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NUMERICAL PREDICTION OF THE MECHANICAL PROPERTIES OF WOVEN FABRIC COMPOSITES

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SUMMARY: The paper presents a numerical procedure to evaluate the mechanical properties of woven fabric composites and the validation of the method against experiments. The numerical approach is founded on the main assumption of periodic distribution of the reinforcements in the composite and on two levels of homogenization. The first homogenization concerns the warp and weft yarns while the second regards a representative volume of the woven fabric laminate with homogenized yarns. An experimental investigation and the relevant numerical predictions were performed on various woven fabric laminates including four types of fibers: carbon, glass, aramid and polyvinylalcohol (PVA).

KEYWORDS: woven fabric, finite element method, experimental investigation, mechanical features.

INTRODUCTION

The application of textile composites in engineering structures has been driven by various attractive aspects like ease of handling, high adaptability, damage tolerance, and, if compared to unidirectional laminates, better out-of-plane stiffness properties. Therefore an in-depth knowledge of the mechanical properties is of fundamental importance. The most known types of fibers are used in woven fabric (WF) laminates on the basis of specific needs and the geometry of the reinforcement (yarn spacing, yarn thickness and shape and the weave type) is chosen to reach the requested mechanical properties. It is therefore important to produce and validate micromechanical models that can predict both the internal stress state of a laminate and the macroscopic behaviour. The micromechanical models for woven fabric composites available in the literature can be grouped into two classes [1]. The models of the first class, that includes analytical methods, such as iso-stress and iso-strain, provide a reasonable estimate of the mechanical properties but the evaluation of the internal stresses is not satisfactory [1, 2]. The second class is based on 'finite element' models of an elementary cell (representative volume RV) of the laminate [3, 4]. The method can provide adequate estimates of the internal stresses, of the local damage and can be used to evaluate the ultimate failure of the composite.

This paper presents a finite element model suitable for the analysis of WF composites subject to any 3D stress state. An homogenization process to pass from a micro to a macro scale and to relate the mechanical properties of the macro element to the properties of the single phases, is applied at two levels. The first level concerns the yarns considered as unidirectional fiber reinforced composites to determine their mechanical properties. The second level of homogenization is applied to the elementary cell of the WF. In this case a 3D FE model is produced and the necessary boundary conditions to ensure the periodicity of the material are applied. In parallel an experimental campaign on WF composite laminates was undertaken. The results are summarised in the first part of the paper. The numerical results are then compared to the experimental values of different laminates in terms of elastic properties.

EXPERIMENTAL INVESTIGATIONS

The mechanical properties of a series of woven fabric laminates including different manufacturing techniques, geometries and types of fibers, have been recently investigated

experimentally [5]. The final objective was the experimental evaluation of the influence of all these parameters on the mechanical properties of the laminates. Four types of fibers and three types of epoxy resins were considered.
In this work only the results of specimens made with plain-weave WF using the bag moulding technique are considered while the results of other materials may be founded in [5]. Four panels were manufactured using the fibers and the matrix reported in Tab. 1 and the woven fabric listed in Tab. 2. The laminates were built with different number of layers that ranged from 3 to 10. It should be noted that all fabrics are balanced and present the same number of fibers and geometry in the warp and weft directions.

		E [GPa]	ν	σ_u [MPa]
Fibers	Carbon	230	0.30	3400
	Glass	76	0.22	2000
	Aramid	80	0.30	2800
	PVA	29	0.30	1400
Matrix	Epoxy SV 312	2.51	0.35	48

Tab. 1 Young's modulus (E), Poisson's ratio (ν) and ultimate tensile strength (σ_u) of the fibers and matrices.

Fibers material	Weight [g/m^2]	Construction [yarn/cm] (in weight [%])		Fiber count [tex]
		Warp	Weft	
Carbon	200	5 (50)	5 (50)	200
Glass	380	6 (50)	6 (50)	320
Aramid	460	6.7 (50)	6.7 (50)	336
PVA	240	6 (50)	6 (50)	200

Tab. 2 Technical characteristics of the reinforcing fabrics.

