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INFLUENCE OF PHYSICAL AND GEOMETRICAL NONLINEARITIES ON FAILURE ANALYSIS OF COMPOSITE PRESSURE VESSELS

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SUMMARY: The present state of art. in the design of composite pressure vessels and pipings is mainly a reflection and a repetition of design procedures utilized for isotropic, metallic pressure vessels. Therefore, the design codes do not use various nonlinear effects encountered in the experimental studies of composite materials. It results simply in the overestimation of the required dimension of composite constructions and on the other hand in the introduction of very high safety coefficients. The latter is mainly caused by the uncertainity in the prediction of failure loads. The aim of the present paper is to discuss the differences arising between the gemetrically and physically linear and nonlinear analysis of buckling, post-buckling and FPF failure behaviour of pressure vessel components.

KEYWORDS : Pressure Vessels, Geometrical Nonlinearities, Physical Nonlinearities, Shells, Plates, Buckling, Post-buckling, Numerical Analysis.

INTRODUCTION

In more than the last three decades, the industry has been pushing rapidly in advance of knowledge on composite materials that may be used by designers of fiber reinforced plastic tanks, vessels and pipings, particularly those in the chemical processing industry. In spite of the fact that these structures are more complicated than isotropic ones, little effort has been spent in studying their mechanical behaviour of these structures. Lacking the knowledge on the directional dependence of mechanical properties of the materials, while at the same time having to construct the containers required, designers almost constantly have to utilize equations developed for istropic metallic vessels to design anisotropic fiber reinforced plastic vessels.

In the design procedures of fiber reinforced plastic tanks, vessels etc. a lot of different factors should be taken into account, such as (see Ref [1]):

- buckling of structural components, i.e. of a compressed cylindrical shells or of domed heads,
- a junction of cylinders with domed heads,
- a junction of nozzles to a pressure vessel body,
- supports and contact problems during the reaction with a pressure vessel body.

The comparison of different approaches used by various design codes (Ref [1]) demonstrates evidently that:

- buckling analysis is conducted in the simplest approach (geometrically linear problem) and the discrepancies with a strict numerical analysis – see Ref [2] is covered by different safety factors,
- a stress concentration arising at the junction of various pressure vessel components is eliminated by additional reinforcement layers of composites and by additional safety factors.

The aim of the present paper is to discuss and solve the above-mentioned problems with the use of the numerical FE approach taking into considerations geometrical and physical nonlinearities (see e.g. Ref. [3]). Using FE approximations two different formulations are compared, the first one that is based on the 2D plate/shell approach (quadrilateral shell FE – first order transverse shear deformation theory) and the second 3D (3D brick FE).

GEOMETRICAL NONLINEARITIES

It is well-known that geometrical nonlinearities in the sense of the large displacement (rotations) theory may change both buckling loads as well as post-buckling behaviour. In our opinion the post-buckling deformations may be responsible for the FPF of structures so that it is necessary to include them in the analysis. Using FE approximations (quadrilateral shell FE – first order transverse shear deformation theory or 3D brick FE) laminated plates and shells having different stacking sequences have been studied in order to establish the effects of geometrical nonlinearities on buckling loads and postbuckling deformations. It has been found that design procedures postulated in codes overestimated buckling loads. For some stacking sequences the differences are even greater than 100%.

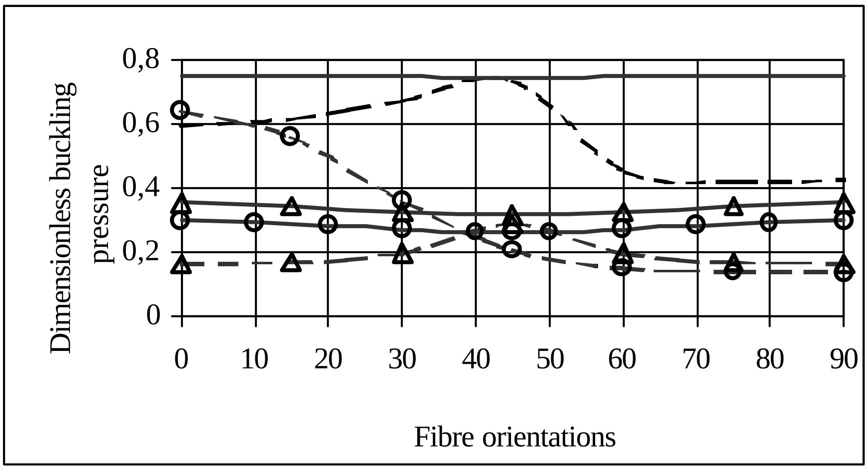


Fig.1 Comparison of buckling pressures for hemispherical domes; (----- unidirectional, —— woven roving, - British Standards, - ASME code)

