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DEVELOPMENT OF A S.C.C. MODEL FOR THE PREDICTION OF THE DURABILITY OF FLEXURAL GFRP BEAMS IN HUMID ENVIRONMENTS

Vincent Pauchard[1,2], Antoine Chateauminois[1], François Grosjean[2] et Pierre Odru[2]

1 - Ecole Centrale de Lyon, Laboratoire IFoS, UMR 7615, 69131Ecully, France
2 - Institut Français du Pétrole, 92582 Rueil Malmaison, France

SUMMARY: The flexural fatigue behaviour of water aged GFRP beams has been analysed in the light of a Stress Corrosion Cracking (S.C.C.) model. This approach was developed in the context of unidirectional glass/epoxy composites, where the initial fatigue damage is known to be mostly associated to the delayed failure of the fibres. Theoretical expressions for the stiffness loss have been derived, which require the determination of the statistical distribution of fibres strength and the assessment of the sub-critical-crack growth rates within the glass filaments. From stiffness loss measurements under monotonic and static fatigue conditions, consistent values of the Weibull modulus, m, and the stress-corrosion parameter, n, have been obtained. Provided that the loading frequency was not to high and that the strain ratio was not to low, these values were successfully used in the model to predict the experimental stiffness loss of the composite beams under cyclic fatigue conditions. A good agreement between the experiments and the theory was also achieved under more complex fatigue loading involving stacking block sequences.

KEYWORDS: Stress Corrosion Cracking, Ageing, Glass/Epoxy, Bending, Stiffness

INTRODUCTION

The fatigue behaviour of unidirectional glass/epoxy composites under bending or tensile loading is known to be largely dominated by the delayed fracture of the fibre reinforcement [1-3]. Often refereed as stress corrosion cracking (S.C.C.), these processes are controlled by the sub-critical growth of fibres surface defects under the combined action of stress and moisture [4-6]. In a previous micro-mechanical investigation [7], we have demonstrated that the fibre failure kinetics within an elementary composite volume subjected to a constant applied strain and a uniform ageing state can be modelled from a combination of the Weibull statistics of fibres strength and classical S.C.C. laws. This approach resulted in a Weibull-based description of the dependency between the fibres survival probability and the loading time. On the basis of an empirical correlation between the stiffness loss of a flexural beam and the associated number of broken fibres within a restricted elementary volume located on its tensile side, it was subsequently demonstrated that the macroscopic stiffness loss of the composite under static or cyclic fatigue conditions could be deduced from the S.C.C. behaviour of the fibres [8].

The accuracy of the prediction was, however, found to be critically dependent upon the estimate of the Weibull modulus, m, and the static fatigue parameter, n, which describe respectively the distribution of the flaw sizes on the fibres and the dependency between the crack velocity and the stress intensity factor. In this paper, an improved methodology is proposed to determine accurately these two parameters from stiffness loss measurements which were carried out under monotonic and static fatigue conditions. On the basis of the calculated values, the validity of the theoretical model is investigated under different cyclic fatigue conditions differing by the frequency, the strain ratio and the maximum applied strain. In addition, some fatigue results obtained under more complex loading, i.e. stacking block sequences, are also considered. It is worth to note that this approach is restricted to the initial stages of the fatigue life, i.e. before the appearance of a significant macroscopic damage in terms of matrix cracking and delamination. Under such conditions, the stiffness loss behaviour can be assumed to be primarily driven by the delayed failure of the unidirectional reinforcement, according to the S.C.C. model.

MATERIALS AND EXPERIMENTAL DETAILS

Materials

The composite material was an unidirectional glass/epoxy composite. The epoxy matrix was based upon a Bisphenol-A epoxy prepolymer (DGEBA) crosslinked using an anhydride hardener (NMA). 2400 Tex E_{CR} glass fibres supplied by Owens Corning were used as a reinforcement. Curved flexural beams corresponding to a portion of the structure under investigation have been elaborated using a continuous filament winding process. The cross-section of the specimen was constant and equal to 10x5 mm^2; the radius of curvature of the beams was *ca.* 700 mm. The average reinforcement volume fraction was found to be 55 %vol.

