CHARACTERISATION AND ANALYSIS OF CARBON FIBRE / EPOXY COMPOSITE TUBES

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SUMMARY

The mechanical behaviour of a carbon fibre / epoxy composite material is identified with tests performed on tubular specimens. [±15°], [±45°], and [±75°] laminates are used to determine the longitudinal, shear, and transverse responses under quasi-static and viscoelastic loading. Experimental results are compared with analytical approaches.

Keywords: damage, viscoelasticity, modelling

INTRODUCTION

A thorough understanding of the properties and behaviour of materials is necessary to improve the design of composite structures. While one can be relatively confident in modelling the elastic behaviour of an undamaged laminate, the effects of viscoelasticity, damage, plasticity, and viscoplasticity are often harder to take into account. Furthermore, each of these behaviours and the parameters used to model them can change with time and damage, making end-of-life behaviour predictions difficult.

Additionally, there is often difficulty in bridging the gap between coupon-level tests and structural-level performance. This is partly due to edge effects resulting in a different stress state in flat-sided coupons, and partly due to differences in the fabrication process itself. As the present study is concerned with tubular structures fabricated by filament winding, it is only logical that the material characterisation be performed using tubular specimens.

In this paper, the behaviour of a carbon fibre / epoxy resin composite material will be investigated. First, a meso-macro model taking into account viscoelasticity, damage, plasticity, and viscoplasticity is presented. Next, the results of a series of quasi-static and creep tests performed on filament wound tubular specimens are discussed. The lay-ups considered are [±15°], [±45°], and [±75°] so as to determine the behaviour in loading in the fibre direction, transverse direction, and under shear stress. The material parameters will be identified, and the model compared with the experimental data.

INTRODUCTION OF THE MODEL

Modelling of composite materials has been the subject of a great deal of research, and an extensive body of literature exists on the subject (see, for example, the WWFE [1]). Models of varying complexity may be purely elastic, or take into account viscoelasticity, stiffness loss due to damage, and permanent deformations. In the current work, the goal is to take into account as many of these phenomena as possible, while maintaining the flexibility necessary to consider different lay-ups and modify behaviour laws as needed. Based on a literature survey, the LMARC model [2] seems best suited to the current application.

In this section, the LMARC model for composite laminate behaviour is presented. The model operates on a meso / macro scale, where the time-dependent ply-level behaviour is determined based on a thermodynamic framework, and the mechanical response of the structure is computed using a non-linear laminate theory based on the Love-Kirchhoff hypothesis.

Mesoscale: Damaged elasto-viscoplastic model

The thermodynamic framework assumes that the mechanical behaviour is obtained when two potentials are defined. The first is the free energy density $\psi=\psi(\varepsilon^e,V^k)$, where ε^e is the elastic strain and V^k are other internal variables used to describe the inelastic strain. The second one is the dissipation potential $\varphi=\varphi(A_k)$, where A_k are the dual variables of V_k. From these potentials the constitutive relations can be obtained.

Reversible Strain

The material coordinate system is defined as given in Figure 1.

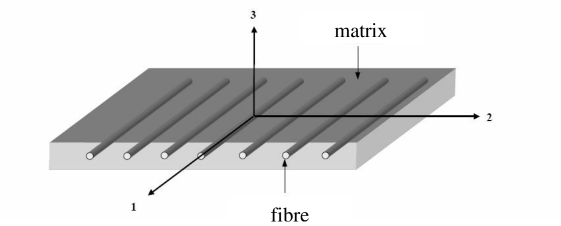


Figure 1: Coordinate system of the lamina

As long as no damage occurs under loading, the strains in the ply are reversible and can be written as the combination of the elastic and viscoelastic strains:

$$\boldsymbol{\varepsilon}^{re} = \boldsymbol{\varepsilon}^{e} + \boldsymbol{\varepsilon}^{ve} \quad (1)$$

where $\boldsymbol{\varepsilon}^e$ is the elastic strain and $\boldsymbol{\varepsilon}^{ve}$ is the time-dependent viscoelastic strain. The free energy density is written as a combination of the elastic and viscoelastic components [3]:

$$\psi = \frac{1}{2}\varepsilon^e : \mathrm{C} : \varepsilon^e + \frac{1}{2}\sum_i \frac{1}{\mu_i}\left(\xi_i : \mathrm{C}_R : \xi_i\right) + \psi^* \quad (2)$$

The $\boldsymbol{\xi}_i$ are a set of elementary viscoelastic strains, each one associated with a relaxation time τ_i and a weight μ_i. $\mathbf{C}$ and $\mathbf{C}_R$ are the elastic stiffness matrix and relaxed viscoelastic stiffness matrix, respectively.