The specimens for the experimental characterisation of the laminate were produced from flat plates cutting five nominally identical specimens per type. Tensile tests, interlaminar shear strength (ILSS), bending and impact tests were performed according to the standards ASTM D3039, D2344, D790M and D5942 respectively.
In this work the attention is focused only on tensile tests. The tensile properties of the laminates were evaluated on two series of five specimens (approximate dimensions 250x25 mm) cut from the same plate in two different directions (0° and 45°). The volume fractions V_f were calculated from the weight of the specimens and the weight per unit area of the dry reinforcement. The void contents were not measured.
The tests were executed at the ambient conditions using a tensile-torsional testing machine (capacity: 100 kN axial, 1000 Nm torsional) available at the Material Testing Laboratory of Polytechnic of Milan. The torsional moment generated during the tensile tests was recorded to detect possible coupling. This quantity is a significant parameter to evaluate possible misalignments of the fibers and can be also adopted as a parameter for a quality control of the manufacturing [5].
The longitudinal and transversal strains were measured by means of a biaxial transducer that was removed from the specimens when the load reached about 50% of the expected ultimate load. The accuracy of the extensometer was also verified by means of electrical strain gauges applied to some specimens and comparing the transversal and longitudinal strains. Nevertheless it must be underlined that the measurement of the transversal deformation is always a delicate matter and even the extensometer can fail because of imperfections of the matrix in the zone of the specimen where the transducer is positioned.

The thickness of each specimen was measured in at least three sections to verify its regularity. The mean values of the elastic modulus, the Poisson's ratio and the ultimate strength were calculated as an average of five results from each series of tests. The standard deviation and the coefficient of variation (standard deviation over mean value) were also calculated to have a measure of the variability and the tests were considered satisfactory when the coefficient of variation was lower than 0.05.
The results of the uniaxial tensile tests for the WF laminates are reported in Tab. 3 where the most significant properties (axial elastic modulus E; Poisson's ratio ν; ultimate stress σ_u) are listed. The values are the average of five tests. The properties of the fabric evaluated on specimens cut at 0° are extensible to the direction 90° because of the full symmetry in the warp and weft directions.

			0°/90° (warp/weft)			45°		
Fibers material	Layers number	V_f %	E [MPa]	ν	σ_u [MPa]	E [MPa]	ν	σ_u [MPa]
Carbon	7	43.6	39359	0.089	624	8271	0.790	153
Glass	5	50.6	18529	0.161	369	5805	0.731	136
Aramid	3	53.6	9858	0.219	331	2804	0.811	---
PVA	4	41.8	8538	0.223	153	2698	0.788	90

Tab. 3 Experimental tensile properties of the woven fabric laminates.

THE NUMERICAL HOMOGENISATION PROCEDURE

The proposed numerical method for three-dimensional analyses of WF composites is presented in this section [6]. The following hypotheses are assumed:
(i) regular distribution of the fibers in the yarns;
(ii) regular arrangement of the reinforcing yarns in the composites.
These hypotheses allow to study the problem within the framework of the micromechanical theory of heterogeneous periodic materials [7] and correspond to two levels of homogenisation. The first level concerns the yarns that are firstly examined by means of an independent model. Each yarns is considered as unidirectional fiber reinforced composite (see Fig. 1a) with fiber volume fraction equal to the packing density (p_d). The mechanical properties of the homogenized yarns (Fig. 1c) are obtained by means of numerical analyses on a representative volume as detailed in [8]. In Fig. 1b the representative volume of a UD composite (hexagonal pattern) is showed. Applications and validation of this approach for unidirectional laminates may be found in [8].

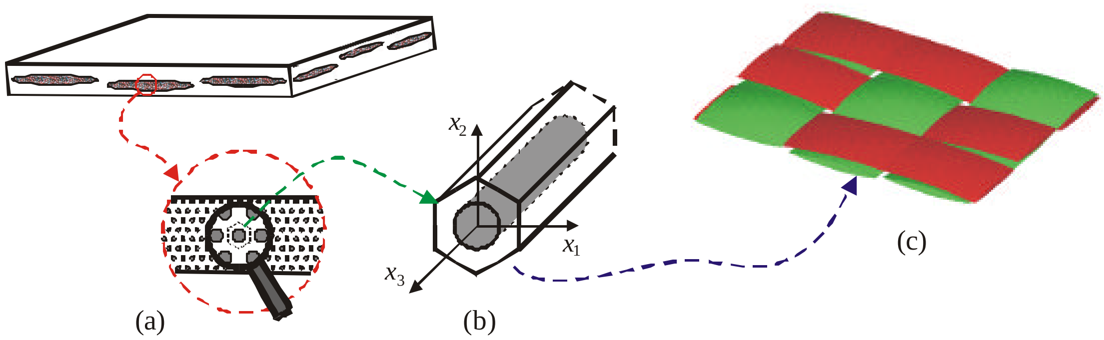


Fig. 1 (a) Woven fabric composite; (b) representative volume of the yarn considered as unidirectional fiber reinforced composites; (c) homogenized yarns.