Three-point bending tests

The initial and residual properties of the specimens have been determined under a three-point bending loading at imposed deflection. A span to depth ratio of 26 mm was selected, which minimised the shear effects. By virtue of the large value of the radius of curvature, the mechanical response of the curved beams was assumed to be similar to that of plane beams. The monotonic loading was performed using an universal Instron testing machine at a rate of 5 mm/min. Static and cyclic fatigue tests were carried out using an hydraulic fatigue machine which operated at frequencies between 0.5 Hz and 10Hz. The experiments were performed at imposed deflection using strain ratios, $R=\varepsilon_{min}/\varepsilon_{max}$, which varied from 0.7 to 1. During the tests, the relative stiffness loss was continuously monitored from the measurement of the maximum load.

Hygrothermal ageing conditions

Monotonic tests have been carried out in ambient environment using pre-dried specimens. These tests were designed in order to determine the strength distribution of the fibres in an 'inert environment', i.e. without any significant contribution of the stress corrosion cracking processes.

Static and cyclic fatigue tests have been carried out using pre-aged specimens which were immersed in a water bath during the experiments in order to avoid moisture desorption. From a preliminary gravimetric study of the water sorption kinetics, a conditioning procedure was designed which consisted in 20 days of immersion at 20°C. This procedure ensured that the saturation was achieved within the superficial composite layers affected by the macroscopic fatigue damage up to a 10% stiffness loss (i.e. about 350 µm in depth). In addition, no significant hydrolysis of the matrix was found to occur during the selected ageing time. The conditioning procedure thus ensured that the composite superficial layers affected by the flexural fatigue damage were homogeneous in terms of ageing.

RESULTS AND DISCUSSION

Description of the stress corrosion cracking model

The delayed failure of a glass filament within a water aged composite is assumed to involve the sub-critical propagation of pre-existing surface flaws. The crack growth rate, v, can be related to the stress intensity factor, K_I, using a power law expression:

$$v = \frac{da}{dt} = AK_I^{\,n} \qquad (1)$$

where $K_I = Y\sigma\sqrt{a}$ is the mode I stress intensity factor, a is the crack length, σ is the applied stress and Y is a shape factor close to $\sqrt{\pi}$. A and n are two parameters assumed to be constant for a given glass within a given physico-chemical environment.

From the integration of equation (1), the time to failure, t_f, of a glass fibre under a given stress loading, $\sigma(t)$, can be expressed as follows:

$$\int_0^{t_f} \sigma(t)^n \, dt = \frac{2K_{IC}^{\,2-n}}{AY^2(n-2)}\sigma_i^{n-2} \qquad (2)$$

where σ_i is the strength of the fibre in an inert environment. For sufficiently high values of t_f (i.e. $t_f >> 1/\nu$), the integral term is approximately proportional to t_f. Equation 2 thus becomes:

$$\lambda t_f = \frac{2K_{IC}^{2-n}}{AY^2(n-2)}\sigma_i^{n-2}\sigma_{max}^{-n} \qquad (3)$$

where σ_{max} is the maximum applied stress and $\lambda \leq 1$ is a numerical constant depending upon $\sigma(t)/\sigma_{max}$ and n.

In the case of a statistical population of fibres surface defects, the distribution of the fibres strength, ε_i, in an inert environment can adequately be described by means of a Weibull [9] expression:

$$P_s(\varepsilon_i) = \exp\left[-\left(\varepsilon_i/\varepsilon_o\right)^m\right] \qquad (4)$$

where m and ε_0 are the respectively the Weibull modulus and a scaling factor taking into account the gage length of the fibres.
By substituting equation (3) into this Weibull expression, one can obtain the lifetime distribution of a statistical population of fibres under monotonic or cyclic loading:

$$P_s(t) = \exp\left[-\lambda^{m/n-2} t^{m/n-2} \varepsilon_{max}^{nm/n-2} C\right] \qquad (5)$$

where C is a constant scaling factor and ε_{max} is the maximum applied strain
In a previous study [8], we have demonstrated that, during the early stage of damage development, the macroscopic stiffness loss of the flexural beam is linearly related to the measured density of broken fibres within an elementary composite volume located on the tensile side of the specimen. Moreover, it was found that, under a static fatigue loading (i.e. R=1), the fibres fracture processes within this elementary volume obeyed to equation (5). Accordingly, the relative stiffness loss, $S(t)/S_0$, can be described by means of an expression derived from equation (5):