The first example deals with the comparison of the numerical (using geometrically nonlinear approach) and analytical predictions proposed by British Standards 2994 and ASME codes – see Fig.1. The analysis is conducted for hemispherical domed end clamped at the edges and treated as the axisymmetric structure. The results are computed for two types of composite materials: unidirectional and woven roving, using 2-D FE approach in the evaluation of buckling pressures (the first order transverse shear deformation theory). As it may be noticed there is a drastic difference between theory and analytical predictions for both types of composite materials considered herein. The discrepancy varies both with fibre orientations and type of the code. More numerical examples confirming the conclusions drawn above are presented and discussed in Ref [2].

It is interesting to consider the effects of the formulation of the problem (understand in the sense of the introduced simplified assumptions) on the values of buckling loads and the post-buckling behaviour for thinwalled 2-D structures made of composite materials. In addition, the effect of fibre orientations are also investigated. In order to explore the above, some numerical calculations were carried out using the NISA II finite element package. Figure 2 shows the geometry of composite structures being considered.

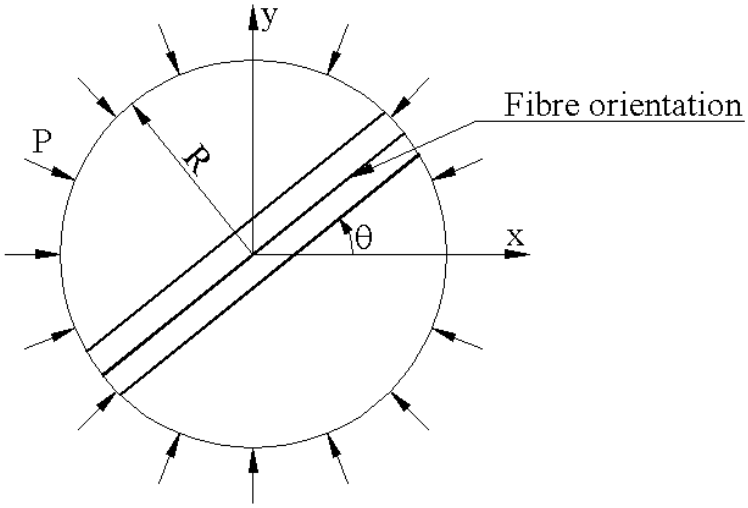


Fig. 2 Geometry of compressed laminated circular plates

The structures considered herein are subjected to an uniform external pressure p – see Fig. 2 and are analysed with the use of three different models, given below:

A. The first order transverse shear deformation 2-D theory with the large deformation option- the numerical finite element approach; the beam/plate is discretised with the use of four noded quadrilateral finite elements NKTP 32.
B. The 3-D theory with the large deformation option – the numerical approach; each of composite structures have been discretised with the use of the eight-noded brick elements NKTP 4 – each of nodes possesses three independent degrees of freedom (displacements u,v,w).

The results discussed herein show the effects of orthotropy on the buckling and post-buckling behaviour of structures but not stacking sequences, since three particular fibre orientations have been investigsted only, i.e è=0^o,45^o and 90^o – see Fig. 2. Let us note that the fibres oriented at è=45^o corresponds to a quasi-isotropic state.
In the FE numerical investigations the circular mesh consisted of 784 plate/brick elements. The identical discretisation have been used for both 2-D and 3-D models.
The results of the numerical computations are demonstrated in Figure 3 in the form of the load-deflection curves. The axial symmetry of the circular plate results in the same magnitude of the buckling loads and in the identical post-buckling behaviour for angle-ply plates with fibres oriented at 0^o and 90^o .
The comparison of the 3-D and 2-D FE results demonstrates evidently that the use of the model A leads always to the overestimation of the buckling loads. The discrepancy decreases as the fibre orientation decreases and the smallest is observed for fibres oriented at 0^o. In the post-buckling range the character of the load-deflection curves is strongly dependent on the type of the FE modelling. The character of the post-buckling curves is almost identical for various fibre orientations considered herein.

