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EXCEL-BASED PROGRAM FOR THE PROGRESSIVE QUADRATIC FAILURE CRITERION

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SUMMARY: Predicting the strength of composite laminates is a difficult task, due to the complex nature of their failures. Different modes of failure exist and interact in a complex manner. One recently developed progressive quadratic failure criterion, which combines the widely accepted Tsai-Wu quadratic failure criteria with a simple progressive damage scheme, has been shown to predict these complex failures with reasonable accuracy. This criterion is for 2-D plane stress condition, but can be easily extended to 3-D. The degradation of a failed ply is done in two steps. The first failure of a ply is considered as a "matrix failure", and the matrix related properties are degraded by the empirical multiplication factor E_m*. The second failure of the same ply is considered as a "fiber failure", and the fibers related properties are degraded by the factor E_f*. The strengths are kept constant except for the compressive strength X', which we assume a power law relation to the normalized shear moduli. The total number of the empirical constants in this criterion is four, including the interaction term for the Tsai-Wu failure criterion. This progressive quadratic failure criterion was programmed into a user-friendly Excel-based program named Mic-Mac/Progressive. This program predicts the failure load and the stress-strain curve of a laminate under combined in-plane loads. It also plots failure envelopes, which shows all the combinations of biaxial or axial-shear stresses at failure. The main inputs are the ply properties, ply orientations, applied loads, and four empirical constants. The default values of the empirical constants provide reasonably accurate predictions, but can also be fine-tuned for a better fit. We have examined the accuracy of our prediction by comparing with numerous test results, and are ready to provide this program free of charge to interested parties.

KEYWORDS: failure criterion, progressive damage, strength prediction, computer program

INTRODUCTION

Predicting the strength of composite laminates is a difficult task, due to the complex nature of their failures. Different modes of failure exist and interact in a complex manner. For this reason, progressive damage failure criteria have been actively studied for the past several decades. Unfortunately, most of the criteria are too complicated to be useful for real design problems. On the other hand, some simple approaches often implemented into the finite element code, are rather too simple and are not accurate enough to be widely accepted.

Most progressive failure criteria consist of two steps. The first step is to determine the initial failure load and the modes of failure. The next step is to apply certain degradation scheme to the material properties according to the failure modes, and then recalculate the failure load and the failure modes. The second step will be repeated until the material can not carry any additional loads. Most of the criteria require recalculations for each small increment in the applied load, and are very computationally intensive. Some criteria even require iterations for each load increment, which can be performed only through complicated software and expensive hardware.

On the other hand, the designers use none of these complicated failure criteria in favor of more empirical and more mathematically simple criteria. In fact, most commonly used failure criteria are the maximum stress, maximum strain, and Tsai-Wu failure criteria, none of which include progressive damage[1]. In many cases, the ply orientations the designer can use are limited to

very few combinations, which are tested intensively. Many of the designs are also limited to initial failure, which can be a fraction of the final failure load. These limitations lead to less efficient designs, additional weight, and loss of manufacturing flexibility. In order to solve these problems, the study of progressive failure criteria should be viewed more as a development of practical design tools, than just as an academic exercise.

Considering all these, we have decided to develop a practical design tool, using simple and reasonably accurate progressive failure criterion. The baseline of this criterion is the most commonly used Tsai-Wu failure criteria, combined with a simple progressive damage scheme. Tsai-Wu failure criterion is linear in the sense that failure load of a certain condition can be calculated without any iteration, making the calculations much simple and quick. The new progressive failure criterion was programmed into an easy to use Excel-based program, which we named "Mic/Mac-Progressive". Excel is software from Microsoft Corporation, and is one of the most popular software available in almost all of the computers in the World. The detail of the new failure criteria and the Excel-based program, and some examples will be shown in the following sections.

TSAI-WU PROGRESSIVE FAILURE CRITERION

One recently developed progressive quadratic failure criterion[2, 3], which combines the widely accepted Tsai-Wu quadratic failure criterion[4] with a simple progressive damage scheme, has been shown to be reasonably accurate by the ongoing work by Hinton, Soden, and Kaddour[5, 6].

Figure 1 shows the outline of this progressive quadratic failure criterion. First stresses in each ply are calculated from the inputs such as material properties, ply orientations, and applied load. The stresses are evaluated according to Tsai-Wu quadratic failure criterion, and we determine which ply fails first and at what load. Then the ply properties are degraded according to the progressive damage scheme. Using these degraded properties for the failed plies, the stresses are recalculated and reevaluated by Tsai-Wu criterion again. The process is repeated until all the plies fail, or no additional load can be carried.

