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PROBABILISTIC ALGORITHMS IN OPTIMISATION PROBLEMS FOR COMPOSITE PLATES AND SHELLS

A.Muc & W. Gurba

Institute of Mechanics & Machine Design, Cracow University of Technology, Kraków, Poland

SUMMARY: In the present various aspects of plated/shell composite structures optimization conducted in conjunction with the FE modelling are discussed. In general, two optimization algorithms are employed herein which belong to the class of probabilistic algorithms, i.e. genetic and simulated annealing algorithms. In details, two optimization problems are solved: stacking sequence optimization and material (thickness) optimization under buckling constraints. The presented problems demonstrate evidently the generality and effectiveness of the proposed method of the analysis.

KEYWORDS: Optimization, Probabilistic Algorithms, FE Modelling, Laminated Plates/ Shells, Buckling

INTRODUCTION

In mechanical, aerospace and other branch of engineering composite materials are increasingly used due to their excellent weight saving and the ease of tailoring. In spite of tremendous progress in analytical capability to analyze the behaviour of composite materials and structures there is a lack of design models which may allow efficient and on the other hand sufficiently accurate tailoring of their specific (anisotropic) properties to specific requirements for structural components. Therefore, the optimum design of composite structures has been a subject of research for many years.
However, in order to deal with the optimization problems of laminated composite thinwalled structures in the structural design problems, at the beginning the following problems should be correctly formulated and established (see Muc [1]):

1. structural model in view of assumptions and hypotheses valid for the analyzed type of governing 2-D equations,
2. optimization model in the sense of FE or other modelling of design structures,
3. optimization algorithms,
4. definitions of design variables,
5. formulations of objective functionals (functions).

It should be emphasized that the majority of the optimization problems have been solved with the use of the classical Love-Kirchhoff (L-K) equations. On the other hand,it is well-known that the higher-ordered shell or plate theories should be employed in the analysis in view of the consistency and the correctness of theoretical results with experimental investigations. The classical theory fails to predict accurately the static and dynamic response, when the structures in question are even moderately thick and /or when they exhibit high anisotropy ratio. In addition, it should be pointed out that the majority of optimization problems have been solved for a very simple boundary and loading conditions and the obtained results cannot be even treated as benchmarks for engineering structures.
In the present paper we intend to discuss possibility of using classical FE codes as the solvers for the external optimization algorithms – see Fig.1. Two types of probablistic algorithms (PA) have been employed herein: genetic algorithms and simulated annealing. The necessity of the

use of PA have been explained for instance in Refs [2,3] and we do not intend to dwell on it. In our opinion the coding of design variables is the most important problem in the pre-

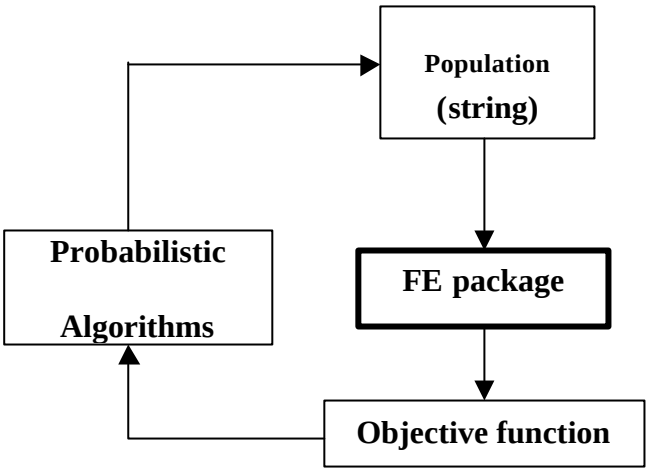

Fig.1 Representation of layout optimization associated with the FE analysis

sent analysis since an optimization procedures should fulfill the assumed generality and flexibility.
In details two different optimization problems have been formulated and solved as discrete (combinatorial) optimization problems:

1. optimal design of the composite topology with different numbers of gens representing fibre orientations, e.g. 0, 45, 90 or 0, 22.5, 45, 67.5, 90 etc.,
2. volume optimization – for a given initial mesh we try to select optimal thickness of individual FE in the mesh.