For plane stress:

$$\mathbf{S} = \mathbf{C}^{-1} = \begin{bmatrix} 1/E_1 & -\nu_{12}/E_1 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix} \tag{3}$$

$$\mathbf{S_R} = \mathbf{C_R}^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \beta_{22}S_{22} & 0 \\ 0 & 0 & \beta_{66}S_{66} \end{bmatrix} \tag{4}$$

The viscoelastic dissipation is given by:

$$\varphi_{ve}^{*} = \frac{1}{2}\sum_i \frac{\mu_i}{\tau_i}\left[\left(-\boldsymbol{\sigma} + \boldsymbol{\chi}_i\right) : \mathbf{S_R} : \left(-\boldsymbol{\sigma} + \boldsymbol{\chi}_i\right)\right] \tag{5}$$

Where $\boldsymbol{\sigma}$ is the stress tensor and with

$$\boldsymbol{\chi}_i = \frac{\partial \psi}{\partial \boldsymbol{\xi}_i} \tag{6}$$

The viscoelastic strain rate can then be written as the sum of the elementary viscoelastic strain rates:

$$\dot{\boldsymbol{\varepsilon}}^{ve} = \sum_i \dot{\boldsymbol{\xi}}_{\mathbf{i}} \tag{7}$$

with

$$\dot{\boldsymbol{\xi}}_{\mathbf{i}} = -\frac{\partial \varphi_{ve}^{*}}{\partial \boldsymbol{\sigma}} = -\frac{1}{\tau_i}\left(\boldsymbol{\xi}_i - \mu_i \mathbf{S_R} : \boldsymbol{\sigma}\right) \tag{8}$$

Because viscoelastic behaviour is mainly due to matrix of the composite material, the form of the time relaxation spectrum (Gaussian, etc.) is dependent on the matrix. For an epoxy matrix the relaxation time spectrum is chosen as triangular (Figure 2) for simplicity, though experience shows it accurately predicts the viscoelastic strain.

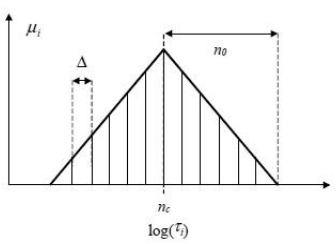


Figure 2: Relaxation time spectrum

Damage

Once the applied loading reaches a certain level, microcracking occurs. These microcracks modify the reversible behaviour of the ply, and they also introduce an irreversible strain in the form of plasticity and viscoplasticity:

$$\boldsymbol{\varepsilon}^{ir} = \boldsymbol{\varepsilon}^{p} + \boldsymbol{\varepsilon}^{vp} \tag{9}$$

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{re} + \boldsymbol{\varepsilon}^{ir} = \boldsymbol{\varepsilon}^{e} + \boldsymbol{\varepsilon}^{in} \tag{10}$$

The elastic stiffness of the ply is modified by adding matrix **H** [4]

$$\tilde{\mathbf{S}} = \mathbf{S} + \mathbf{H} \tag{11}$$

where **H** is linked to the density of microcracking by a self consistent model and has the components:

$$\mathbf{H} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & H_{22} H(\sigma_2) & 0 \\ 0 & 0 & H_{66} \end{bmatrix}, \begin{cases} H(\sigma_2) = 0 \ si \ \sigma_2 < 0 \\ H(\sigma_2) = 1 \ si \ \sigma_2 \geq 0 \end{cases} \tag{12}$$