$$\frac{S(t)}{S_0} \approx \exp\left[-t^{m/n-2} \varepsilon^{mn/n-2} \lambda^{m/n-2} K\right]. \qquad (6)$$

where S_0 and $S(t)$ are respectively the initial stiffness and the stiffness at time t. K is constant scaling factor.
The S.C.C. nature of the dependency of the stiffness loss upon time, maximum applied strain and loading ratio can been verified by plotting the experimental stiffness loss curves in a Weibull representation (i.e. $\log(\ln(S/S_0))=f(\log(t))$. In reference [8], the corresponding plots were found to be linear and their slopes (~m/n-2) were independent on the strain level, according to the theory. The prediction of the effects of the loading ratio was, however, found to be critically dependent upon the determination of the corresponding activation parameter, $m/(n-2)\log(\lambda)$ (cf equation (5)).
The application of the stress corrosion model thus relies upon an accurate determination of the values of the microscopic parameters, m and n, which characterise respectively the statistical population of fibres defects and their sub-critical crack growth rates. Although these parameters can be determined from in situ microscope observation of the fibres failures [7], an alternative method based upon macroscopic load measurements would be highly desirable. Such an approach is detailed below on the basis of the stiffness loss measurements which were carried out under monotonic and static fatigue conditions.

Determination of the Weibull modulus from monotonic loading experiments

The determination of the Weibull modulus, m, was based upon the analysis of the load-displacement behaviour of the non-aged composite beams under monotonic loading. Figure 1 shows an example of the composite response in a load-displacement diagram and in a Weibull plot. In the latter case, the logarithm of the reverse of the relative stiffness loss, $\ln(S/S_0)$, has been represented as a function the strain, ε, in a Log-Log plot. The observed linear relationship shows that the stiffness loss can be adequately described using the following Weibull expression:

$$S/S_0 \approx \exp\left[-\left(\frac{\varepsilon}{\varepsilon^*}\right)^m\right]. \tag{7}$$

where ε^* is a scaling factor.

The Weibull modulus was identified from a linear fit of the experimental data reported in the log-log plot, Fig. 1b. This calculation yield to a value of m=4.3, which is close to the value (m=3.9) determined from in situ observation using similar glass fibres [7].

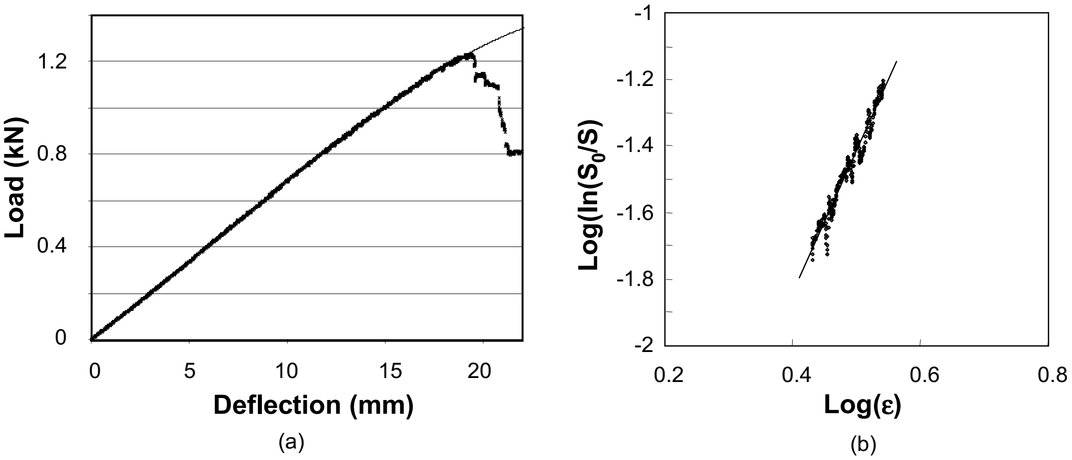

Figure 1 : Three-point bending behaviour of the non-aged composite beams under monotonic loading. (a) Load-displacement response; (b) Relative stiffness loss in a Weibull plot.