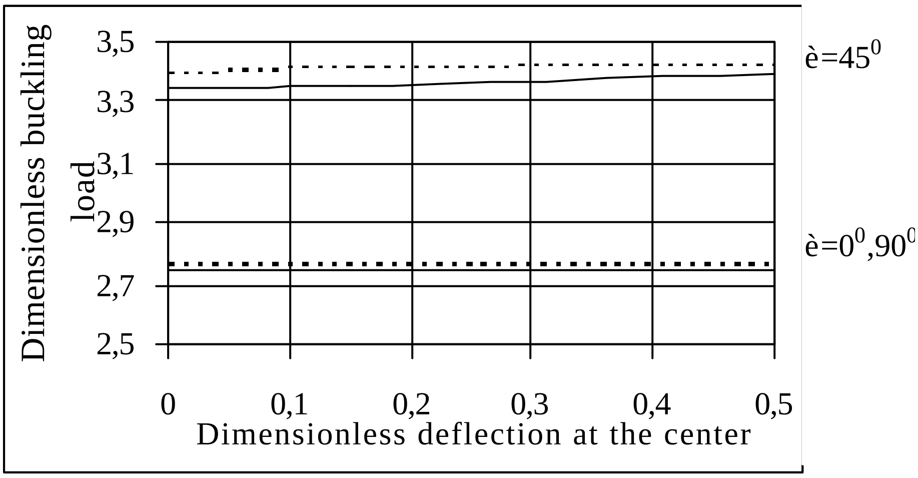
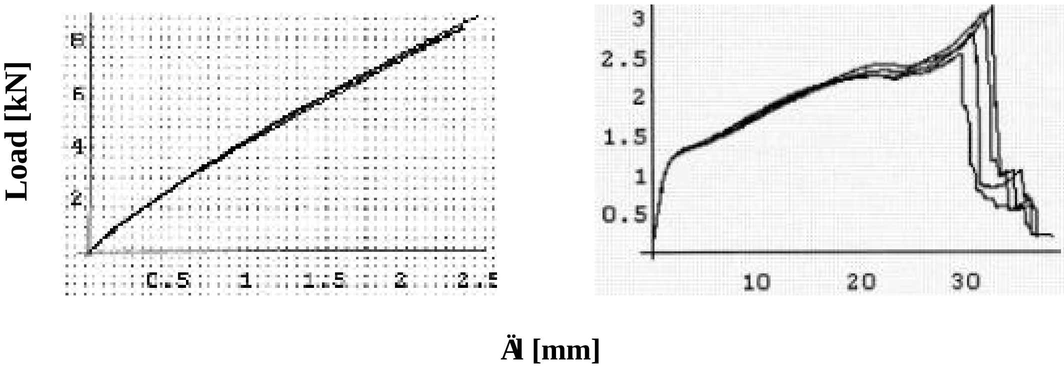


Fig.3 Post-buckling load-deflection curves of circular plates (R/t=10, —— 3-D FE analysis, ------- 2-D FE analysis)

PHYSICAL NONLINERARITIES

Modelling of the phenomenon

A great number of composite materials demonstrates physical nonlinearities on the *ó – å* curves – see e.g. Fig.4. Their origin is obvious – arising of the microcraks inside the matrix without any visible macrocraks. It leads directly to the stiffness degradation and results in the decrease of the appropriate value of the stiffness matrix. As it may be seen in Fig.4 those effects are directly connected with the fibre orientations



a) 0°

b) 45°

Fig.4 The *ó – å* curves for aramid/ epoxy resin – Muc *et al.* [4]

The character of the s - e plot imposes automatically that it is necessary to implement herein the plastic flow theory used in the physical description of isotropic materials.
The analysis in this area of composite mechanics begins in 1973 from the work of Hahn, Tsai [5]. They have introduced a quadratic term of the stress component s_6 to describe physical nonlinearities. Further works in this area have been made by Dvorak & Bahei-El-Dina [6]. Sun i Chen [7] have proposed the use of one parametrical plastic flow potential in the following form:

$$f = (\sigma_2^2 + 2a_6\sigma_6^2)/2 \tag{1}$$

Owen, Li [3] have applied the plastic potential similar to the Huber-Mises-Hencky yield condition which has six parameters of anisotropy:

$$f = a_1(\sigma_1 - \sigma_2)^2 + a_2(\sigma_2 - \sigma_3)^2 + a_3(\sigma_3 - \sigma_1)^2 + 3(a_4\sigma_4^2 + a_5\sigma_5^2 + a_6\sigma_6^2) \tag{2}$$