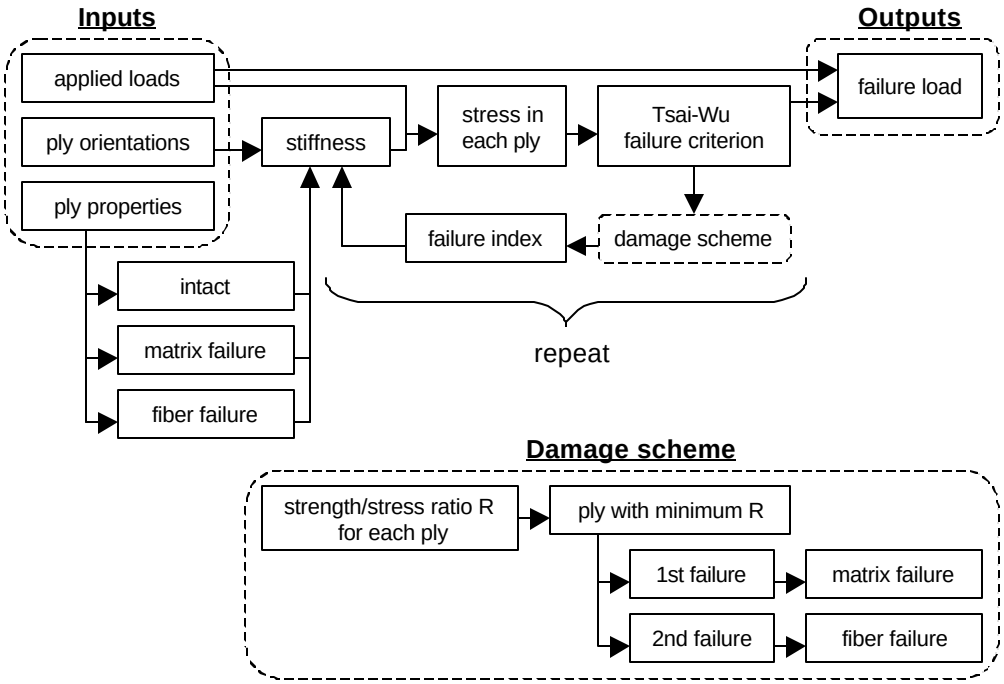


Figure 1. Outline of the progressive quadratic failure criterion

Tsai-Wu quadratic failure criterion is used to determine the strength/stress ratio R for the intact and the degraded states. This calculation is linear in the sense that R can be calculated without any iteration for each given condition, intact or degraded. Therefore, the numbers of required calculations will be much fewer than the other mathematically intensive criteria.

Tsai-Wu criterion has been known to have good agreement when predicting the initial failure[7], but is often not used in progressive failure criteria since it does not give the failure mode as other criteria claim. Most progressive failure criteria need a clear distinction between the failure modes, which will determine what degradation scheme should be applied. The philosophy behind why Tsai-Wu failure criterion does not give failure modes are, first of all, failure modes are usually very complex and can not be clearly distinguished, and second, because failure criteria are inherently empirical and there is always a danger in trying to claim physical reasoning. Following this philosophy, the progressive Tsai-Wu quadratic criterion uses a progressive damage scheme that is different from other progressive failure criteria.

Different failure criteria use different numbers of failure modes, ranging from one in Tsai-Wu criterion up to nine for some complicated criterion[8]. Progressive quadratic criterion uses two failure modes, the fiber failure and the matrix failure. Rather than having two different paths for these two failure modes, the degradation of a failed ply is done in two steps. The first failure of a ply is considered as a "matrix failure", and the matrix related properties, E_y, G_s, and ν_{xy} are degraded by the empirical multiplication factor E_m*. The second failure of the same ply is considered as a "fiber failure", and the properties, E_x, E_y, G_s, and ν_{xy} are degraded by the factor E_f*. The strengths are kept constant except for the compressive strength X', which we assume a power law relation to the normalized shear moduli shown in the following equations.

$$X'_{degraded} = X' \times (Gs_{degraded} / Gs)^n \quad (1)$$

$$\text{for matrix failure } X'_{degraded} = X' \times (E_m^*)^n \quad (2)$$

$$\text{for fiber failure } X'_{degraded} = X' \times (E_f^*)^n \quad (3)$$

The total number of the empirical constants in this criterion is four, namely E_m*, E_f*, n in the above equations, and the interaction term F_{xy}* required for the Tsai-Wu failure criterion. Tsai[9] has shown that the value of F_{xy}*=-0.5 leads to reasonable agreement in most materials, as shown later in Figure 7.