Different numerical examples dealing with optimization problems for cylindrical shells have been solved to illustrate the effectiveness of the proposed numerical methods.

PROBABILISTIC OPTIMIZATION ALGORITHMS

Genetic algorithms is one of possible probabilistic search techniques. It is based on a simulation of natural evolution - see e.g. Refs [4-6]. However, the effectiveness of the optimization procedure is strongly affected by various factors, such as : a size of a population, the crossover and mutation probabilities and the selection methods. In this way, after one searching procedure of optimal configurations for laminated structures even if we obtain any solution it is impossible to verify whether the solution gives the global optimal solution or not. In addition, it should be emphasized that for multilayered laminated structures various stacking configurations may correspond to the identical values of the stiffness matrix. In order to verify the correctness of the optimal solutions it is necessary to repeat the genetic searching algorithms changing the randomly selected factors mentioned above or to employ completely different probabilistic (or not) optimization scheme.
Another probabilistic search technique is based on the physical phenomenon (the Boltzmann distribution) and it is called as the simulated annealing method (SA). According to that method the acceptance of the string is estimated after the computation of the value exp(-DF/T), where DF is the cost deviation of the objective function F and T is a temperature. The temperature T is a parameter of the optimization problem and at high temperatures all strings in the population are equiprobable. At low temperatures the best configurations are only reachable. A broader discussion of these problems can be found e.g. in Ref. [7].

In the present work three various optimization probailistic algorithms have been adopted in order to verify the effectiveness and the correctness of the found optimal solutions:

1) the classical genetic algorithms having randomly selected one point of the cross-over in the string , whereas the selection procedure is based on the rank method,
2) the classical simulated annealing method (SA),
3) the combined simulated annealing and genetic algorithms; the lack of the appriopriate selection methods is the main disadvantage of the SA procedures, thus it is possible to adopt in all SA procedures various selection rules introduced and verified rather numerically than theoretically in GA; the used herein version of the selection method is the same as mentioned above, i.e. the rank strategy.

It should be mentioned that our experience in the application of the GA and S.A. shows evidently that the second method requires less arbitrarily chosen parameters, and on the other hand it converges almost always to the correct results.