Determination of the stress corrosion parameter

According to the determinist nature of the relationship between the number of broken fibres and the relative stiffness loss during the early stages of damage development, the various values of these two parameters can be associated univocally. If it is assumed that the populations of fibres within two different composite specimens are statistically representative of the same distribution of flaws, the strength and lifetime data obtained during a monotonic and a static fatigue experiment can be ranked in increasing order in order to generate a time to failure *vs.* strength curve. As it will be detailed below, the initial crack velocity of the flaws can be derived from a derivation of this curve. The basis for this calculation is the expression of the time to failure as a function of the crack velocity :

$$t_f = \int dt = \int da/v \qquad (8)$$

By taking into account that $K_I = Y\sigma_{app}\sqrt{a}$, equation (8) can be rewritten as follows:

$$t_f = \int_0^{tf} dt = \frac{2}{\sigma_{app}^2 Y^2} \int_{KI0}^{KIC} \frac{K_I dK_I}{v(K_I)} \qquad (9)$$

Which gives by differentiation with respect to K_{I0}:

$$\frac{dt_f}{dK_{I0}} = \frac{-2K_{I0}}{\sigma_{app}^2 Y^2 v(K_{I0})} \qquad (10)$$

It is noted that $K_{I0} = Y\sigma_{app}\sqrt{a_0}$ and $K_{Ic} = Y\sigma_i\sqrt{a_0}$, equation (10) can be rewritten in the following form:

$$v(K_{I0}) = \frac{2K_{IC}^2}{\varepsilon_i^2 EY^2} \frac{d\varepsilon_i}{dt_f}. \qquad (11)$$

which demonstrates that the differentiation of the strength versus lifetime relationship can be used to generate a v(K) curve. In this study, the strength data, ε_i, corresponding to a given experimental lifetime, t_f, have been generated using an expression derived from equation (7):

$$\varepsilon_i = \varepsilon^* \left[\ln\left(1 - S_0 / S(t)\right)\right]^{1/m}. \qquad (9)$$

where $S(t)/S_0$ is the relative stiffness loss measured at the time t under a static fatigue condition. The resulting crack-propagation curve is presented in Figure 2.

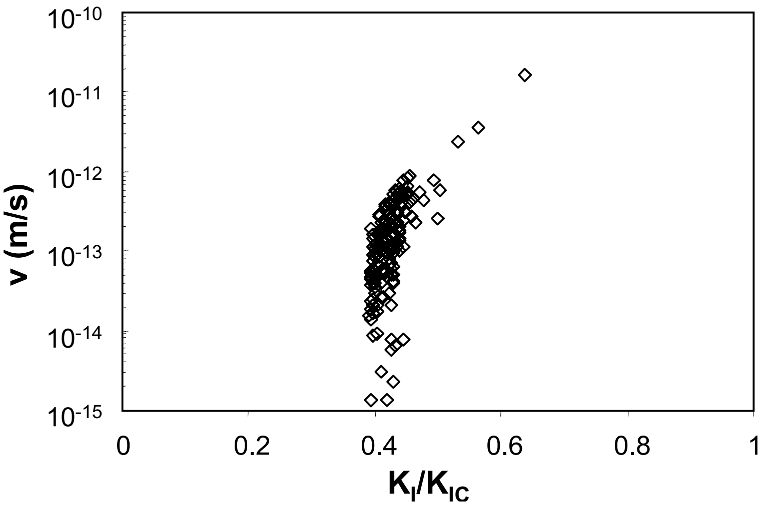

Figure 2: Calculated sub-critical crack growth rates within the glass fibres as a function of the stress intensity factor (Data generated from a static fatigue experiment at ε_{max}=1.2 %)

It exhibits the classical features of the v(K) curves established using bulk glasses, namely a power law dependency of the crack velocity above a non-propagation threshold. From these results, the value of the static fatigue parameter, n, was found to be equal to 14, which is a value consistent with previous reported results for bulk glasses [10].

Prediction of the cyclic fatigue behaviour

From the knowledge of the Weibull modulus and the static fatigue parameter, it was attempted to predict the stiffness loss of the flexural beams under cyclic fatigue conditions at various frequencies and strain ratios. For each set of experimental condition, the value of the shift factor, λ, (cf equation (3)) was calculated from the numerical integration of the following expression:

$$\lambda = \frac{1}{t}\int_0^t \left(\frac{\varepsilon(t)}{\varepsilon_{max}} \right)^n dt \, . \qquad (10)$$

The experimental data obtained under different cyclic fatigue conditions have been compared to the theoretical prediction. For a sinusoidal loading with a strain ratio of 0.7 (Fig.3), a good agreement was obtained whatever the value of frequency within the range 0.5-5Hz.