The two step failure process may not seem physical, but when we view the actual failure of a laminate, we rarely observe a case where the fibers are broken but the matrix is intact. Even when the material is loaded in the fiber direction, slight misalignment in the fiber orientation will cause the matrix to fail first and then the fiber. Mathematically, when the fiber stress is very high, the matrix failure is followed instantly by the fiber failure of the same ply, and will be no different from assuming fiber failure from the beginning. As we mentioned before, the failure modes are very complex and highly interacting, and drawing a clear line to divide the failure mode may not be any more physical than our two step process. We would also like to repeat the underlying philosophy that failure criteria are only empirical.

EXCEL-BASED PROGRAM

This progressive quadratic failure criterion has been programmed into a user-friendly Excel-based program named Mic-Mac/Progressive. This program predicts the failure load and the stress-strain curve of a laminate under combined in-plane loads. It can also plot failure envelopes in three planes, the s_x-s_y plane, the s_x-τ_{xy} plane, and the s_y-τ_{xy} plane. The main inputs are the ply properties, ply orientations, applied loads, and four empirical constants. The default values of the empirical constants provide reasonably accurate predictions, but can also be fine-tuned for a better fit. The effects of these empirical constants can be easily examined using a simple command in the program.

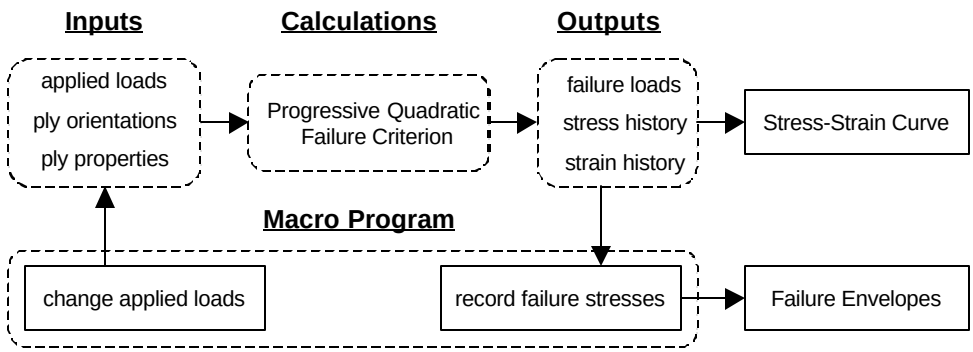
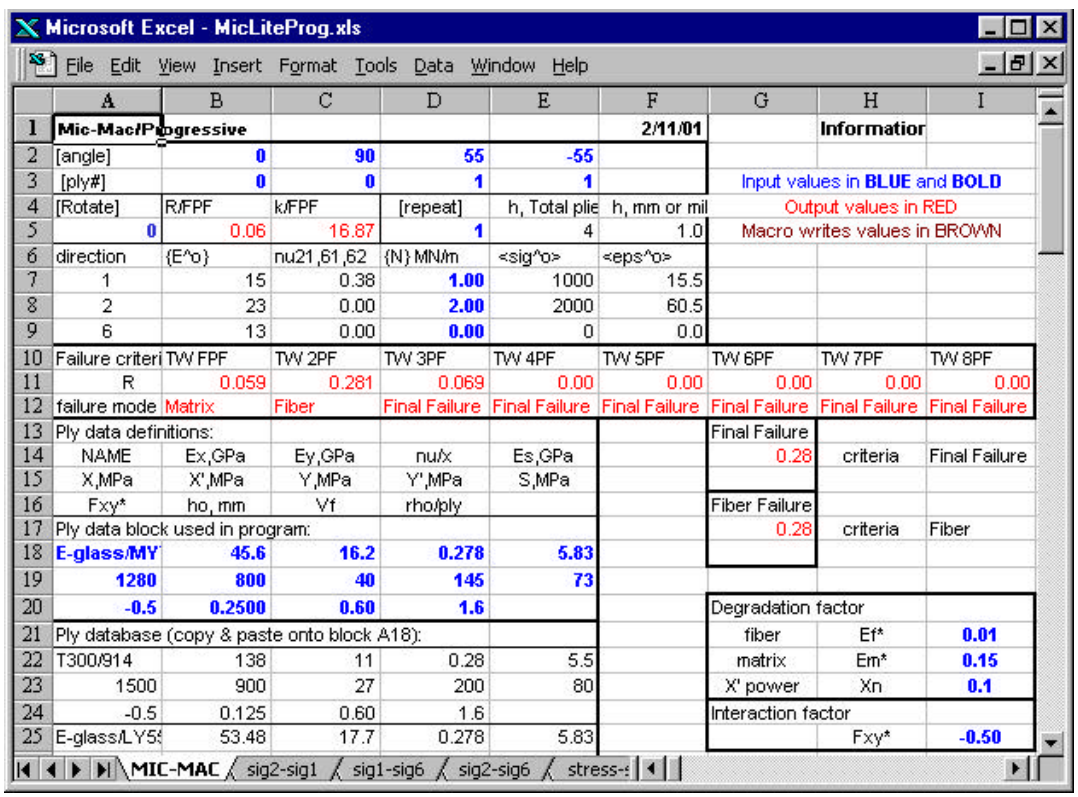


