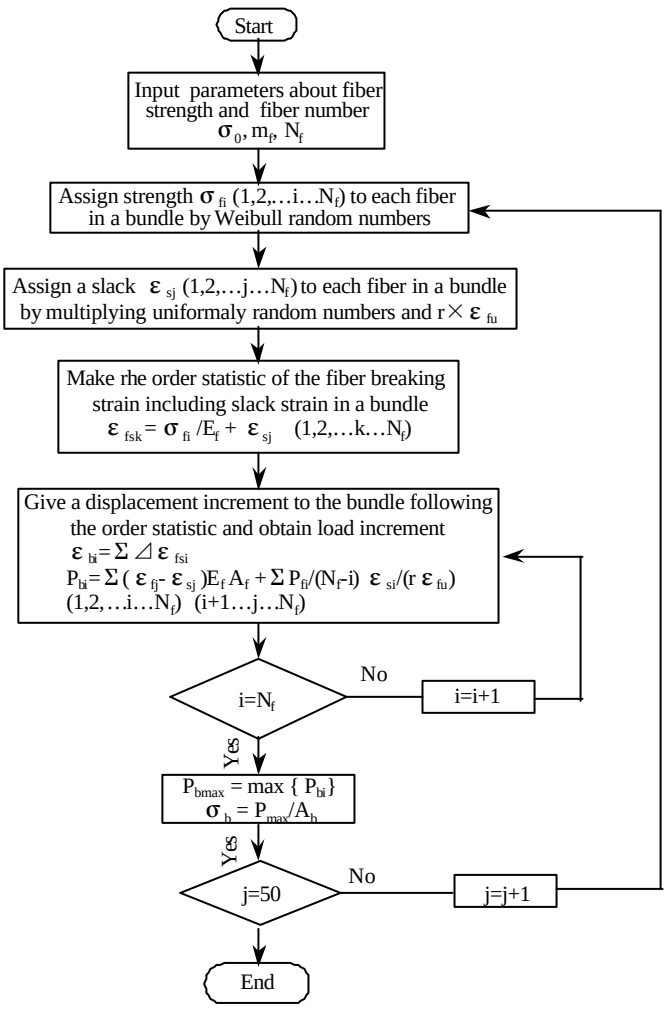
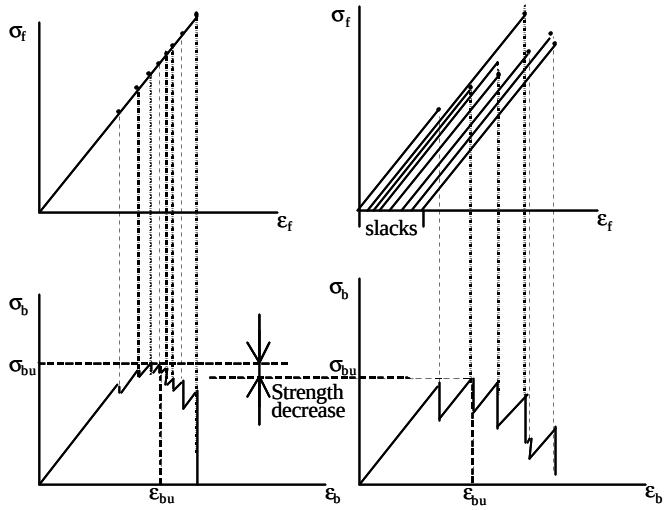


