NON LINEAR MODEL FOR EVALUATION OF DAMPING IN POLYMER COMPOSITE

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Abstract
In present paper a nonlinear model has been proposed for the prediction of damping of viscoelastic materials. Hysteretic loop has been obtained for the evaluation of damping of the polymer materials. Results are obtained for nonlinear modified four parameter model for viscoelastic materials. This model is be used for prediction of loss factor for different fiber volume fraction of the fiber reinforced composite. Variation of strain with respect to time has been evaluated.

1 Introduction

Damping capacity is a measure of a material’s ability to dissipate energy during mechanical vibration under cyclic loading [1]. When utilized effectively in a structural application, this property allows undesirable noise and vibration to be passively attenuated and removed to the surroundings as heat. Vibration damping is essential to the attainment of performance goals for engineered systems that can exhibit significant structural dynamic response. Passive structural damping can be increased most predictably through the use of materials with known damping properties. Because of the potential for practical payoffs, some research efforts have pursued the development of structural materials (typically composites) with increased damping properties.

The damping of conventional materials used in conventional ways does not generally provide sufficient energy dissipation for many types of engineering structure. Composite materials are used for improved tailoring of properties. Polymers generally display linear behavior in the region of low and intermediate stress. Hysteretic loop remain elliptical. However significant nonlinearity had been shown by most materials at high strain. Many authors[2-3] have evaluated nonlinear damping by different methods. The non-linear behavior of polymeric fiber composite materials has been studied theoretically in terms of a model developed for elastic plastic materials, and generally valid for elastic–plastic response. This theory is focused on the current material state and connects material anisotropic response with identifiable directions in the present material state [4]. Pooler and Smith [5] studied the effect of time and temperature on a commercial wood plastic composite by means of series of creep and recovery tests. A prony series was used to describe the material time dependant compliance where times are shifted with stress and temperature to describe nonlinear response and temperature dependence. Jeary [6] discussed the mechanism causing damping and concluded that fracture at both microscopic and macroscopic scale is the dominant mechanism for energy release. A generalized nonlinear damping model is considered as basis of prediction of damping values at the design stage.

In this paper a nonlinear model has been proposed for the prediction of damping of viscoelastic materials. Hysteretic loop has been obtained for the evaluation of damping of the materials. Results are obtained for nonlinear four parameter model for viscoelastic materials. This model is used for prediction of damping of the fiber reinforced composite. Further Variation of strain with respect to time has been evaluated.

2 Mathematical model

Mathematical model development for rheological behavior of solid is to permit the realistic results to obtained from mathematical analysis of complicated structure under various conditions such as sinusoidal, random and transient loading. In 1784 Coulumb recognized that the mechanism of damping operative at low stresses may be different than those at high stresses even today major emphasis is placed on linear models of damping because of its accuracy
at low stress regime and computationally more economical than nonlinear ones. The linear model for viscoelastic material is Maxwell model which consist of a spring and dashpot in mechanical series as shown in Fig 1(a) and further Kelvin –Voigt model which comprise of a spring in parallel with dashpot as shown in Fig1(b). After two parameter, three parameter and four parameter model has been developed that predict more accurately the real behavior of viscoelastic solid as shown in Fig 1(c) and (d).The differential equation given in Eq. (1) represent the four parameter shown in Fig 1 (d) with nonlinear parameter to predict the nonlinear behavior of viscoelastic solid. In this paper matrix is assumed as viscoelastic material and modeled with this four parameter and using this model polymer composite is than modeled through bridging model [7]. This model is analysed under cyclic loading and the results are obtained and plotted in form of hysteresis loop.

2.1 Hysteresis loops for vibrating systems:

In study of vibration hysteresis loop is very often used as measure of energy dissipation. A hysteresis loop is obtained by recording the magnitude of a force verses the displacement brought about by its action. The area enclosed by the loop is proportional to the amount of energy dissipated in the system within one period as given by Eq. 1(b).

\[
\text{Strain energy} = \int_{\varepsilon=0}^{\varepsilon=\varepsilon_a} \sigma d\varepsilon \quad (1a)
\]

\[
\text{Dissipative Energy} = \int \sigma d\varepsilon \quad (1b)
\]

\[
\sigma + p_1 \sigma + p_2 \sigma = q_1 t^2 \left(\varepsilon_1^2\right) + q_2 \varepsilon_2 \quad \ldots (2)
\]