Figure 2. Flow chart of the Mic-Mac/Progressive

The input screen of Mic-Mac/Progressive is shown in Figure 3. The inputs, which are shown in **Bold**, are the ply properties (such as E_x, E_y, G_s, ν_{xy}, X, X', Y, Y', and S), ply orientations, applied stress resultants (loads per unit width), and four empirical constants. The default values of the empirical constants (E_m*=0.15, E_f*=0.01, n=0.1, and F_{xy}*=-0.5) provide reasonably accurate predictions, but can also be fine-tuned for a better fit.



Microsoft Excel - MicLiteProg.xls

File Edit View Insert Format Tools Data Window Help

	A	B	C	D	E	F	G	H	I
1	Mic-Mac/Progressive					2/11/01		Information	
2	[angle]	0	90	55	-55				
3	[ply#]	0	0	1	1		Input values in BLUE and BOLD		
4	[Rotate]	R/FPF	k/FPF	[repeat]	h, Total plie	h, mm or mil	Output values in RED		
5	0	0.06	16.87	1	4	1.0	Macro writes values in BROWN		
6	direction	{E^o}	nu21,61,62	{N} MN/m	<sig^o>	<eps^o>			
7	1	15	0.38	1.00	1000	15.5			
8	2	23	0.00	2.00	2000	60.5			
9	6	13	0.00	0.00	0	0.0			
10	Failure criteri	TW FPF	TW 2PF	TW 3PF	TW 4PF	TW 5PF	TW 6PF	TW 7PF	TW 8PF
11	R	0.059	0.281	0.069	0.00	0.00	0.00	0.00	0.00
12	failure mode	Matrix	Fiber	Final Failure	Final Failure	Final Failure	Final Failure	Final Failure	Final Failure
13	Ply data definitions:						Final Failure		
14	NAME	Ex,GPa	Ey,GPa	nu/x	Es,GPa		0.28	criteria	Final Failure
15	X,MPa	X',MPa	Y,MPa	Y',MPa	S,MPa				
16	Fxy*	ho, mm	Vf	rho/ply			Fiber Failure		
17	Ply data block used in program:						0.28	criteria	Fiber
18	E-glass/MY	45.6	16.2	0.278	5.83				
19	1280	800	40	145	73				
20	-0.5	0.2500	0.60	1.6			Degradation factor		
21	Ply database (copy & paste onto block A18):						fiber	Ef*	0.01
22	T300/914	138	11	0.28	5.5		matrix	Em*	0.15
23	1500	900	27	200	80		X' power	Xn	0.1
24	-0.5	0.125	0.60	1.6			Interaction factor		
25	E-glass/LYS	53.48	17.7	0.278	5.83			Fxy*	-0.50

MIC-MAC / sig2-sig1 / sig1-sig6 / sig2-sig6 / stress-

Figure 3. Input screen for Mic-Mac/Progressive

The example shown above is for E-glasss/MY750 material with [55/-55]s ply orientations. The empirical constants are all default values. The applied stress resultants are $N_1=1\times10^6$N/m in the fiber direction and $N_2=2\times10^6$N/m transverse to the fiber. The results are initial failure (First Ply Failure) at strength/stress ratio of $R_{FPF}=0.059$, and final failure (Last Ply Failure) at $R_{LPF}=0.281$. This translates to initial failure at $N_1=0.059\times10^6$N/m and $N_2=0.118\times10^6$N/m, and final failure at $N_1=0.281\times10^6$N/m and $N_2=0.562\times10^6$N/m.