Fig.2 Flow chart of the present simulation

and sectional area, respectively. Also, subscript f, s, b, fu and *max* mean fiber, slack, bundle, fracture and maximum, respectively. For example, σ_{fi} expresses the strength of each constituent fiber in a fiber bundle, and is obtained from equation (3). The procedure of the present simulation is as follows. Firstly, strength of each constituent fiber was made after inputting the parameters about SiC fiber. Secondly, to simulate tensile test of the fiber bundle, which have slacks for the constituent fibers and compare with experimental results, the slack was added to the breaking strain for each constituent fiber. The slacks were given by multiplying $r \times \varepsilon_{fu}$ to uniformly random numbers in (0,1). Where, ε_{fu} is the average breaking strain of the constituent fibers and equal to the average strength divided by elastic modulus of the constituent fiber, σ_{fu}/E_f. r was the constant, which shows the level of the slacks, and substituted by 0.55 according to pre-simulation, but by zero for the case of fiber bundle without slacks. Thereafter, the order statistic of the all fiber-breaking strains including the slacks for the constituent fibers of a bundle was made. Then the displacement increment was given to the fiber bundle at the step by following the order statistic of the all fiber-breaking strains including the slacks. At the same time, the load increment was given. After a fiber-breaking, the load, which was loading on that fiber, was distributed the rest fibers, which have a smaller slack strain than the bundle strain at that step. Stress and train of the fiber bundle can be calculated out by using accumulating the increment of displacement and load at each step, respectively. The stress dividing the maximum load by sectional area of the fiber bundle was defined as the strength of fiber bundle. The simulation was carried out at 50 times for each fiber bundle, repeatedly. Relationship between the breaking process of a fiber bundle and the constituent fibers with or without the slacks is shown as in Fig.3.



(a) Without the slacks (b) With the slacks

Fig.3 Schematic drawing of relationships between the breaking process of a bundle and the constituent fibers without or with slacks.

RESULTS AND DISCUSSION

Figure 4 gives stress-strain curves of SiC fiber bundles from tensile test. Though the bundle strain was calculated by using displacement of the crosshead during tensile test, it is can be seen clearly that the slop becomes gentle with increasing the number of the constituent fiber. It hints that the slacks of the constituent fiber go up with increasing the number of the constituent fiber. Also, till the maximum stress (bundle strength), there are abrupt drops of the stress due to the fibers breaking at some locations on the curves. Relationship between bundle strength and the number of the constituent fiber is shown in Fig.5. From this

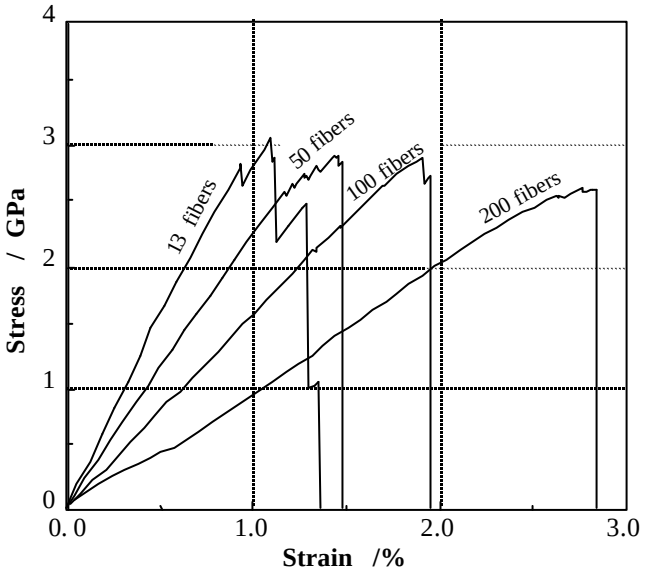


Fig.4 Stress-strain curves of SiC fiber bundles from tensile test.

figure, it can be seen that the bundle strength decreases with increasing the number of the constituent fiber, but the rate becomes slow when the fiber number is over 100. Although the bundle strength obtained from the simulation shows larger values than the experimental ones in the case of no slacks, almost agrees with experimental ones in the case of introducing the slacks. Also, relationship between Weibull shape parameter of bundle strength and the number of the constituent fiber is shown in Fig.6. Weibull shape parameter goes up with increasing the fiber number, and gets the values between