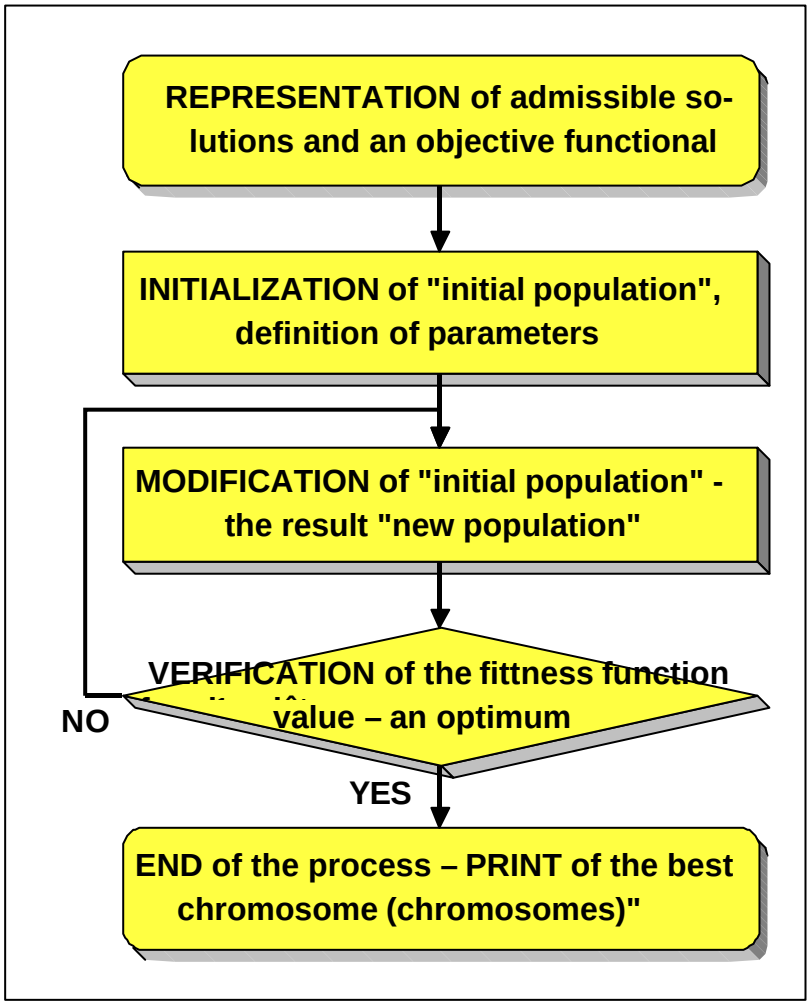

Fig. 2 Flowchart of probabilistic algorithms

Comparing those two methods it is worth to note that the number of iterations does not correspond to the total CPU time since for the SA method a lot of computational operations such as cross-over or mutation is simply eliminated. In our opinion the fundamental advantage of the probabilistic search techniques is based on the starting points in an optimization procedure and then on the proper choice of the selection methods. The

comparison of the results demonstrates that the elimination method of the worst members in the population should be as simple as possible what is equivalent to the reduction of parameters describing the physical (SA) or evolution process (GA).
Both optimization procedures have identical representation exhibited in Fig.2. The difference is described by the term „modification of the initial population" that is connected with the use of the GA or S.A. procedures. However, in our opinion, the effectiveness and the correctness of the optimization procedures is also strictly associated with the proper coding of the design variables. Particularly for composite structures it becomes a crucial point in the unique evaluation of optimal solutions.

CODING OF DESIGN VARIABLES

Stacking sequence optimization

In the stacking sequence optimization each design variable represents the prescribed fibre orientation. As the total number of possible angle variations is denoted by the natural number K > 1 then fibre orientations may be written in the following form:

$$\grave{e}_k = \eth\,(k-1)/(2K) \qquad , k=0,1,...,K \tag{1}$$

Therefore the simplest coding method of fibre orientations is based on the natural number representation, i.e. each natural number k describes the fibre orientation given by the above equation. Thus, the chromosome is defined by a sequence of natural numbers where the total number of terms in the chromosome is equal to the total number of the layers in the laminate, i.e. :

$$\text{Chromosome} = \{1,0,0,2,3,3,1,1\} \tag{2}$$

In the above relation it is assumed that K = 3 and the total number of layers (NLAY) is equal to 8. The introduced above coding procedure for the laminate sequence is identical to the existing in the FE modelling used by the NISA II. Therefore the preparation of the input data for the FE evaluation of the objective function is reduced only to the replacement of one line corresponding to the laminate sequence.
However, the coding method given by eqs (1), (2) is not a general one since it does not take into account other than fibre orientations types of design variables characterizing ply properties, i.e. thicknesses of each individual plies or material properties in the case of hybrid composites. Since the analysis of thicknesses of individual plies as design variables are less interesting we focuse our attention on the material properties only. Let us assume that the laminate is made of three types of layers having different mechanical properties in the sense of Young's moduli, Kirchhoff's moduli and Poisson's ratios and the type of the material is denoted by I, II and III, respectively. Of course, each layer having different material properties may have different fibre orientations described similarly as previously by eq (1). Now, the set of design variables cannot be represented by the chromosome (2) because each gen does not possess an information about mechanical properties. In such a case the chromosome should be modified to the matrix form given below:

$$\text{Chromosome} = \begin{bmatrix} II & 0 & 0 & 0 & 0 & III & 0 & 0 \\ 0 & II & II & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & III & 0 & 0 & I \end{bmatrix} \tag{3}$$

In eq (3) the number of columns corresponds to the total number of layers in the laminate, whereas the number of rows to possible fibre orientations, i.e. k=0 to the first row, and k=3 to

the last one. Let us note that in each of the columns one term only is not equal to zero. The written above example of stacking sequence coding corresponds directly to the representation (2) since K=3 and NLAY=8.

