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DESIGN PROCEDURES OF COMPOSITE STRUCTURES VIA FUZZY SET APPROACH

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SUMARY: The variability of properties in composite materials and structures is an origin of a scatter in failure loads. In the present paper we propose a non-stochastic description of uncertainity effects that allows us to evaluate lower and upper bounds of failure loads understood in the sense of buckling, FPF and delaminations. The vertex method is introduced and discussed herein to evaluate failure loads in a numerical manner. Then, we demonstrate the results of fatigue tension and four-pont bending tests being the introduction for buliding Fuzzy Knowledge Base. Some experimental results dealing with the acoustic emission analysis of fatigue tests are also shown here.

KEYWORDS: Buckling, Delaminations, FPF, Fuzzy Sets, Fatigue Tests, Acoustic Emission, Laminated Plates

ITRODUCTION

The use of fibre-reinforced composite materials in modern engineering structural design has become a common practice. However, since more design variables typically exist when composite materials are employed and the manufacturing processes for producing composites are more complex, more variability can exist in a design procedures with composites compared to conventional materials. Thus, the variability of properties that occurs in composite structures leads directly to a random field of variables describing constructions. Therefore, there is a fundamental and still open question: how many and which of the scatter parameters can (or should) be incorporated in the design process and how can we manage to take into account the existing variability of parameters. Thus, the majority of available references in literature discussing design problems of composite structures is devoted to the analysis conducted in a pure deterministic way since the classical statistical analysis cannot solve uniquely the problem of multidimensional statistical distributions.
The objective of the present paper is twofold:

- to demonstrate the applicability of the fuzzy set theory to the analysis of the limit load carrying capacity of structures in the conjunction with the FE computations – the vertex method (Ref [1])
- to present various experimental results dealing with the fatigue behaviour of composie structures in order to demonstrate the necessity of using fuzzy set description and to mention also the necessity of building rather Fuzzy Knowledge Base describing experiental data than to evaluate statistical distributions.

The work is a continuation and extension of our investigations in this area – see Refs [2-5]. Different mechanical and geometrical properties describing the analysed structure are assumed to be fuzzy. In general, we use the triangular membership functions.

The presented examples illustrate the advantages and disadvantages of the proposed fuzzy set approach to design problems of composite structures. The proposed methodology can be easily extended to the FE examination of various failure modes (e.g. delaminations etc.) and can be used to different structures subjected to complicated loading and boundary conditions.

THE FUZZY SET THEORY AND COMPOSITE MATERIALS

Data are generally described by defining subsets of the given space. Let consider a certain space for instance of all integers IN. In the space IN the feature “less than 10” is characterized by the set:

$$A = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9 \} \subset \mathrm{IN} \tag{1}$$

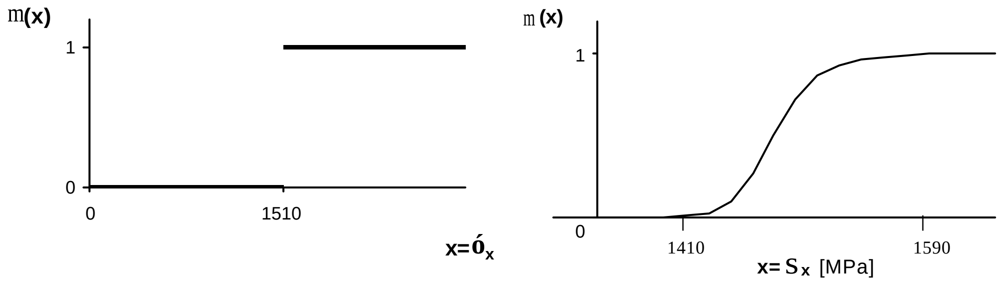
Another representation of the data “integer less than 10” is the definition of the characteristic function Π_A in the following way:

$$\Pi_A : \mathrm{IN} \rightarrow \{0,1\}$$
$$\Pi_A(\eta) = \begin{cases} 1, & \text{if less than } 10 \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