The second homogenization level, summarized below, deals with a fictitious woven fabric composite (WF) containing the homogenized yarns (Fig. 2a). The regular distribution of the reinforcements in the material allows us to extract a representative volume (RV) (see Fig. 2b) of the WF to investigate the response of the homogenized material (Fig. 2c) in terms of macroscopic (or global) quantities.

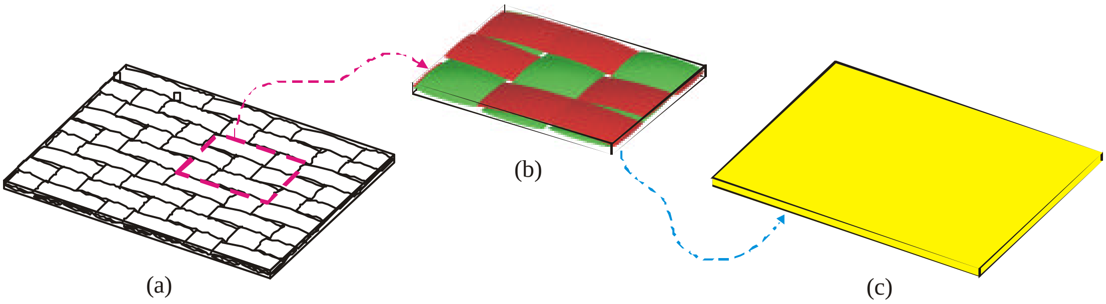


Fig. 2 (a) Woven fabric composite with homogenized yarns; (b) representative volume; (c) homogenized material.

The global constitutive law links macroscopic stresses and strains, whose components are collected in vectors $\boldsymbol{\Sigma}$, $\mathbf{E}$ respectively (bold symbols denote matrices and vectors). They are defined as volumetric averages of the relevant microscopic variables $\sigma(\mathbf{x})$, $\varepsilon(\mathbf{x})$ which are functions of the position vector $\mathbf{x}$ in the RV. The macroscopic constitutive law describes the global response of any RV belonging to an ideal infinite medium subjected to uniform boundary conditions, either in stresses or displacements. For composites consisting of non-linear components, this law has to be determined in incremental form.
The local fields of stress $\sigma(\mathbf{x})$ and strain $\varepsilon(\mathbf{x})$ must reproduce the periodicity of the heterogeneous material. Such property is satisfied if the following relation between the displacement and the strain fields holds [7]:

$$\mathbf{u}(\mathbf{x}) = \mathbf{u}^0 + \boldsymbol{\Omega}\cdot\mathbf{x} + \mathbf{E}\cdot\mathbf{x} + \tilde{\mathbf{u}}(\mathbf{x}) \tag{1}$$

where $\mathbf{u}^0$ represents a rigid displacement of the RV and Ω is the anti-symmetric tensor related to the small rigid rotation of the RV. The pure strain modes of the RV are described by the last two terms in (1): a constant term (the macroscopic strain $\mathbf{E}$) and a V-periodic term, with zero average value, associated with the V-periodic part $\tilde{\mathbf{u}}$ of the microscopic displacement field (being V the volume of the RV).
The incremental problem to define the macroscopic constitutive law, can be expressed as follows:

given $\dot{\boldsymbol{\Sigma}}$, find $\dot{\mathbf{u}}\ (\Rightarrow \dot{\mathbf{E}})$ such that:

$$\operatorname{div}\dot{\boldsymbol{\sigma}} = \mathbf{0} \quad \text{in } V \tag{2a}$$

$$\dot{\mathbf{t}} = \dot{\boldsymbol{\sigma}}\cdot\mathbf{n} \quad \text{anti-periodic on } \partial V \tag{2b}$$

$$\dot{\boldsymbol{\Sigma}} = \frac{1}{V}\int_V \dot{\boldsymbol{\sigma}}\, dV \tag{2c}$$

$$\dot{\tilde{\mathbf{u}}} = \dot{\mathbf{u}} - \dot{\mathbf{E}}\cdot\mathbf{x} \quad V-\text{periodic} \tag{2d}$$

$$\dot{\mathbf{E}} = \frac{1}{V}\int_V \dot{\boldsymbol{\varepsilon}}\, dV \tag{2e}$$

$$\dot{\boldsymbol{\sigma}} = F(\dot{\boldsymbol{\varepsilon}}(\dot{\mathbf{u}})) \quad \text{in } V \tag{2f}$$