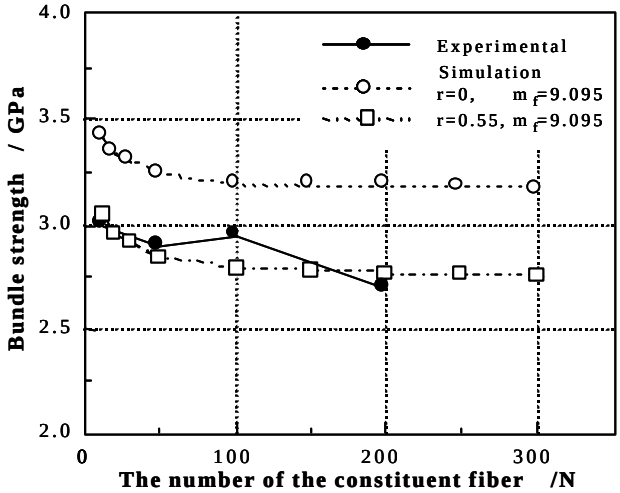


Fig.5 Relationship between bundle strength and the number of the constituent fiber.

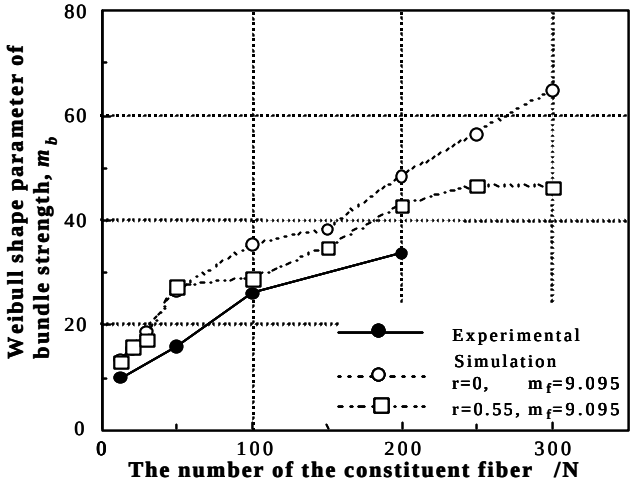


Fig.6 Relationship between Weibull shape parameter of bundle strength and the number of the constituent fiber.

experimental ones and the case of no slacks for the case of introducing the slacks. From those results, it can be understood that there are slacks for the constituent fibers in the experiment. It indicates that the present simulation is applicable to simulate tensile test of the bundles. However, the simulation is only carried out for no slacks hereafter to compare with Coleman theory because there is no slack problem in that theory.

To compare with Coleman theory, the relationship between the bundle strength and the number of constituent fiber was examined by the simulation and changing Weibull shape parameter of the fiber strength. The results are showed in Fig.7. Bundle strength was expressed by the ratio of bundle strength from the simulation to one from Coleman theory. It is interesting that bundle strength decreases and converges to Coleman theory with increasing the number of the constituent fiber. Besides, bundle strength converges to Coleman theory fast with increasing Weibull shape parameter of the constituent fiber strength. Figure 8 shows the relationship between the bundle strength expressed

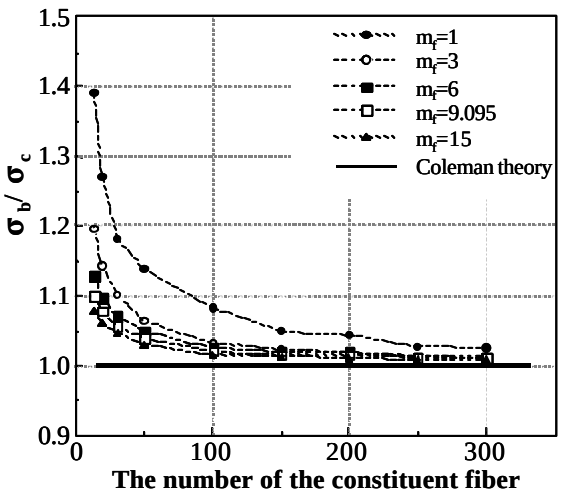


Fig.7 Ratio of bundle strengths from the simulation and from Coleman theory.