The function Π_A yields the value 1 for each element of the space IN that belongs to the set A and the value 0 for each element that does not. The above representation is commonly called as a crisp set. However, this concept can not be used directly as we intend to characterise the typical property for composite materials as e. g.: the failure of CFRP under tension occurs as the tensile stress ó$_x$ equal to the ultimate value 1510 [MPa] . The characteristic function of this set is depicted in Fig. 1a . A problem arises if the linguistic term “the failure under tension” has to be described. It is well-known that from the micromechanical point of view the failure starts from microcracks in the matrix for the stress values much lower than 1510 [MPa] . In addition, the value 1510 is usually an average value characterizing rather a scatter of random values of macrocracks appearing at the stress level 1510 . Therefore, for some specimens one can observe the final (macroscale) failure as ó$_x$ is equal to 1590 or to 1410 [MPa] . A possible solution to this problem is to generalize the definition of the characteristic function in a way that it yields values from the interval [0, 1] and not just the two values of the set {0, 1} . This leads to the notion of a fuzzy set.
A fuzzy set μ of X is a function that maps the space X into the unit interval, i. e.:

$$\mu : X \rightarrow [\ 0, 1\ ]\ . \tag{3}$$

The value μ(x) denotes the membership function of x to the fuzzy set μ . Fig. 1b shows (subjectively defined) a membership function of the fuzzy set μ describing the linguistic meaning of the term “the failure under tension”. In this way one can describe various properties, e g. strains , tensile (compressive) ultimate stresses etc.. The membership function μ(x) is called as the vertical representation. The use of fuzzy sets to formally represent vague data is often done in an intuitive way because in many applications there is no model that provides a clear interpretation of the membership degrees, although we want or we try to base on various experimental data.



a) a crisp set b) a fuzzy set

Fig. 1 The representation of the term “the failure occurs ..”.

Of course, there are different possibilities to determine and represent a membership functions characterizing a fuzzy set. If the subspace C contains only a finite member of elements, a fuzzy set μ of X will be defined by specifying for each element $x \in X$ its membership degree $\mu(x)$. If the number of elements is very large or a continuum is chosen for X then $\mu(x)$ can be better defined by a function that can use parameters which are adapted to the actual modelling problem. For instance, as we want to represent the term "Young's modulus is equal to 200 GPa" in the sense of a fuzzy set having a finite amount of the experimental data we can select one of different representations given in Figure 2.

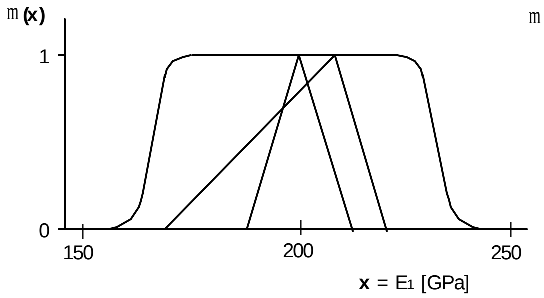


Fig. 2 Various fuzzy representations of the term "Young's modulus is equal to 200 GPa"

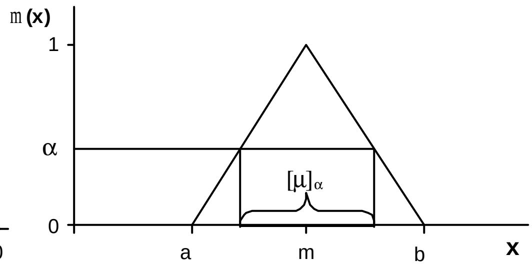


Fig. 3 Definition of α-cuts

There was presented one of possible fuzzy set representations. Another approach is the so-called horizontal representation of fuzzy sets. This is introduced by using their α-cuts instead of the membership functions $\mu(x)$ which are called vertical representation.
Let $\mu \in F(x)$ and $\alpha \in [0, 1]$. The set is called the α-cuts of μ:

$$[\mu]_\alpha = \{ x \in X \mid \mu(x) \geq \alpha \} \tag{4}$$

Let μ be the triangular function on IR given in Fig. 3 . The α-cuts of μ are in this case defined as follows:

$$[\mu]_\alpha = \begin{cases} [a + \alpha \cdot (m - a), b - a \cdot (b - m)] & \text{if } 0 < \alpha \leq 1 \\ \mathrm{IR} & \text{if } \alpha = 0 \end{cases} \tag{5}$$