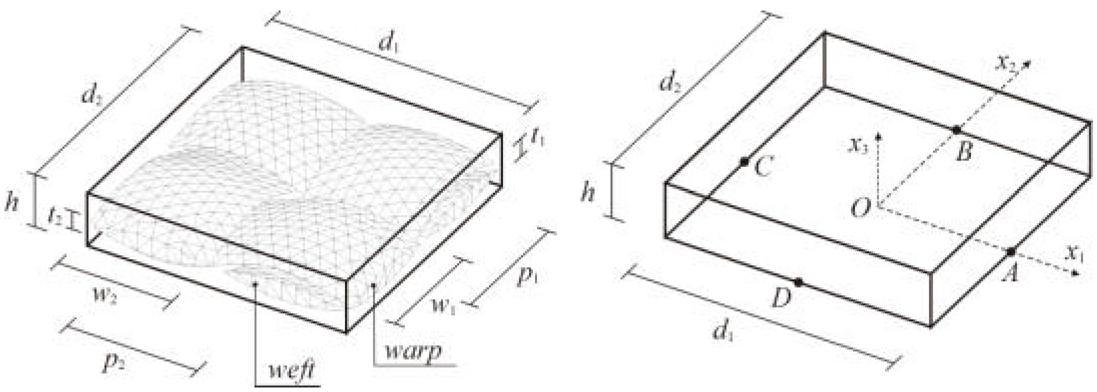
where V and ∂V indicate the volume and the boundary of the RV respectively, and $\mathbf{n}$ the outward unit-normal on ∂V. Eqn. (2f) represents the microscopic incremental constitutive law.
The solution of the incremental problem (2) is obtained by means of a model based on the displacement formulation of the finite element method.
The main task to implement problem (2) in a FE code based on the displacement formulation, is the assignment of the boundary conditions to ensure that the displacement field complies

with Eqn. (1). Referring to the RV singled out from a plane woven composite in Fig. 2a, such conditions can be written as:

$$\mathbf{u}^{H} - \mathbf{u}^{K} = \mathbf{u}^{J} - \mathbf{u}^{Y} \tag{3a}$$

$$\mathbf{u}^{P} - \mathbf{u}^{\hat{P}} = \mathbf{u}^{A} - \mathbf{u}^{F} \tag{3b}$$

In Eqn. (3a) J and Y are one of the midpoints pairs (A, C), (B, D); H and K are points corresponding in the periodicity belonging to the sides $x_1=d_1/2$ and $x_1=-d_1/2$ respectively if (A, C) are considered, or belonging to the sides $x_2=d_2/2$ and $x_2=-d_2/2$ if (B, D) are considered (see Fig. 3b). In Eqn. (3b), P is any point on the side $x_3=0$ and $\hat{P}$ is the corresponding point on the side $x_3=h$.



(a) (b)

Fig. 3 The RV geometry and the FE model of the reinforcement; (b) characteristic points of the finite element model.

The macroscopic displacement gradient are related to some displacement components on ∂V by employing the periodicity condition (3a,b) and accounting for the general expression of the displacement field (1), namely:

$$\mathbf{E} + \boldsymbol{\Omega} = \begin{bmatrix} \dfrac{u_1^A - u_1^C}{d_1} & \dfrac{u_1^B - u_1^D}{d_2} & \dfrac{u_1^F - u_1^A}{h} \\ \dfrac{u_2^A - u_2^C}{d_1} & \dfrac{u_2^B - u_2^D}{d_2} & \dfrac{u_2^F - u_2^A}{h} \\ \dfrac{u_3^A - u_3^C}{d_1} & \dfrac{u_3^B - u_3^D}{d_2} & \dfrac{u_3^F - u_3^A}{h} \end{bmatrix} \tag{4}$$