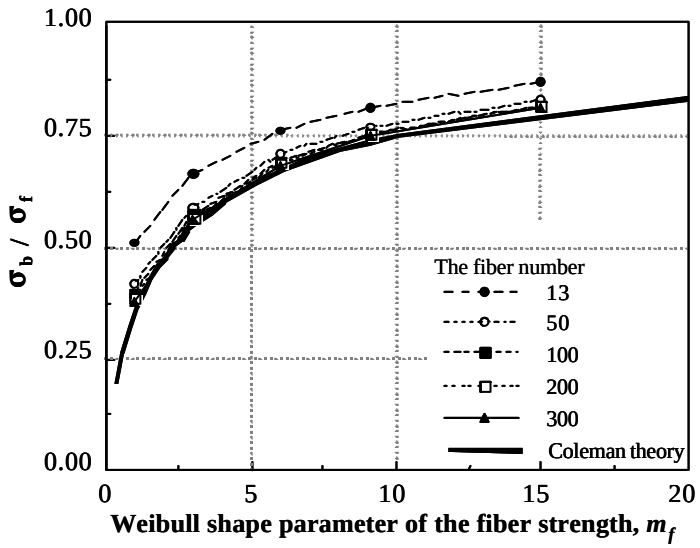


Fig.8 Ratio of bundle strength and the fiber strength from the simulation and Coleman theory.

by dividing the bundle strength by the constituent fiber strength and Weibull shape parameter of the constituent fiber strength. Bundle strength goes up with the Weibull shape parameter as Coleman theory even when the fiber number is countable, and approaches to Coleman theory with the number of the constituent fiber. According to the suppositions and method of Daniels [2] and Coleman [3], coefficient of variation (C.V.) of bundle strength was got as equation (4).

$$C.V. = [\exp(\frac{1}{m_f}) - 1]^{1/2} N_f^{-1/2} \quad \ldots\ldots (4)$$

Here N_f is the number of the constituent finer. From this equation, it is understood that the scatter of bundle strength is only related to Weibull shape parameter of the fiber strength and the number of the constituent fiber. Coefficient of variation of bundle strength from the simulation and equation (4) is shown as Fig.9. Coefficient of variation of bundle strength goes down and approaches to the calculation value from equation (4) with increasing the number of the constituent fiber.

In order to discuss relationships between the breaking process, the breaking accumulation of the bundle against the number of the constituent fiber, the fiber stresses when the fibers break during loading the bundle were read out in the order number of fiber breaking on every simulation for each fiber bundle. Then average fiber stress and Weibull shape parameter were calculated by Weibull distribution for each fiber breaking. Relationships between the order number of fiber breaking and the fiber stress, Weibull shape parameter are shown in Fig.10 and Fig.11, respectively. The fiber stress and Weibull shape parameter show an increase tendency with the order number for the bundles. That is, the more the accumulating fiber-breaking is, the higher fiber stress needed for the breaking is, and the lower the fiber stress scatter is. Therefore, the more the accumulating fiber breaking is, it becomes more obvious that bundle strength is controlled by the fiber, which has a high strength and low strength scatter. Figure 12 shows the relationship between the number of constituent fiber and count of breaking-fiber till appearing maximum stress for the bundles. It can be seen that count of breaking-fiber goes up with increasing the number of the constituent fiber and with Weibull shape parameter m_f of the

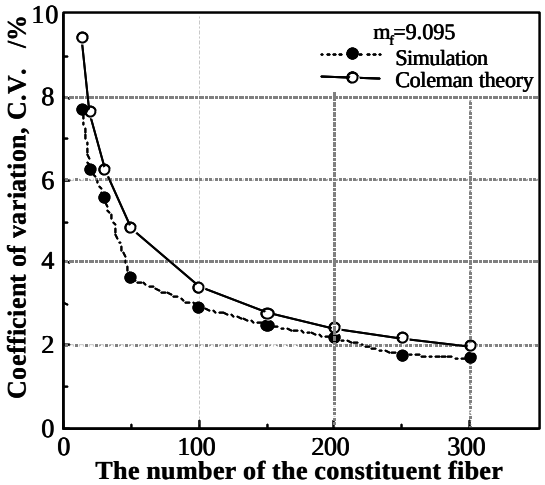


Fig.9 Coefficient of variation of bundle strength from the simulation and Coleman theory.