Material optimization

Using the FE approximation to evaluate the objective functional it is possible to change (redefine) various groups of data corresponding to: (i) the existence (or not) of a finite element (or groups of them) in the initial mesh – the reduction of the volume, (ii) the variations of thicknesses at nodes of a finite element etc. – it may also change the total volume (an increase or a decrease of it). Thus, having a 2-D structure representig plate/shell each finite element is described as the term of the 2-D matrix:

$$a(i,j) \tag{4}$$

Let us assume that the thickness of the individual FE is characterized by the natural number k multiplied by a real constant, i.e.:

$$a(i,j) = 0.1 * k(i,j)\ ,\ \ k(i,j)^{o}\ \{1,2,...K\} \tag{5}$$

Both the value of the real multiplier (now it is taken to be equal to 0.1) and the total number of k values in the sequence are completely free. There is also an equivalent representation of the thickness distributions in the given mesh by real numbers. It is assumed in advance that the thickness of the structure can belong to the following interval:

$$t \in [\,a,b\,],\quad a = \{0,0,0,0,0,0\},\quad b = \{1,1,1,1,1,1\} \tag{6}$$

The lower and upper bounds of the interval are expressed by the strings of the 0 and 1 numbers presented in Eq. (6). Using the binary representation the interval [a,b] may be divided into $2^6 - 1$ subintervals. Increasing the number of positions in the string one can divide the initial interval into subintervals with the required density and in this way express the various thicknesses in the mesh. The thicknesses are piecewise constant on the FE used in the analysis.

COMBINATORIAL OPTIMIZATION PROBLEMS

Let us start from the simplest problem of the stacking sequence optimization for mul-tilayered, laminated 2-D structures, such as plates or shells. The total thickness of the 2-D composite structure is equal to t. It is constructed of an arbitrary number N of orthotropic layers of thickness t/N. The laminate is assumed to be symmetric, balanced. As the result only N/4 ply orientations are required to describe the laminate configuration. Thus, our optimization problem is formulated as follows:

$$Max_S\, Min_{m,n}\, \lambda_b(s;m,n) \tag{7}$$

where s denotes the set of design variables, and m,n are the wavenumbers in buckling, λ_b is the objective function, computed directly in the FE analysis and denotes a buckling load parameter . The symbol s corresponds to a set being a bit string representation of possible solutions (see eqs (1), (2)) to the optimization problem (7).

Now, using the coding of the node thickness defined by eqs (4), (5) it is possible to represent the toatal volume of the disretized 2-D laminated structure as follows:

$$V = 0.1A\sum_{i=1}^{I}\sum_{j=1}^{J} k(i,j) \tag{8}$$

where A denotes the area of each individual FE (identical for each i,j), and t is a plate or shell thickness.
With the use of the above definition the material optimization problem can be written in the following explicit form:

$$\underset{a(i,j)}{Min} \quad V[a(i,j)] \tag{9}$$

where the pair of coefficients (i,j) characterizes the properties of the ij-th element. In addition, the problem (9) is subjected to different equality (or inequality) constraint conditions. They form is prescribed for the particular optimization problems. Design variables have to be selected among a(i,j) given, discrete variables.
It should be emphasized that the formulated optimization problems (7), (9) have few characteristic features given below:

— it is a discrete optimization problem,
— it is strongly dependent on the type of the FE discretization and the particular classes of 2-D FE are (according to our experience) the most convenient in our analysis,
— since the optimization problems (7), (9) are discrete problems they should satisfy the general rules and theorems of the combinatorial analysis.

An augmented objective functional (instead of V – see eq (9)), used to rank the designs with constraints, is defined as follows:

$$V^* = V + p_1\boldsymbol{a} - p_2\boldsymbol{b} \tag{10}$$

where á means the violation of the most critical constraint, whereas â is a margin of the same constraint; they are both positive. p_1 is a penalty parameter, and p_2 is a bonus parameter for constraint margins.