Rigid translations of the model are suppressed by prescribing $\mathbf{u}^{O}=\mathbf{0}$ (see Fig. 3b). Rigid rotations can be avoided by imposing the skew symmetric tensor $\boldsymbol{\Omega}$ to vanish, setting:

$$\frac{d_1}{d_2}u_1^B - \frac{d_1}{d_2}u_1^D + u_2^C - u_2^A = 0 \tag{5a}$$

$$\frac{d_1}{h}u_1^F - \frac{d_1}{h}u_1^A + u_3^C - u_3^A = 0 \tag{5b}$$

$$\frac{d_2}{h}u_2^F - \frac{d_2}{h}u_2^A + u_3^D - u_3^B = 0 \tag{5c}$$

The six components of the symmetric tensor $\mathbf{E}$ can be explicitly written as:

$$E_{11}=\frac{u_1^A-u_1^C}{d_1};\quad E_{22}=\frac{u_2^B-u_2^D}{d_2};\quad E_{33}=\frac{u_3^F-u_3^A}{h};\qquad \text{(6a,b,c)}$$

$$E_{12}=\frac{u_2^A-u_2^C}{d_1};\quad E_{13}=\frac{u_3^A-u_3^C}{d_1};\quad E_{23}=\frac{u_3^B-u_3^D}{d_2}\qquad \text{(6d,e,f)}$$

Relations (6) provide the kinematic boundary conditions that allow the simulation of any prescribed macroscopic strain through only some free nodal displacements (u_1^A, u_2^A, u_3^A, u_1^C, u_2^C, u_3^C, u_2^B, u_3^B, u_2^D, u_3^D, u_3^F).
Prescribed macroscopic stresses acting along the orthogonal axes x_1, x_2, x_3 of the model can also be imposed through kinematic boundary conditions using the following scheme:

$$u_1^C=-u_1^A \text{ prescribed } \Rightarrow \Sigma_{11};\quad u_2^D=-u_2^B \text{ prescribed } \Rightarrow \Sigma_{22}\qquad \text{(7a,b)}$$

$$u_3^F=-u_3^A \text{ prescribed } \Rightarrow \Sigma_{33};\quad u_2^C=-u_2^A \text{ prescribed } \Rightarrow \Sigma_{12}\qquad \text{(7c,d)}$$

$$u_3^C=-u_3^A \text{ prescribed } \Rightarrow \Sigma_{13};\quad u_3^D=-u_3^B \text{ prescribed } \Rightarrow \Sigma_{23}\qquad \text{(7e,f)}$$

In the numerical simulations a macroscopic component (stress or strain) is prescribed and the remaining ones are evaluated by the volumetric averages (2c,e) of the microscopic counterparts in the Gauss points.

APPLICATIONS AND COMPARISONS

The procedure was validated predicting the elastic properties of the woven fabrics presented in the experimental section above. The mechanical properties of the composite components are listed in Tab. 1. The geometrical features of the representative volumes are detailed in Tab. 4 with reference to Fig. 3a. These geometric parameters were obtained by microscopic observations that provide also the yarns morphology as shown in Fig. 4.

Fig. 4 Microscopic observation of the aramid WF composite.

The fiber volume fraction of the yarns considered as unidirectional fiber reinforced composite, in the first homogenisation step, is assumed equal to the packing density factor p_d=0.8 for carbon, glass and PVA and p_d=0.75 for aramid reinforcement. The fiber volume fractions V_f of the yarns in the WF laminates, in the second homogenisation step, are listed in Tab. 3.

Fibers material	d_1=d_2 [mm]	h [mm]	w_1=w_2 [mm]	t_1=t_2 [mm]	p_1=p_2 [mm]
Carbon	4.10	0.35	1.80	0.14	2.05
Glass	3.24	0.38	1.35	0.15	1.62
Aramid	3.40	1.20	1.60	0.48	1.70
PVA	3.38	0.70	1.50	0.24	1.69

Tab. 4 Geometrical features of the considered RV (see Fig. 3a)

The finite element meshes employed in the first homogenisation step were made with 8-nodes solid elements as already experienced in [8]. The elastic features of the transversal isotropic homogenized yarns obtained in the first step are detailed in Tab. 5 (x_3 is the fiber axis, while x_1-x_2 indicates the plane orthogonal to the fibers, see the hexagonal RV in Fig. 1b).

Four-node tetrahedrons finite elements were used in the discretization of the woven fabric composite RV in the second homogenisation step. The details of the meshes are reported in Tab. 6. As an example, the FE mesh of the reinforcements of the carbon woven fabric composite is drawn in Fig. 3a.