THE VERTEX METHOD – NUMERICAL ANALYSIS

Let us introduce N fuzzy parameters describing material or geometric parameters of a composite structure considered. These fuzzy parameters are denoted by $x_1, x_2, \ldots, x_N$. The membership functions are discretised using several α-cuts – eqn (4) . Considering the left and right end points of the α-cuts intervals $[\mu]_\alpha$ (see Fig. 3) for all fuzzy parameters one can find the total number of the combinations $N_{c/\alpha}$ per α-cut in the following form:

$$N_{c/\alpha} = \begin{cases} 2^N & \text{for } 0 \leq \alpha < 1 \\ 1 & \text{for } \alpha = 1 \end{cases} \tag{6}$$

An output response denoted by p is an unknown function of the input fuzzy parameters x_i (i=1, 2, …, N), so that:

$$p = f\,(x_1, \ldots, x_N) \tag{7}$$

Using the α-cut concept combined with the binary representation (6) of the fuzzy parameters x_i (i=1, 2, …, N) the relation (7) can be rewritten in the abbreviated form:

$$p = f\,(C_{\alpha,j}),\;\; j = 1, 2, \ldots, N_{c/\alpha} \tag{8}$$

Since the output response p as a function of fuzzy parameters is a fuzzy set the corresponding interval in p is obtained from the relation (7):

$$[p_\alpha^L, p_\alpha^R] = [\min_{\lambda,j} f\,(C_{\lambda,j}), \max_{\lambda,j} f\,(C_{\lambda,j});\;\; \lambda \ge \alpha,\;\; j = 1, 2, \ldots, N_{c/\alpha} \tag{9}$$

As it may be seen the relation (9) allows to obtain a scatter of the output parameters an then to built the appropriate probability distributions and reliability functions by sweep of α-cut at different possibility levels.
In order to conduct the computations and to evaluate the upper and lower bounds of the output response (9) it is necessary to determine the deterministic method of the definition of the function f given in eqn (7). It can be defined in a pure analytical way or alternatively in a pure numerical way. As it may be noticed the vertex method resembles here the Monte Carlo simulation method where the output response has also the deterministic, so that unique form.
The function f existing in eqn (7) may describe an arbitrary failure criterion for composites, e. g. buckling, delamination, the first-ply-failure etc., whereas the symbol p denotes the corresponding value of the failure load.

FUZZY SET ANALYSIS OF LIMIT LOAD CARRYING CAPACITY

For composite structures the concept of the limit load carrying capacity (LLCC) have been introduced by Muc *et al.* [6]. Briefly speaking the fundamental idea of the LLCC is based on the evaluation of the lower bound envelopes of different failure loads corresponding to the analysed composite structure subjected to the prescribed loading and boundary condition and having uniquely defined laminate topology. As different failure loads we understand failure loads corresponding to various failure modes encountered in the analysis of composite structures, i.e. delaminations, matrix cracking (the Fisrst-Ply-Failure – FPF), gloabl or local buckling fibres debonding etc. In details, in the present work three types of failure modes are considered: delaminations, FPF and global buckling of structures. The results of the analysis are illustrated on the example of compressed rectangular plate.

Delaminations

Let us consider buckling problem of a rectangular plate having an ellipsoidal crack located at the plate centre. The total crack area is denoted by the symbol A. It is assumed that the crack area may vary dependly on the loading (cyclic) conditions. Analysis of delamination initiation is carried out either by mechanics of materials approach [7,8] or a fracture mechanics approach [9-11]. Since the delamination growth can be attributed to a mixed mode of fracture, the criterion for the initiation of the delamination growth was selected as follows [12]:

$$RB = g_1^\delta + g_2^\gamma, \qquad g_1 = \frac{G_I}{G_{Ic}}, \quad g_2 = \frac{G_{II}}{G_{IIc}} \tag{10}$$