Physical explanation of this result is as follows. At R_{FPF} the stresses and strains in one of the plies reach a level that is considered as a failure by the Tsai-Wu criterion. Then the matrix-controlled properties of the ply are degraded, and the stiffness is recalculated. Because of this change in the stiffness, the load is redistributed among the plies, and the stresses in each ply change. Tsai-Wu failure criterion is applied again to this new stress state, and R is recalculated. In the example above, the second failure occurs in the same ply, so we consider it as a fiber failure. The process is repeated until all the plies failed, or when the material can no longer carry any additional load. In this case, the load level at the third failure is less than that of the second, and therefore we consider the second failure as the final failure, and use the second value of R as R_{LPF}.

Figure 4 is an example of a stress-strain curve for the input shown in Figure 3. These curves are plotted by connecting the stresses and strains at each incremental failure. In this example, the first set of dots is the stresses and strains at failure of intact laminate, and the second set of dots is for the degraded state. The third set of dots was not plotted since the stress level was lower than the second set. In each of these steps, the calculation is performed linearly as if we are loading a material with specific properties (intact or degraded) linearly up to failure. By connecting these dots rather than making a staircase stress-strain curve, we are assuming a gradual failure inside the "failed" ply rather than an instantaneous failure. We believe this is a fair assumption, since the stress-strain curves from the tests match well with our prediction.

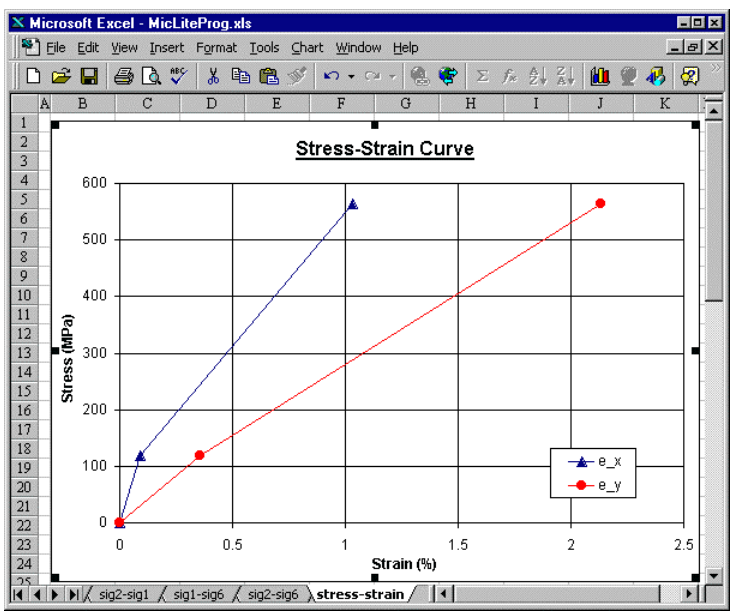


Figure 4. Stress-strain curve from Mic-Mac/Progressive

Another function of this program is plotting failure envelopes with a single command. Failure envelope shows the strength of a lamina or a laminate at all combined stress state, and is typically plotted in biaxial stress plane (σ_x-σ_y) or axial-shear stress plane (σ_x-τ_{xy} or σ_y-τ_{xy}). The biaxial failure envelope of a lamina intersects σ_x axis and σ_y axis at values equal to longitudinal tensile strength, longitudinal compressive strength, transverse tensile strength, and transverse compressive strength. For a laminate, the envelope will be a combination of the individual envelopes for the lamina. Note that this will not be a simple addition of the curves, due to the stress redistribution after the degradation of the failed ply.

Figure 5 is an example of a failure envelope for the same ply properties and orientations as in Figure 3. The initial failure envelope in the figure is equivalent to the envelope for Tsai-Wu failure criterion without the progressive damage. The progressive damage scheme allows the envelope to expand further out, leading to a larger failure envelope shown in the figure as final

failure envelope. In this laminate, the fiber failure always led to final failure, and therefore the fiber failure envelope is identical to the final failure envelope. The bulges in the tension-tension condition and compression-compression condition show that this laminate can continue to carry load after initial matrix failure in these stress states. Comparison between this predicted envelope and the published test results will be shown in the next section.