The anisotropy parameters a_1 ..... a_6 are taken from experimental data.
In flow theory the material behaviour is described by three conditions: the initial yielding condition (described above), the flow rule and the hardening rule. Let the total strain component is the sum of the elastic strain and the plastic one:

$$\{d\varepsilon\} = \{d\varepsilon^{el}\} + \{d\varepsilon^{p}\} = [S]\{d\sigma\} + \{d\varepsilon^{p}\} \tag{3}$$

The elastic strain increment is given from the Hook law, whereas the plastic one $\{d\varepsilon^p\}$ is derved from the flow rule:

$$\{d\varepsilon^{p}\} = d\lambda \frac{\partial f}{\partial\{\sigma\}} \tag{4}$$

and $d\lambda$ is a proportionality constant. After some manipulations that parameter may be expressed as follows:

$$d\lambda = \frac{\left(\frac{\partial f}{\partial\{\sigma\}}\right)^T [C]}{\frac{4}{3} f H_p + \left(\frac{\partial f}{\partial\{\sigma\}}\right)^T [C]\left(\frac{\partial f}{\partial\{\sigma\}}\right)}\{d\varepsilon\} \tag{5}$$

H_p is a plastic modulus. Using the above relations, one can obtain finally the incremental elastic-plastic flow rule that has the similar manner as for isotropic materials, i.e. :

$$\{d\sigma\} = [C](\{d\varepsilon\} - \{d\varepsilon^{p}\}) = [C]\left(1 - \frac{\partial f}{\partial\{\sigma\}}\frac{d\lambda}{d\{\varepsilon\}}\right)d\{\varepsilon\} = [C^{ep}]d\{\varepsilon\} \tag{6}$$

Numerical results

For the isotropic materials it is obvious that physical nonlinearities (understood in the sense of plasticity) may lead to the unloading of the structure. Therefore, we propose to use a plastic flow law in the deformation analysis of the pressure vessel elements in order to evaluate the real stress concentration factors. In the previous section it has been demonstrated that using the stress-strain characteristics and the flaw potential f it is possible to build the appropriate physical nonlinear relation corresponding to the realistic nonlinear physical behaviour (e.g. Fig. 4) of each individual ply in the laminate. The proposed above approach may be easily implemented into the FE code in order to model e.g. a stress concentration effect between the saddle support and the cylindrical shell.
It should be mentioned that the analysis of stress concentration effects for vessels lying on the saddle supports is a significant in the design of pressure vessels has got a considerable attention in the literature (but in the elastic range only) – see e.g. Refs [8-12].
Now, let us consider a ring placed on the saddle support to observe the effects of stress concentration at the circumferential direction, at the horns of the saddle. The plane stress FE (NKTP =1) have been used herein. The ring has been discretized with the use of 768 FE,

whereas the steel support – 2560 FE. It is assumed the the cylinder is made of the glass mat/epoxy resin and loaded by an internal uniform pressure.

Figure 5 is a plot of deformed and undeformed shapes of the ring being in the elastic-platic state of deformations. Figure 6 demonstrates the effects of the stress concentrations at the horn of the saddle. The comparison of the stress concentrations effects in the elastic and elastic-plastic range shows that the plastic flow reduces the values of the maximal stresses by a factor 47% , of course, for the identical value of the internal pressure.

In our opinion, the presented example shows evidently the necessity of taking into account of plastic effects in the design process. On the other hand, it illustrates also the simplicity of modelling plastic effects in the stress/strain analysis of multilayered laminated composite structures.