Damage variables D_I and D_{II} are defined as the variation of the transverse modulus E_2 and shear modulus G_{12}, respectively:

$$D_I = -\frac{\Delta E_2}{E_2} = 1 - \frac{S_{22}}{S_{22} + H_{22}} \tag{13}$$
$$D_{II} = -\frac{\Delta G_{12}}{G_{12}} = 1 - \frac{S_{66}}{S_{66} + H_{66}}$$

Furthermore, using a homogenisation method it can be shown that D_{II} is dependent on D_I:

$$D_{II} = 1 - S_{66} \left[S_{66} + \frac{D_I}{(1 - D_I)^{1/2}} (S_{11} S_{22})^{1/2} \right]^{-1} \tag{14}$$

Thus, only D_I is necessary to describe the effect of microcracking on the elastic behaviour. It is now possible to introduce a damage criteria [5]:

$$f^D = -Y - R_D - Y_c \leq 0 \tag{15}$$

with Y defined as the derivative of the free energy density with respect to the damage variable D_I:

$$Y = \frac{\partial \psi}{\partial D_I} \tag{16}$$

$$R_D = \alpha (D_I)^P \tag{17}$$

where α and P are material parameters that describe the evolution of damage and Y_c is the damage yield point. The damage in the ply can thus be determined for any combination of stresses. Furthermore, the effect of damage can be taken into account in the analysis of inelastic strains using an equivalent stress, given as:

$$\tilde{\boldsymbol{\sigma}} = \mathbf{S}^{-1} : \tilde{\mathbf{S}} : \boldsymbol{\sigma} \tag{18}$$

Irreversible Strain

Two types of plasticity are considered here. The first occurs mainly under shear loading, and is the result of internal friction occurring between the microcrack surfaces.

The criteria used for this phenomenon is:

$$f_P = \left[(\tilde{\boldsymbol{\sigma}} - \mathbf{X}) : \mathbf{M} : (\tilde{\boldsymbol{\sigma}} - \mathbf{X}) \right] - Z_c \tag{19}$$

with

$$\mathbf{M} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{20}$$

$$\mathbf{X} = \mathbf{X_1} + \mathbf{X_2} \tag{21}$$

$$\dot{\mathbf{X}}_\mathbf{1} = \delta_1 \dot{\boldsymbol{\varepsilon}}^p + \gamma_1 \mathbf{M} : \mathbf{X_1} \dot{\bar{\boldsymbol{\varepsilon}}}^p \tag{22}$$

$$\dot{\mathrm{X}}_2 = \delta_2 \dot{\varepsilon}^p \tag{23}$$

X is the kinematic hardening as a function of accumulated plastic strain. The plasticity limit Z_c is related to the damage yield point and is found to be:

$$Z_c = 2Y_c \sqrt{E_1 E_2} \tag{24}$$

The other type of plastic strain is related to the increase in volume of the material due to voids created by microcracking. This damage plasticity creates an additional strain in the transverse direction, and its amplitude is directly linked to the density of microcracking. A linear relation between D_I and this plastic strain is proposed:

$$\dot{\mathbf{X}}_\mathbf{3} = \delta_3 \dot{D}_I \tag{25}$$

A viscoplastic shear strain due to delayed friction phenomenon is also considered in the model. The dissipation potential has the following form:

$$\varphi_{vp}^* = \frac{K}{\eta + 1} \left\langle \sqrt{(\tilde{\boldsymbol{\sigma}} - \mathbf{X_3}) : \mathbf{M} : (\tilde{\boldsymbol{\sigma}} - \mathbf{X_3})} - Z_c \right\rangle^{\eta+1} \tag{26}$$

$$\dot{\mathbf{X}}_\mathbf{3} = \delta \dot{\varepsilon}^{VP} \tag{27}$$

From mesoscale to macroscale

Several options exist for considering non-linear behaviour in composite laminates. In the current work, an extension of Classical Lamination theory is used. Based on the Love-Kirchoff hypothesis, it assumes that a section perpendicular to the middle plane of the laminate remains perpendicular under loading.