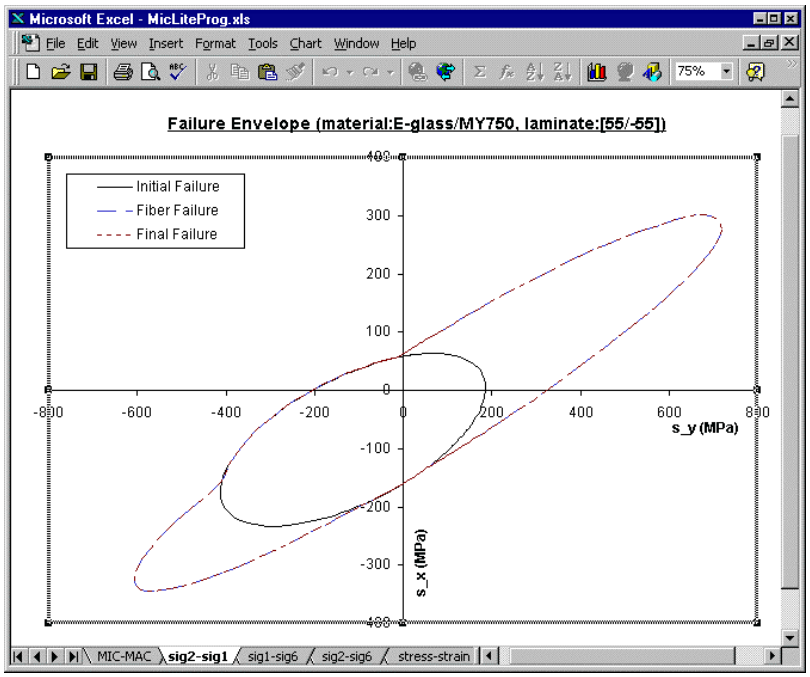


Figure 5. Failure envelopes from Mic-Mac/Progressive

The third function of this program is the parametric studies of the four parameters. With a single command, failure envelopes using three different values for a chosen parameter can be plotted. This allows us to study the sensitivity of the parameters on the failure envelope, and to optimize these values.

Figure 6 shows the example of the parametric study using the same material as before, changing the values of the matrix degradation factor E_m*. As we increase E_m* up to the default value of 0.15, we observe that the failure envelope becomes smaller in the tension-tension region and larger in the compression-compression region. These results are not trivial, and can not be studied without the help of computer programs such as the one introduced in this paper.

Each parameters affect the failure envelope in different ways. Typically, the matrix degradation factor E_m* has largest effect, controlling the size of the envelope. The power law factor n for compressive strengths also affect the envelope to some extent. The effect of fiber degradataion factor E_f* is small and limited to certain conditions. The interaction factor E_{xy}* should be determined in the lamina level. Note that these parameters are not arbitrary factors that can be adjusted to fit any data. The overall shape of the failure envelope and the stress-strain curve are determined, and these factors only allow fine tuning the prediction.

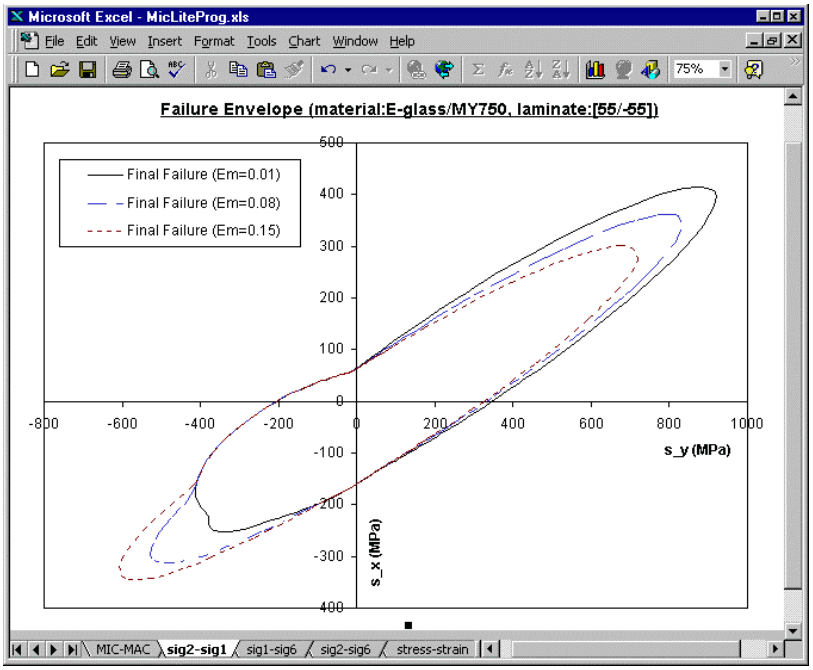


Figure 6. Parametric study using Mic-Mac/Progressive

In this section, we have shown the capabilities of our program, which are (1) prediction of complex stress-strain curve during progressive failure, (2) creation of failure envelopes for combined stress state, and (3) parametric study using failure envelopes.

EXAMPLES AND COMPARISON

In this section, we will show examples of the prediction created from the program and the comparison with the published test results. All the tests are performed by tubular specimens subject to combination of axial load, shear load, and internal/external pressure.