	Carbon	Glass	Aramid	PVA
E_1=E_2 [MPa]	28434	32645	18060	13521
E_3 [MPa]	184545	59315	60756	23707
G_{12} [MPa]	10644	12730	6646	5034
G_{13}=G_{23} [MPa]	9787	13261	6449	5020
ν_{12}	0.335	0.282	0.312	0.343
ν_{13}=ν_{23}	0.047	0.140	0.092	0.176
ν_{31}=ν_{32}	0.308	0.255	0.310	0.308

Tab. 5 Elastic features of the yarns obtained in the first homogenisation step.

Fibers material	Number of nodes	Number of elements
Carbon	2262	11265
Glass	4418	17662
Aramid	1527	5893
PVA	4736	17873

Tab. 6 Nodes and elements number of the finite element meshes employed.

The experimental elastic moduli, measured in direction 0°, are depicted in Fig. 5 with their standard deviations, for the four woven fabric composites considered. The values of the elastic modulus obtained with the present numerical procedure, are also presented in Fig. 5 for comparison. A good agreement between experimental and numerical values is evident. On the contrary, some differences were detected in terms of Poisson's ratios (Fig. 6). Nevertheless it must be underlined that the experimental measurement of the transversal deformation is always a delicate matter. This problem will be certainly treated in future developments of the research.

Fig. 7 shows the deformed mesh of the homogenized yarns in a shear test of the PVA specimen. This permits to verify the effectiveness of the imposed periodicity boundary conditions on the RV of the WF composite to reproduce the periodicity.

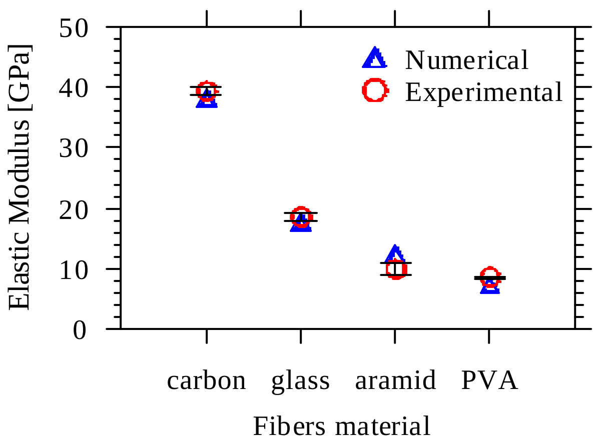


Fig. 5 Comparison of the numerical and experimental elastic moduli for traction in 0° direction.

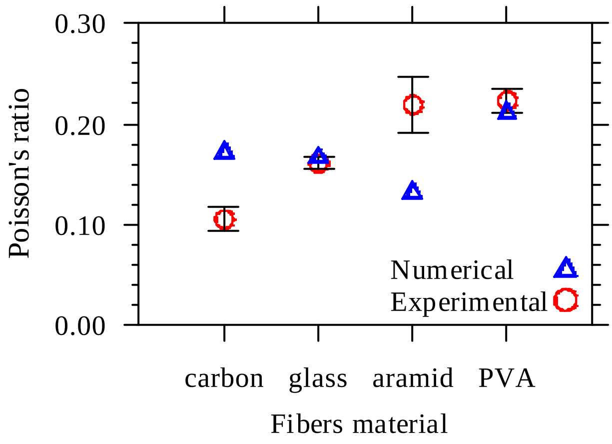


Fig. 6 Comparison of the numerical and experimental Poisson’ s ratio for traction in 0^o direction.

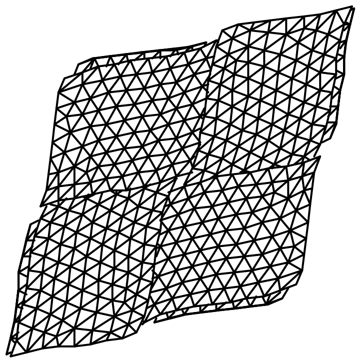


Fig. 7 Deformed mesh of the yarns from shear test in the plane x_1-x_2 of PVA specimen

CONCLUSIONS

A numerical procedure to evaluate the mechanical properties of woven fabric composites has been presented and validated against experimental results. The method is based on the homogenization technique for periodic media. This is first applied to the warp and weft yarns by means of FE models and allows us to determine the mechanical properties of the yarns. The second level of homogenization is studied using an elementary volume of the woven fabric and imposing the proper boundary conditions to ensure the periodicity. All the components of the stiffness matrix may be obtained. The model has been validated simulating different tensile experiments of flat specimens made with various types of fibers.

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