In a linear analysis, the classical matrices **A**, **B**, and **D** relate the middle plane strain $\boldsymbol{\varepsilon}$ and curvature $\boldsymbol{\rho}$ to the generalized force **N** and moment **M**:

$$\begin{bmatrix} \mathbf{N} \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon}_\mathbf{0} \\ \boldsymbol{\rho} \end{bmatrix} \tag{28}$$

Here, **A**, **B**, and **D** are constant. The extension to non-linear behaviour is made by writing eq (29) in an incremental form:

$$\begin{bmatrix} \Delta\mathbf{N} \\ \Delta\mathbf{M} \end{bmatrix} = \begin{bmatrix} \mathbf{A}^* & \mathbf{B}^* \\ \mathbf{B}^* & \mathbf{D}^* \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{\varepsilon}_\mathbf{0} \\ \Delta\boldsymbol{\rho} \end{bmatrix} \tag{29}$$

ΔM and **ΔN** are increments of the generalized force and moment for a given time step, and $\Delta\boldsymbol{\varepsilon}_0$ and $\Delta\boldsymbol{\rho}$ are the incremental mid-plane strain and curvature. Matrices **A***, **B***, and **D*** are the laminate tangent stiffness dependent on loading history, and can be assessed using the Newton-Raphson method.

EXPERIMENTAL RESULTS

Carbon fibre / epoxy resin tubes were fabricated by filament winding and cured. The internal diameter, thickness, and length are 60 mm, 1.2 mm, and 300 mm, respectively. The lay-ups considered are [±75°], [±45°], and [±15°]. These were chosen to facilitate the identification of transverse, shear, and fibre direction properties, while attempting to minimize specimen fragility and remaining within the limits of the fabrication process.

Uniaxial quasi-static and creep tests were performed. For the quasi-static tests, a series of progressive loadings and unloadings were applied, as schematised in Figure 3a. This makes it possible to determine the evolution of stiffness and accumulation of irreversible strains as a function of increasing stress. A similar principal was used for the creep tests (Figure 3b). The specimen was loaded and unloaded and the damage level determined before being held at the previously applied load for 2 hours. At the end of the test, the specimen was allowed to relax, and the cycle repeated for a higher load level. Thus, the viscoelastic behaviour is observed at different stress and damage levels.

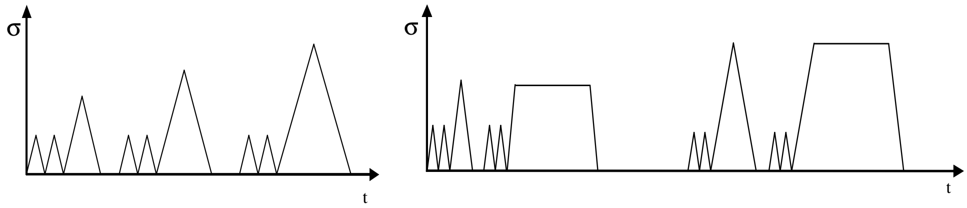


Figure 3 a) Quasi-static progressive loading and b) Creep test progressive loading

Discussion of results

Based on the results of the [±15°] lay-ups, the fibre-direction behaviour was found to be linear elastic, as would be expected. A slight reduction in stiffness was observed with the progressive loading, which could be attributed to matrix damage. Additionally, a small amount of inelastic strains occurred. No axial viscoelastic behaviour was observed, although creep was observed in the circumferential direction. The strength of this lay-up in the axial direction is greater than 1000 MPa (the capacity of the machine used in this study). However, the near total loss of stiffness in the circumferential direction qualifies as a structural failure.

The behaviour of the [±75°] specimens is elastic and fragile. The stress strain curves are linear, and neither damage nor plasticity was observed with increasing loading. Fracture occurred locally at approximately 42 MPa when a single filament delaminated and unravelled down the length of the specimen. Even though the matrix is viscoelastic, very little time dependent strain was observed during the creep tests. This is probably due to the relatively low stress levels (25 MPa – 30MPa) that had to be used, as failure occurred during the creep tests at stresses lower than the strength found during the quasi-static tests.