where RB is the failure index, G_{Ic} and G_{IIc} are the critical strain energy release rates corresponding to mode I and II fracture, respectively. ä and ã are coefficients – it was found that ä=ã=1 provides the best fit to the experimental results. A delamination would start to propagate when RB 1.
A comparison of fatigue tests with different stress ratios R indicates that fatigue damage manifests itself most severly at R=-1 (see e.g. Ref [13]) and this value was taken in the analysis. In addition, it has been observed that under the stress ratio R=-1 at comparatively low stress amplitydes matrix cracks develop rather early , and then at increasing cycle numbers precipitate localized delaminations. In order to take into account ply degradation due to matrix cracking it is assumed the simplest degradation model of the material properties

describing plies oriented at 90^0, i.e.: E_1=âE_1 , E_2 = 0, G_{44} = 0, where â is a degradation factor. The more complicated model considering fuzzy set approach to the analysis of intralaminar crack propagation is discussed in Ref. [4] but now we restrict our investigations to the interlaminar crack behaviour only.

In the numerical analysis five parameters are considered to be fuzzy, namely, the degradation factor â and four parameters describing the failure criterion (eqn (10), i.e. ã, ä and G_{Ic}, G_{IIc} . Russel and Street [14] measured fracture toughness of common composites and obtained the scatter in experimental data equal to ±5 % . In our analysis the variability (understood as the fuzziness) of these parameters is taken to be equal to ±10 %. The identical variability is assumed for the rest of parameters, i.e. â, ã and ä. Their nominal values corresponding to á=1 (Fig. 3) are taken as follows: â = 0.6, ã =1.0, ä =1.0.

The vertex of the triangular membership functions is always located at the middle of the intervals describing variations of the parameters. It should be emphasized that the form of the membership function is a hypothesis and the triangle plotted in Fig.3 can be replaced by other function satisfying experimental conditions and results or simply the assumptions of the theoretical modelling. It is assumed that the analysed herein plate is made of the carbon/epoxy and their static material properties are following: E_1 = 134 GPa, E_2= E_3 = 10.2 GPa, G_{12}=G_{13}= 5.52 GPa, G_{23} = 3.43 GPa, í $_{12}$=í $_{13}$ = 0.3, í $_{23}$ = 0.49.

The different binary combinations $C_{á,j}$ formed by failure coefficients ã, ä and G_{Ic}, G_{IIc} result in different failure envelopes. For the nominal values of the above parameters the dimensionless failure curve is a straight line (á = 1). The failure envelopes being a lower and an upper bounds at á = 0 are not straight lines and in addition they are not symmetric with the respect to the nominal valuea at á = 1. The presented example of the sensitivity study demonstrates the synergistic effects of different fuzzy parameters. Although relatively small ranges of the variability in the input failure coefficients were selected ,± 10%, the resulting variability in the output failure envelopes is quite large. However, the symmetry or not of the upper and the lower bounds at á = 0.0 with respect to the nominal curve (á = 1.0) depends completely on the type of the fuzzy parameter. As it may be seen in Fig.4 the abovementioned curves describing lower and upper bounds are symmetric if we analyse the variability (fuzziness) of the degradation factor â parameter. It should be emphasized that such a behaviour is possible only as the buckling mode is identical for the whole ranges of parameters considered in the numerical analysis. Since it was investigated that the buckling mode for the compressed plate had the symmetric shape it was possible to obtain symmetry in the plot drawn in Fig.4.

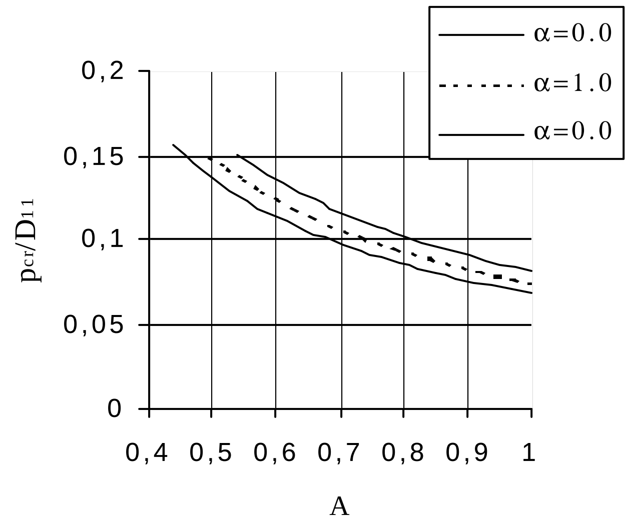


Fig. 4 Dimensionless buckling loadsof a rectangular plate with ellipsoidal crack

First-ply-failure

The classical FPF envelopes (e.g. the Tsai-Wu criterion) are described with the aid of variety of material constants characterizing different modes of failures (tension, compression etc.).