Figure 7 is the biaxial failure envelope of E-glass/MY750 [0°] lamina. Since progressive damage does not apply to a lamina, this case is equivalent to using Tsai-Wu failure criterion. Tsai-Wu criterion assumes interaction between the two axial stresses, and creates slanted elliptical envelope as shown in the figure. On the other hand, failure criteria that do not assume interaction create rectangular or diamond shaped envelopes. The test result by Al-Khalil *et al*[10] shows clear sign of the interaction, and agrees well with the prediction by Tsai-Wu criterion using the default interaction factor Fxy*= -0.5.

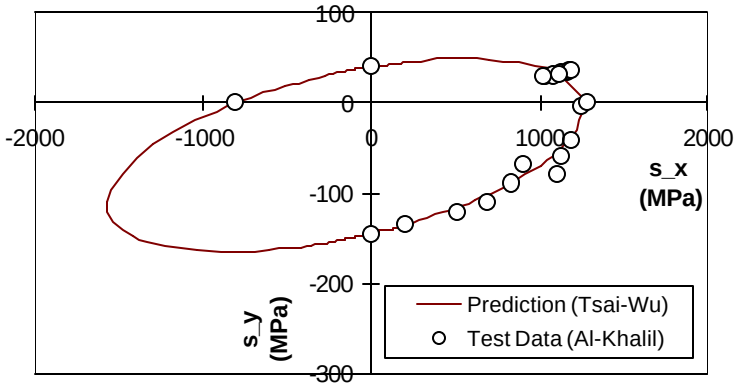
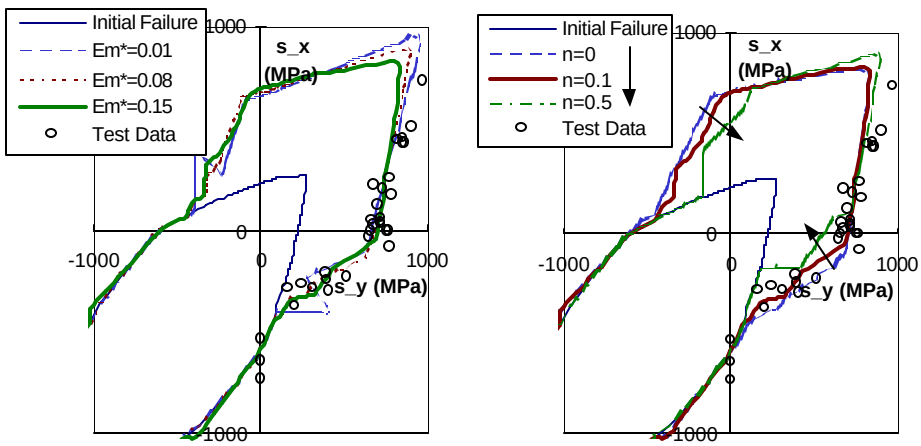


Figure 7. Failure envelope for E-glass/MY750 [0°] lamina

Figure 8 is the biaxial failure envelopes of AS4/3501-6 [0°/45°/-45°/90°] CFRP laminate. Several failure envelopes are plotted using different values of the two of the four parameters to show the sensitivity of these parameters. In both figures, the dark curves are the failure envelopes using the default values. The figure to the left shows the envelopes for different values of matrix degradation factor E_m*, and the figure to the right for different values of the power law factor of compressive strength. n=0 is equivalent to assuming no strength degradation. The default values of E_m*=0.15 and n=0.1 show good agreement with the test results by Swanson *et al* [11, 12, 13].

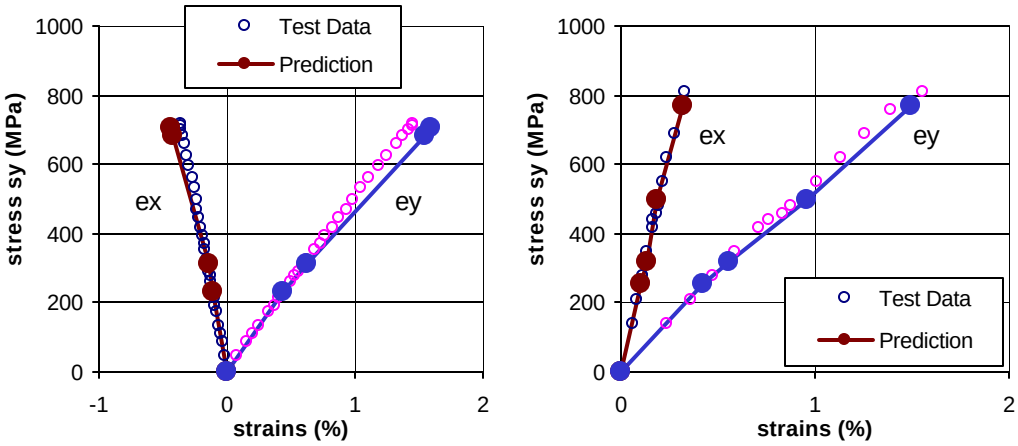
The two other parameters not shown in Figure 8 are the fiber degradation factor E_f*, and the interaction factor F_{xy}* for Tsai-Wu criterion. The fiber degradation factor E_f* had very little effect on the failure envelope. The interaction factor F_{xy}* has more effect on the failure envelope, but the value should be determined in the lamina level.