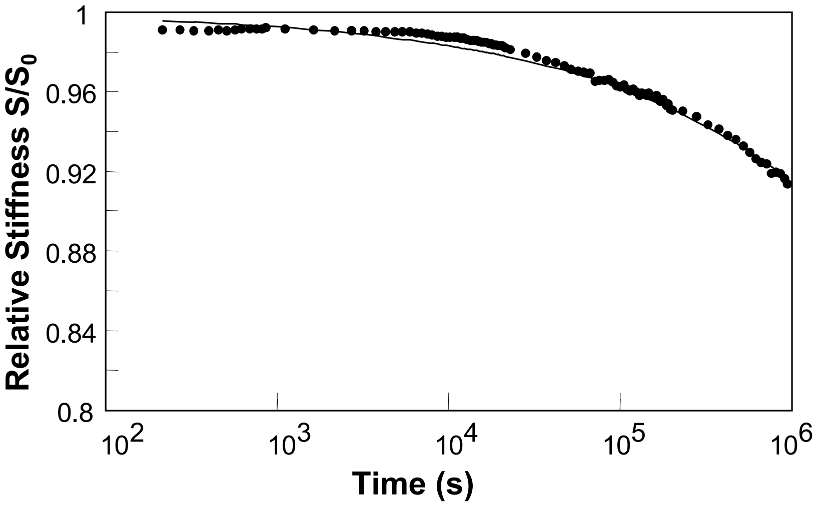

Figure 3: Relative stiffness loss of the aged composite as a function of time under a cyclic fatigue condition. ε_{max}=1.4%; f= 0.5 Hz; R=0.7. Solid line : theoretical prediction.

For a sinusoidal loading under a strain ratio of 0.3 (Fig. 4), the agreement was found to be acceptable at low frequencies (1Hz), but higher loading frequencies (9Hz) resulted in a significant deviation from the theoretical prediction. In this later case, the observed acceleration of the stiffness loss with frequency could be due to degradation processes within the matrix or the interphase, which are not taken into account in the S.C.C. model.

Provided that the strain ratio is not to low and that the frequency is not to high, these results demonstrate that the S.C.C. model can provide a reliable estimate of the initial stiffness loss under constant cyclic fatigue conditions. In real applications, the material is, however, submitted to more complex fatigue loading where the frequency, the maximum applied strain and the strain ratio can vary during the lifetime of the structure. In order to test the ability of the model to take into account such a complex loading, a specific fatigue procedure based upon stacking block sequences has been designed. The latter consisted in the repetition of three successive sinusoidal loading sequences which were characterised by different frequencies, strain ratios and maximum applied strains. The sequences were defined as follows:

- Sequence 1 : 40 seconds, ε_{max}=1.4 %, R=0.7, f=1 Hz
- Sequence 2 : 40 seconds, ε_{max}=1.2 %, R=0.9, f=3 Hz
- Sequence 3 : 20 seconds, ε_{max}=1.9 %, R=0.8, f=5 Hz

The theoretical fatigue behaviour under this complex loading was derived from equation (3), where the parameter, λ, was estimated by integration of equation (10) over the complete loading sequence. The comparison of the experimental and theoretical stiffness loss data (Fig.5) shows that the S.C.C. model can satisfactorily integrate the effects of stacking block sequences, provided that within each block the loading conditions ensure that a S.C.C. behaviour is obeyed.