NUMERICAL RESULTS

The cylindrical panel subjected to axial compression at the x- direction have been modelled with the use of 24 quadrlateral four noded finite elements (NKTP 32, NORDR 1). Each of them possesses five degrees of freedom, so that the first order transverse shear deformation theory is applied. Two various boundary conditions of panels have been applied: (i) the panel is simply-supported at all edges (to compare and verify with analytical solutions – see Muc [8]) and (ii) the panel is clamped at the edges parallell to the x – axis and simply-supported at the other edges. The results of computations are presented in Table 1.

Boundary conditions	Geometrical ratio L/R				
	1.0	1.5	2.0	2.5	3.0
Simply-supported	$[1,1,1,3,3,1]_S$	$[1,1,2,2,3,1]_S$	$[1,1,2,2,3,1]_S$	$[1,1,2,2,2,1]_S$	$[1,2,2,2,2,1]_S$
Clamped	$[1,1,1,1,3,3]_S$	$[1,1,2,2,2,3]_S$	$[1,1,2,2,2,3]_S$	$[1,2,2,2,3,3]_S$	$[1,2,2,2,3,3]_S$

Table 1. Optimal strings for compressed multilayered cylindrical panels

As it may be seen the type of boundary conditions has a great influence on the optimal stacking sequences. They are also dependent on the geometrical ratios of the cylindrical panel. It is woth to mention that the similar optimal strings have been obtained using both GA and S.A. optimization procedures.
The next example deals with the optimization problem of the thickness distribution for the cylindrical panel subjected to the action of a normal concentrated force P located at its center. Therefore the optimization problem (9) is subjected to the following constraint:

$$l_{cr} = l_{admis} \tag{11}$$

where $ë_{cr}$ denotes the buckling load parameter. $ë_{admis}$ means the initial buckling load parameter computed for the constant thickness distribution over the area of the structure.

Fig. 3 Optimal thickness distribution for a quarter of the cylindrical panel (the projection on the x-y plane – the concentrated force is located at the left, lower corner of the panel)

In the numerical example that value have been evaluated for following geometrical parameters of the panel: t=0.5, R/t = 160, L/R = 2, f/R = 0.5, where t is a shell thickness, R – the radius, L – the total length and f – the shallowness parameter. Assuming that the Young modulus is equal to 30 [GPa] and the value of the conentrated force P = 10 [N] the buckling load parameter $ë_{admis}$ is equal to 3.519. Using the augmented optimization problem (10) and introducing one base point only – at the place where the external force is located, the optimal thickness distributions have been obtained. The optimization results are plotted in Fig.3 and the final volume is equal to 40% of the initial one. Probably it is possible to obtain further reduction of the volume (thickness) by the increase of the thickness at the base point over the admissible value t = 0.5. It should be mentioned that at the beginning of the optimization process the shell has been divided into four rectangular parts with two admissible thicknesses only, i.e. K = 2 in eq (5) and the parameter 0.1 has been replaced by the number 0.25. Finally, a quarter of the structure have been divided into 512 triangular FE, K was equal to 10 and the parameter 0.1 in eq (5) was replaced by the number 0.05.

CONCLUDING REMARKS

In this paper the components for the application of probabilistic algorithms procedures in the conjunction with the finite element analysis for layout optimization (understood in a broader sense) of composite structures have been introduced. The major modulus, i.e. the calculation of the objective function is (or may be) treated as a black box. The accuracy and correctness of the optimization procedures is mainly dependent on the PA procedures.

Based on the above studies and examples, the following conclusions can be drawn:

1. Proper coding of design variables and then selection of a new population are essential for optimization

2. With the regard to shape optimization the correct geometrical construction of convex hulls seems to be essential in the generation of cubic Bezier splines
3. Optimization procedures do not require any sensitivities studies what seems to be a great advantage over the classical optimization methods
4. There is no necessity to change the initial mesh using any adaptive procedures

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