Test data: by Swanson et al

Figure 8. Failure envelopes of AS4/3501-6 [0°/45°/-45°/90°] laminate

Figure 9 shows the stress-strain curves of the same material for the biaxial load of σ_y/σ_x=20/1 and σ_y/σ_x=2/1, respectively. The comparison of the predicted and measured stress-strain curves is more severe test of the prediction, but we can observe that the prediction agrees well with the test results from Christoforou[12], and from Trask[13].



Loading: σ_y/σ_x=20/1
Test Data: by Christoforou

Loading: σ_y/σ_x=2/1
Test Data: by Trask

Figure 9. Stress-strain curves of AS4/3501-6 [0°/45°/-45°/90°] laminate

Detailed comparison of the prediction and the test results[2] has been performed for an ongoing work organised by Hinton et al[5, 6]. The predictions show good agreement with the test results in many cases, but there are few cases where good agreements are not achieved. One example is the case under pure shear condition, where nonlinearity of the G_s plays an important role. The progressive Tsai-Wu failure criterion is based on a linear theory and can not deal with this nonlinear behavior. Although it is possible to modify the program to deal with this nonlinearity, linear theory enables simple, quick, and sufficiently accurate calculation, which we consider to be more important.

Another obvious limitation is that residual stress due to the curing is not included. This can be easily added into the Excel program. It is useful to point out that curing stress are offset by moisture absorption over a long-term exposure in air. In fact, we have shown that typically 0.5% moisture offsets residual stress due to the cure temperature of 100°C above room temperature[3].

OTHER FAILURE CRITERIA

Mic-Mac/Progressive uses progressive quadratic failure criterion for the prediction, but the program can be easily modified to use other failure criteria. In fact, we have another program that performs first ply failure predictions of five different failure criteria. This program is also available free of charge, and has been used to perform comparison of different failure criteria[7].

For anyone accustomed to Excel, it is not difficult to modify Mic-Mac/Progressive to use different failure criteria provided they meet certain conditions. One condition is that the criteria should be linear up to failure in each intact or degraded state. Most initial failure criteria and some progressive failure criteria meet this condition. One of our ongoing research is based on the modified version of Mic-Mac/Progressive with a different progressive failure criterion. Interestingly, the resulting predictions are not much different from those introduced in this paper.

Through these studies, we observed that there are small differences in the predictions using different failure criteria. On the other hand, there are large differences in the complexity of the calculations required to obtain these almost identical predictions. This finding also supports our arguement of the importance of simple and practical failure criteria.

CONCLUSION

We have shown that our program, Mic-Mac/Progressive can easily create predictions that agree well with the test results. The failure load and stress-strain curves are calculated instantly, and failure envelopes can be plotted by a single command. The program runs on Microsoft Excel, which is one of the most popular software available in most computers.

The accuracy of the prediction was shown to be reasonably good for number of cases, and can be further improved by fine-tuning the four empirical factors. The sensitivity study of these empirical factors can be easily performed by another single command in this program. We believe that our initial choices of the factors (F_{xy}*=-0.5, E_m*=0.15, E_f*=0.01, n=0.1) are sufficient for preliminary application, given the good agreement with the test results.

There are limitations in the failure criterion, and subsequently in the prediction from this program. One such limitation is the inability of the criterion to deal with the nonlinear behavior of the material, which is an inevitable consequence of a linear theory. But on the other hand, linear theory enables simple, quick, and reasonably accurate prediction, which we consider as more important as practical design tool.

With the simplicity of the criterion, availability of the hardware to run the program, and the reasonable accuracy we have shown, we believe that this program will be a valuable tool in many aspects of design. We are happy to provide this program free of charge to anyone interested. The program can be downloaded at www.thinkcomp.com after a free registration.

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