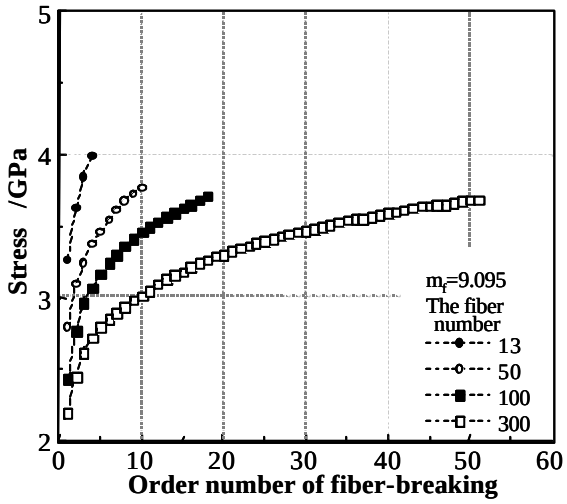


Fig.10 Relationship between fiber stress and order number of fiber-breaking during loading the bundle.

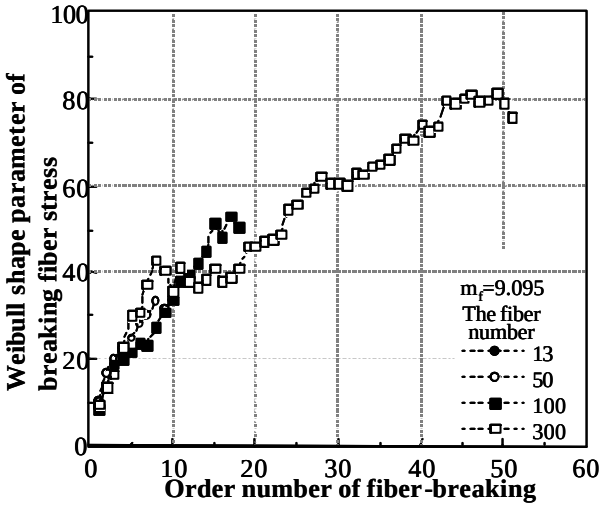


Fig.11 Relationship between Weibull shape parameter of fiber stress and order number of fiber-breaking during loading the bundle.

constituent fiber strength. That is, the larger the number of the constituent finer is, the more the accumulating fiber-breaking is. From the above results, the breaking process and the accumulation of the bundle are related to the number of the constituent fiber and Weibull shape parameter m_f of the constituent fiber strength.

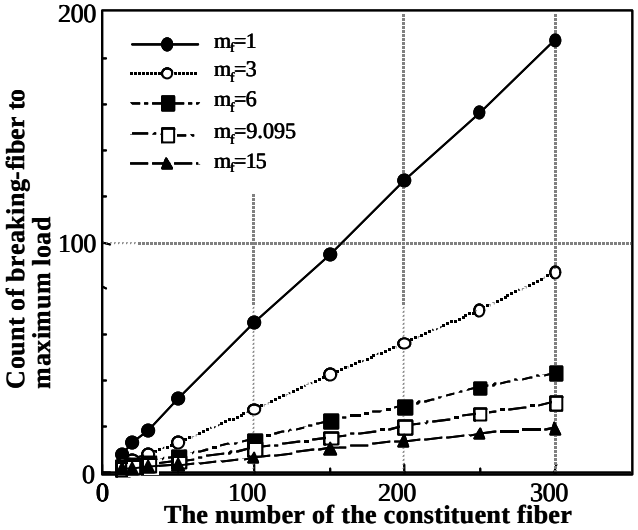


Fig.12 Relationship between count of braking-fiber to maximum load of bundle and the number of the fiber in the bundle.

CONCLUSIONS

Bundle strength decreases and Weibull shape parameter of bundle strength goes up with increasing the number of constituent fiber. Bundle strength converges to Coleman theory with the number of constituent fiber, and converges fast when Weibull shape parameter of constituent fiber strength has a large value. Also, bundle strength goes up with increasing Weibull shape parameter of constituent fiber strength as Coleman theory even when the number of constituent fibers is countable. In addition, the breaking process and the accumulation of fiber-breaking of the bundle are related to the number of constituent fiber and Weibull shape parameter m_f of constituent fiber strength.

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