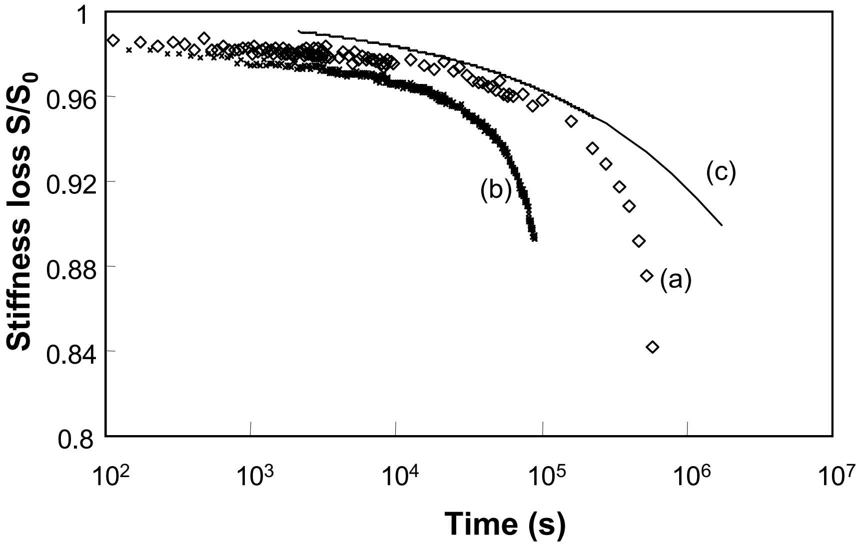

Figure 4: Relative stiffness loss of the aged composite as a function of time under a cyclic fatigue condition. ε_{max}=1.4%; R=0.3. (a) f=1 Hz; (b) f=9 Hz; (c) theoretical prediction.

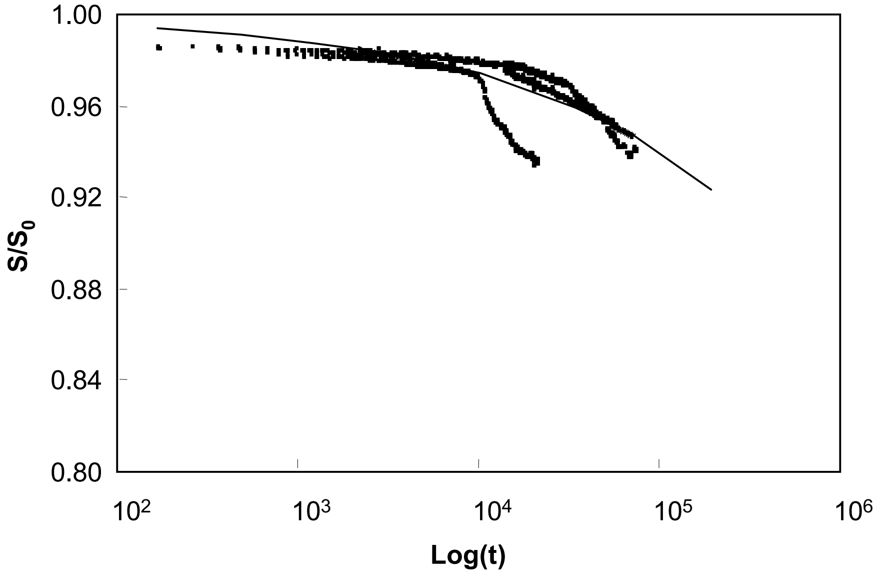

Figure 4: Relative stiffness loss of the aged composite under stacking block fatigue sequences. Solid line : theoretical predictions.

CONCLUSION

A theoretical S.C.C. model has been developed in order to predict the stiffness losses of aged unidirectional glass/epoxy beams under a flexural fatigue loading. The application of this model was restricted to the initial stages of the fatigue life, i.e. when the damage mostly consists in the accumulation, at the microscopic level, of delayed fibres failures under the combined action of stress and moisture. It was demonstrated that a monotonic test to failure and a static fatigue test were sufficient to provide consistent values of the parameters which characterise the statistical distribution of fibres strength (i.e. the Weibull modulus, m) and the dependence of sub-critical crack-velocity upon the stress intensity factor (i.e. the stress corrosion factor, n). Using these data, the model was validated within the context of the prediction of the stiffness loss of the aged flexural beams under cyclic fatigue loading, provided that the frequency was not to high and the strain ratio was not to low. These restrictions were found to be necessary in order to ensure that the evolving contribution of the matrix and the interface to delayed fibres failures are minimised. Within this frame, it was also demonstrated that the model can provide a acceptable prediction of the stiffness loss under more complex loading involving stacking block sequence. This latter result confirms the S.C.C. nature of the fatigue behaviour of the aged composites, in the sense that the stiffness loss is mostly governed by the time spent at a given strain level, independent on the strain rate.

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