The [±45°] lay-up demonstrates a great deal of viscoelastic behaviour, a 25% reduction in stiffness with increasing load, and significant amounts of irreversible strain

(Figure 4). Stiffness loss begins at approximately 65 MPa, and final rupture occurs around 140 MPa.

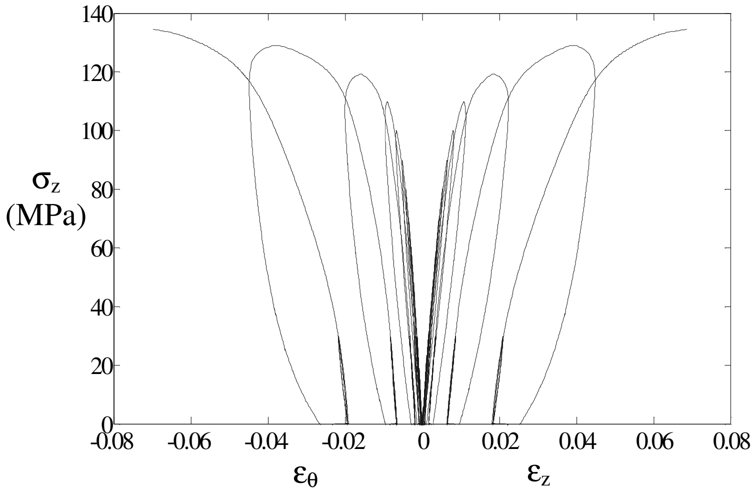


Figure 4: Results of a quasi-static test for a [±45°] specimen

During the creep tests (Figure 5), the modulus measured at the end of each loading was different than the modulus measured at the beginning, indicating damage progression during the test. This is particularly noticeable for the creep test at 115 MPa. Finally, the creep tests at the same load level – 65 MPa and 85 MPa - but different levels of damage exhibited markedly different behaviour.

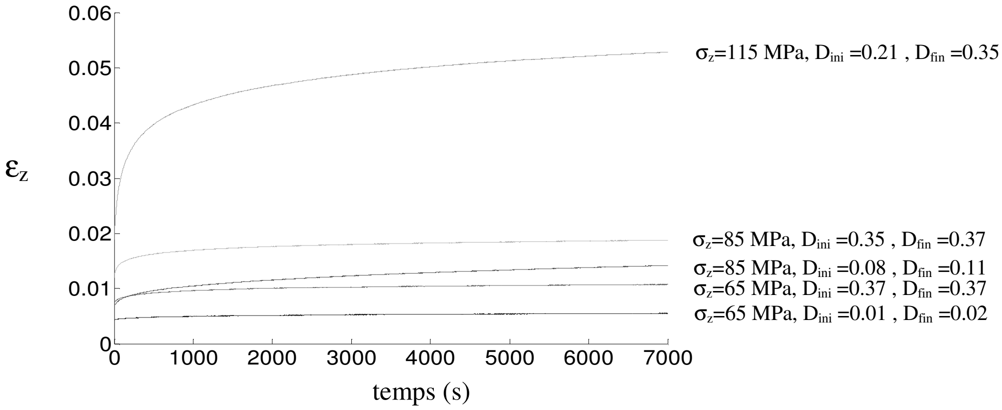


Figure 5 : Results of a creep test for a [±45°] specimen

Identification of Parameters and Comparison with Model

The LMARC Model has mainly been used to describe the behaviour of glass-epoxy composite laminates [2-5]. Here, its accuracy in predicting the behaviour of carbon fibre/epoxy composites is investigated. The material properties are obtained from the experimental data using a reverse optimisation to identify the properties in the fibre direction, transverse direction, and in shear, as well as the associated damage, viscoelasticity, plasticity, and viscoplasticity parameters. A Levenberg-Marquardt optimisation is used to this end [6]. The resulting parameters are given in Table 1.