Where

\[
p_1 = \frac{\mu_m + \mu_f}{E_m - E_f} \quad p_2 = \frac{\mu_f}{E_m + E_f}
\]

\[
q_1 \cdot q_2 = \mu_m \cdot q_2^2 \quad \mu_f
\]

Solving nonlinear equation (2) the stress and strain is obtained and plotted in Figs. (2-6). Loss factor has been obtained by hysteretic loop plotted by using the equation (3) [1].

\[
\eta = \frac{D}{2 \Pi U} = \left[\int_{\varepsilon=0}^{\varepsilon=\varepsilon_a} \sigma(t) dt \right] \int_{\varepsilon=0}^{\varepsilon=\varepsilon_a} \sigma d\varepsilon \quad \ldots (3)
\]

2.2 Two Phase Bridging Model

Two phase bridging model [7] for unidirectional fiber reinforced composite has been discussed. In two phase model elastic stresses in matrix material are correlated with the fiber by bridging matrix. The bridging matrix represents the load sharing capacity of one constituent phase in the composite with respect to the other phase. The composite is considered to be transversely isotropic.

\[
\{d\sigma_i^m\} = \left[ A_{ij}^m \right] \{d\sigma_j^f\} \quad (4)
\]

Where \{d\sigma_j^f\} = \{d\sigma_{1f}, d\sigma_{2f}, d\sigma_{3f}, d\sigma_{12f}, d\sigma_{23f}\}^T

Figure 1. Two and three and four parameter model .
(a) kelvin model, (b) Kelvin-Voigt model, (c) Three parameter model, (d) Four parameter model
\[ A_{ij} \] = bridging matrix

and suffix \( f \) and \( m \) are fiber and matrix respectively. A quantity without suffix represents the composite.

3 Results and Discussion:

Using the following material properties, stress strain are evaluated for different fiber volume fraction from \( V_f = 0.3 \) to \( V_f = 0.6 \). Fig. 2 shows the variation of stress with respect to strain for viscoelastic matrix. strain varies linearly for very low values of stress and it takes sharp variation at higher stresses.

Material properties

\( E_f = 72.4 \text{GPa}; \ E_m = 3 \text{ GPa}; \ E_v = 3 \text{ GPa}; \)
\( \mu_m = 21000 \text{GPa}\); \( \mu_v = 3 \text{ GPa}; \ w = 2\pi; \)

Fig. 2. Variation of stress with respect to strain for epoxy

Fig. 3 illustrate the variation of longitudinal stress \( \sigma \) with respect to strain for fiber volume fraction equal to 0.3. Here stress is more and strain is less as with the addition of fiber object become more stiff and hence strain in reduced which leads to lower loss factor. Fig. 4 shows the dependence of strain with stress for fiber volume fraction \( V_f = 0.4 \). Strain is further reduced with the increases of fiber as object is getting more stiffer and stiffer. Fig. 5 predict the variation of longitudinal stress with respect to strain for fiber volume fraction \( V_f = 0.5 \). Fig. 6 shows the variation of stress with respect to strain for fiber volume fraction equal to 0.6. Stress is more and strain is less in comparison to the less fiber contain. Loss factor is not reduced much because of its dependence on matrix properties. It may be possible that at higher strain matrix will be dominant in nature.
Fig. 5. Variation of longitudinal stress with respect to strain for fiber volume fraction $V_f=0.5$

Fig 6. Variation of longitudinal stress with respect to strain for fiber volume fraction $V_f=0.6$

Fig. 7 shows the complete cyclic variation of stress and strain for 20 cycles. It indicates that the slope of hysteresis loops for each cycle decreases as the number of cycle increases. Further the area of hysteresis loop is larger for higher cycles which corresponds to higher level of damping in reference to higher level of strain. Fig. 8 depicts the variation of stress in the composite materials subjected to cyclic loading with the passage of time. Here the strain level increases with time which affects the energy dissipated in polymer composites.

**4 Conclusions:**

Logitudinal loss factor of polymer composite has been evaluated using the hysteretic loop methods. It is very simple method for finding out the nonlinear damping behavior of composite materials. Because of cyclic loading damping behavior of
materials is mainly depend on Viscoelastic matrix when fiber is assumed as elastic.

References