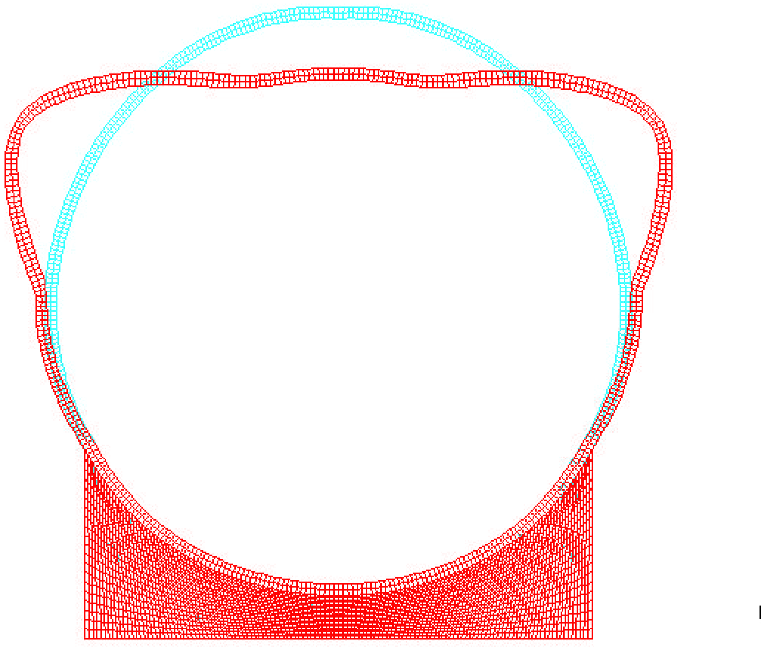


Fig. 5 Deformed and undeformed shapes of the ring (not to scale) in the elastic-plastic range

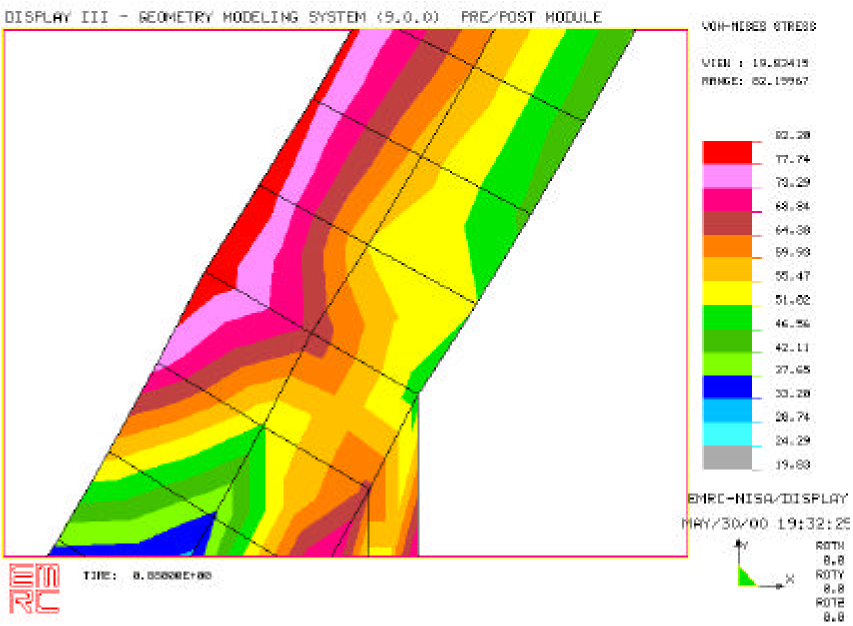


Fig.6 Stress concentration around the horn of the saddle (a fragment of the structure)

CONCLUSIONS

The necessity of taking into account of both geometrical and physical nonlinearities in the design of pressure vessels made of fibre reinforced plastics is pointed out in the present paper. Studying the presented above approaches, methods and results the following conclusions can be drawn:

— In the design of pressure vessels and pipings made of fibre reinforced plastics it is necessary to conduct FE analysis and in addition to take into account directional dependance of the material properties
— The linear predictions of buckling pressures for pressure vessel components availaible in the design codes do not reflect satisfactorily directional dependance of buckling pressures since geometrical nonlinearities should be considered
— There is almost no difference in the predictions of buckling and post-buckling behaviour in the numerical models including 2-D (first order transverse shear deformation theory) and 3-D finite elements
— It is possible to describe nonlinear physical behaviour in each plies of the laminate using the real stress-strain nonlinear characteristics and the flaw law presented in the paper
— Taking into account nonlinear physical relations one may reduce significantly stress concentrations arising during the contact between the saddle support and the cylindrical part of the vessel; the similar effects may be obtained at the junction of nozzles

In summary, the present study shows evidently that the use of present design codes result in the wrong prediction of the pressure vessel components thicknesses due to the incorrect application of experimental data demonstrating the existence of both physical and geometrical nonlinearities. In our opinion, the proper conjunction of numerical analysis with the experimental data may result in the further weight savings of pressure vessels and in this way a better optimization of such structures.

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