Those values may be treated as fuzzy ones and in this way the fuzzy set theory allows us to buid the lower and upper bounds of the failure envelopes for each fibre orientations of plies in the laminate. Assuming that five strength constants are fuzzy parameters such envelopes have been built and the plot is presented in Fig. 5. For compressive deformations the uncertainity of failure locations (at plies having fibres oriented at 0^0 or 45^0) is the characteristic feature of the resulting curves

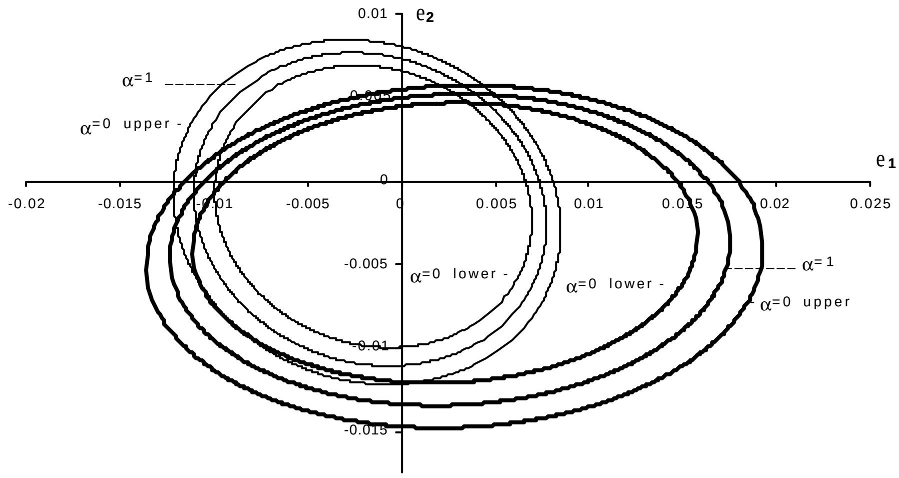


Fig. 5 Upper and lower bounds of FPF envelopes – the Tsai-Wu criterion

(—0^0, —45^0)

Global buckling

To evaluate global buckling loads numerical studies are performed on compressed square plate. The numerical FE analysis is carried out in the elastic geometrically linear range only, with the use of the four noded quadrilateral shell elements (NKTP 32) employing the first order transverse shear deformation plate/shell theory. The geometric and material characteristics are following: E_x = 280 GPa, E_y = 12 GPa, G_{xy} = 7GPa, G_{xz} = 0.6G_{xy}, G_{yz} = G_{xy}, ν_{xy} = 0.28, t/a = 0.1, where t is a plate thickness and a is a plate length, respectively. Four parameters have been considered as fuzzy variables: the total thickness t, the Young moduli E_x, E_y and the Kirchhoff modulus G_{xy}.

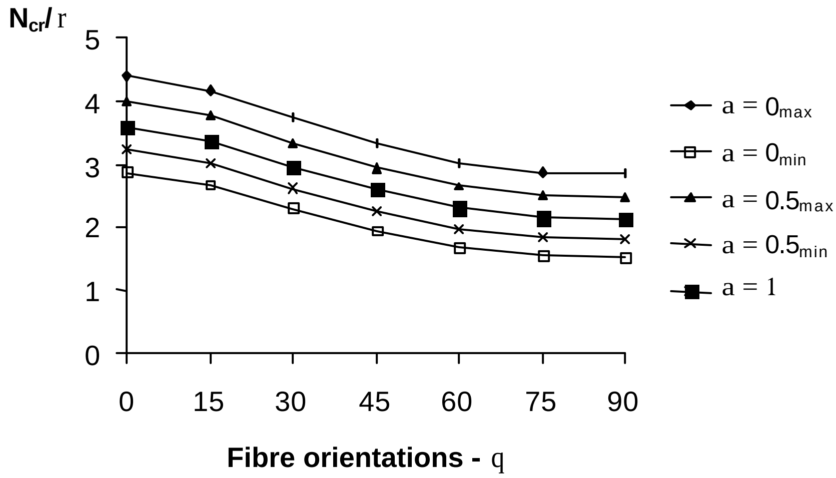


Fig. 6 Distributions of buckling loads for compressed angle-ply plates

In the present study it is assumed that the membership functions of the fuzzy parameters are triangular as shown schematically in Fig. 3 (see also eqn (5)) where:

$$m = \frac{a+b}{2}, \quad a = 0.9 \cdot m \quad b = 1.1 \cdot m \tag{11}$$

and m is an average value for each of the fuzzy parameters, for instance it can be evaluated from the experimental data. As it may be seen from the relation (11) the variability (fuzziness) is taken to be equal ±10%, which falls within typical ranges of scatter in experimental data for static tests.

The distributions of buckling pressures versus the angle of fibre orientations at α=0, 0.5 and 1.0 are plotted in Fig. 6. The upper and lower bounds (α=0 and 0.5) of the curves drawn for α=1.0 are not symmetric. The interval (9) is strongly dependent on the fibre orientations as well as on the wavenumber in buckling.

FUZZY KNOWLEDGE BASE - EXPERIMENTAL RESULTS

A great number of experiments both for the static and fatigue loading have been conducted in order: (1) to demonstrate the scatter and variability of the results for composite materials, (2) to build fuzzy knowledge base with the use of experimental data. Below some of them are reported and in general they exhibit different results in the sense of the fatigue life value. One material have been tested woven roving glass/epoxy resin. The fibres have been oriented at 0^0 and 45^0.

Tension of a rectangular specimen with a circular hole

Figure 7 demonstrates geometry of specimens used in the analysis. The results are presented in Table 1 and in Fig.8.

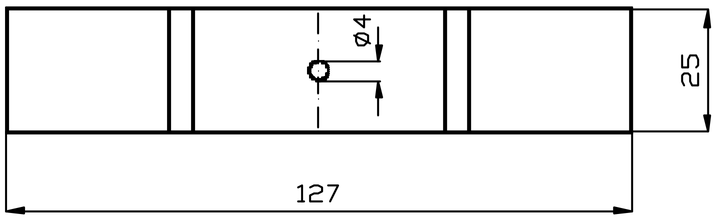


Fig. 7 Geometry of a rectangular specimen with a circular hole

0°	Number	1	2	3	4	
	Force – the mean value [kN]	6.8	6.0	6.0	6.0	
	Amplitude of the force [kN]	±1	±1	±1	±1	
	Stress [MPa]	105.46	98.45	98.04	106.67	
	Fatigue life	352	3025	11012	17617	
45°	Number	3	4	5	6	7
	Force – the mean value [kN]	3.0	3.3	3.3	3.3	3.3
	Amplitude of the force [kN]	±0.4	±0.5	±0.5	±0.5	±0.5
	Stress [MPa]	58.54	54.31	63.36	73.53	65.42
	Fatigue life	100000	56950	16285	24	1397

Table1 Results and geometry of specimens (average: length = 127 mm, width = 25.5 mm, thickness =2.35 nmm)

a) b)

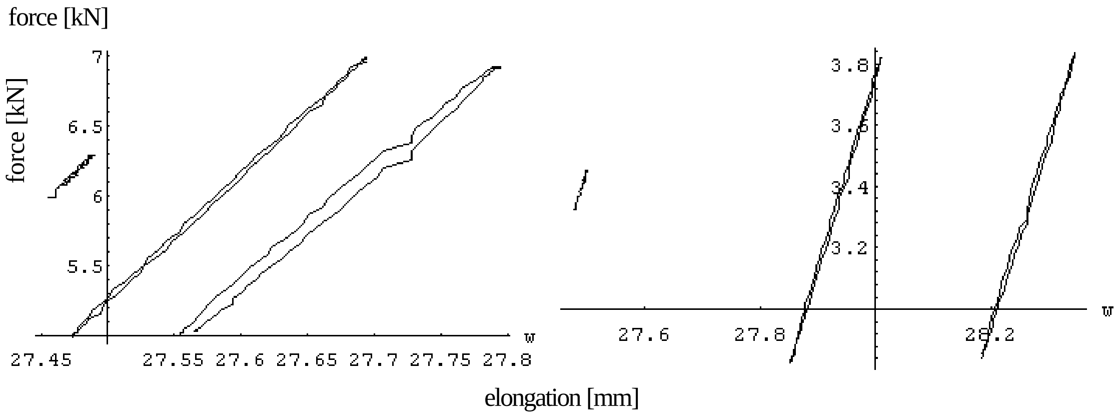


Fig. 8 The ó-å fatgue characteristics:a) specimen No.4–angle 0°,b) specimen No. 4– angle 45°

Four-point bending of a rectangular specimen

Next experimental data have been obtained from four-point bending fatigue tests. The results are presented in Table 2 as well as in Fig.9. The results demonstrate the appearance of the combined failure mode and and due to that effect even bigger than previously scatter of the fatigue lives.

Number	6	7	8	9	10	11	12
Force [N]	700	600	550	550	550	550	550
Amplitude of force [kN]	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Deflection [mm]	9.29	7.95	8.06	9.45	9.36	7.92	7.50
Fatigue life	1489	7717	13887	9330	4610	17225	5147

Table 2 Experimental results and geometry of rectangular specimens (average: length =140 mm, width =13.05 mm, thickness = 5.58 mm)

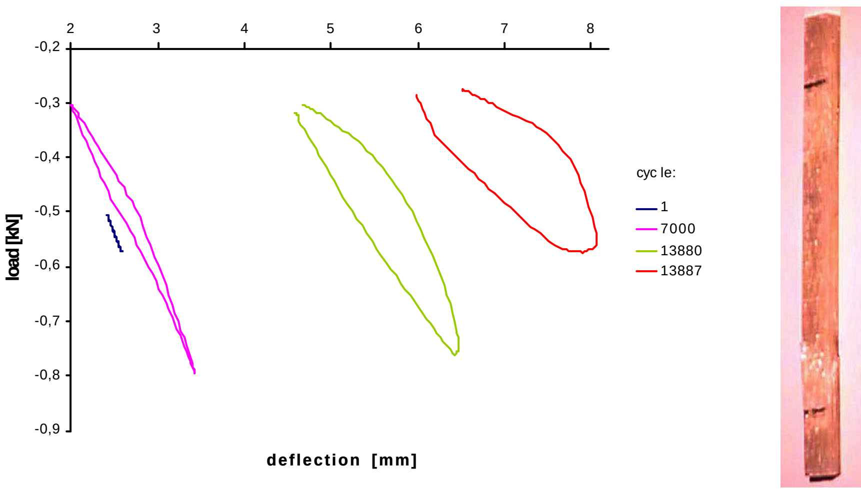


Fig. 9 The characteristics load – deflection of the specimen No. 8 and the photograph of the specimen after failure

Acoustic emission

In order to examine failure mechanisms under fatigue loading the acoustic emission method have been also used in the analysis of the specimens subjected to four-point bending. Acoustic emission refers to the release of sound energy at the microscopic damage sites created by the application of external loads. The sound energy released by the evolution of various types of damage travels through the material as transient elastic stress waves. The surface displacements caused by the stress waves can be transformed into electric signals using piezoelectric transducers placed on the top surfaces. If necessary the acoustic emission signals received by the transducer are first amplified by a preamplifier mounted close to the transducer. The signals from the preamplifier are filtered and then further amplified before being analyzed. Filtering is required to eliminate extraneous mechanical and electrical noise. The acoustic emissions can be monitored, recorded and processed to locate and map the critical defects in the material.

Acoustic emission monitoring is carried out using oscilloscope. The acoustic waveforms are detected by a thin piezoelectric transducer which has a wide band frequency (100 kHz – 1 MHz) placed and coupled on the specimens with silicon oil. A high-pass filter with 90 kHz is used to obtain the amplitude distribution which are recorded during the test.

The results are presented in Figs. 10-12. They show evidently the gradual matrix failure and it is associated with the delamination fatigue failure.

Due to the lack of the space the experimental results will be presented in the separate paper together with the broader discussion of the effects and results.

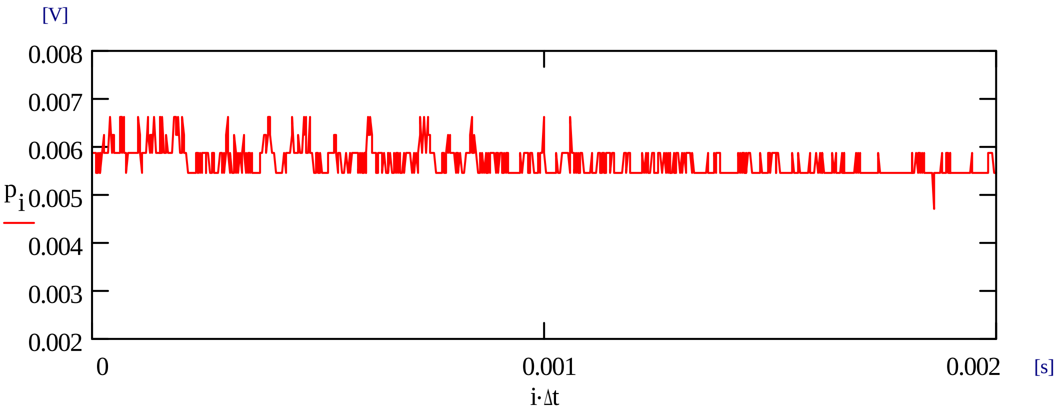


Fig. 10 Signal associated with 7000 cycles of specimen No. 8

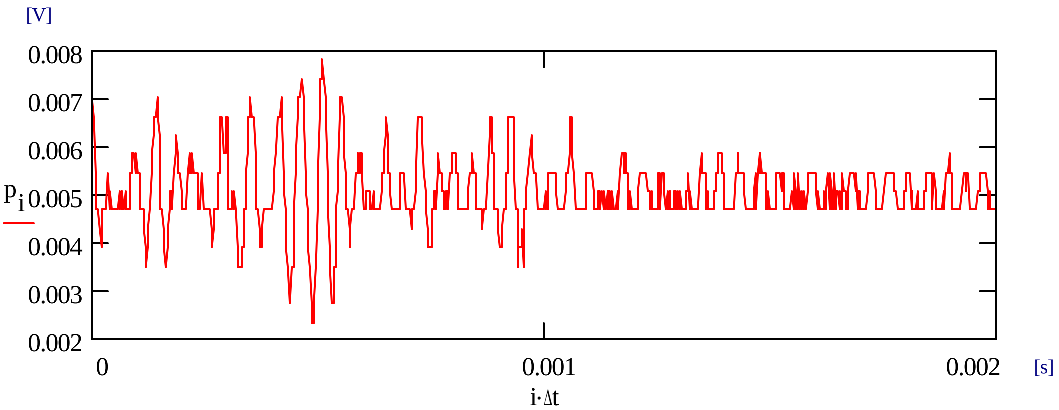
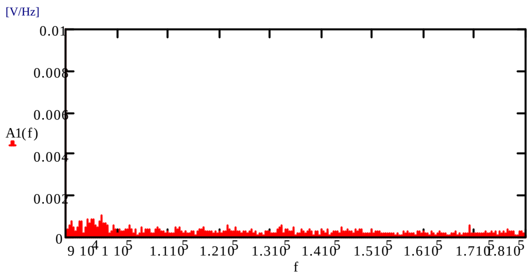
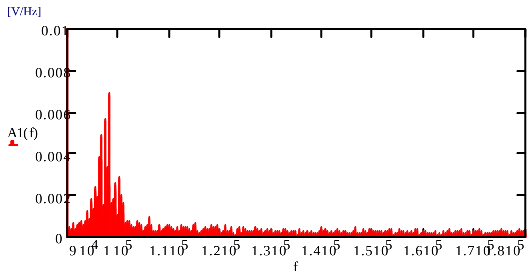


Fig. 11 Signal associated with failure of specimen No. 8



a)



b)

Fig. 12 Frequency plots – specimen No. 8 : a) N= 7000 cycles, b) prior to failure

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