Table 1: Material parameters

Property	Value		Property	Value
E_{11}	138 GPa		$Y_i^{[\pm 75]}$	0.002 MPa
E_{22}	9.20 GPa		α	0.11 MPa
v_{12}	0.34		P	0.99
G_{12}	5.35 GPa		δ_1	8921 MPa
N_c	3.313 s		δ_2	3453 MPa
N_0	4.989 s		γ	157
β_{22}	.13		δ_3	0.003 MPa
β_{66}	.38		K	2.57E-11
Y_c	.0645 MPa		η	7366
$Y_i^{[\pm 15]}$	0.003 MPa		δ	3.93 MPa
$Y_i^{[\pm 45]}$	0.046 MPa			

Parameters E_{11}, E_{22}, and G_{12} are obtained with the initial stress strain curves from the [±15°], [±75°], and [±45°] quasi-static tests, respectively. The influence of the Poisson ratio v_{12} is most easily determined from the [±15°] and [±75°] tests.

The lack of damage progression in the [±75°] would seem to indicate a damage yield point superior to the maximum value reached by the damage variable Y; for this lay-up at rupture (42 MPa), Y is 0.082 MPa. This value is superior, however, to the damage initiation point observed in the [±45°] lay-up, which occurred at a Y equal to 0.017 MPa. Some of this difference can be explained by residual stresses from the manufacturing process, which contribute to an initial Y; these values can be found using a thick-walled tube calculation (see, for example [7]) and taken into account by:

$$Y_c = Y_c^* + Y_i \tag{30}$$

The parameters α and P are determined from the evolution of damage in the [±45°] lay-up. The damage D_I versus the axial stress is given in Figure 6.

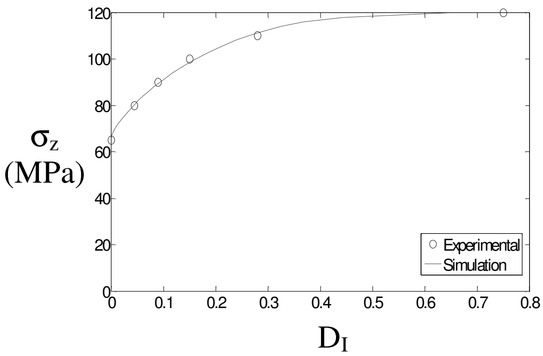


Figure 6: Evolution of damage versus applied axial stress in a [±45°] specimen

The parameters δ_1, δ_2, and γ for the plasticity in shear are likewise determined from the permanent deformations observed in the [±45°] lay-up. The parameter δ_3 for the damage plasticity is chosen as a small, non-zero number as no plasticity was observed in the [±75°] lay-up. The viscoplastic parameters are chosen to compensate for any remaining permanent deformation.

Influence of damage on viscoelastic behaviour

The viscoelastic parameters for the virgin material are determined from the [±15°] and [±45°] creep tests, as very little creep was observed from the [±75°] tests. Two difficulties arise once damage is introduced. The first is damage progression during the test, that is to say a reduction in stiffness due to the propagation of microcracks while the specimen is held at the creep stress. The current damage kinetics do not consider this effect, though a dissipation potential could be added to take it into account. The second problem is the evolution of the viscoelastic properties β and τ with damage. Simply taking into account the reduced stiffness of the laminate by using equation (18) is insufficient for predicting the creep behaviour once damage is present. In order to avoid over-complicating the model and adding additional parameters, a simple modification of equation (4) is proposed:

$$\mathbf{S_R} = \mathbf{C_R}^{-1} = \frac{1}{(1-D_I)^2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & \beta_{22}S_{22} & 0 \\ 0 & 0 & \beta_{66}S_{66} \end{bmatrix} \quad (31)$$

Figure 7 gives a comparison of the viscoelastic modelling of a [±45°] lay-up at three different damage levels – D_I=0, D_I=0.20, and D_I=0.30 – for the same load, with and without the modification. Experimentally, β_{66} is found to change from 0.41 to 0.95, an evolution which can be taken into account with equation (31). The effect of damage on the relaxation time appears to be minimal. While this approach is somewhat empirical in nature, β_{66} evolves more quickly than D_{II} and no direct link was found. Additionally, the lack of damage progression in the [±75°] lay-up results in an incomplete understanding of the effect of D_I on β_{22}. Further research is needed in this area.

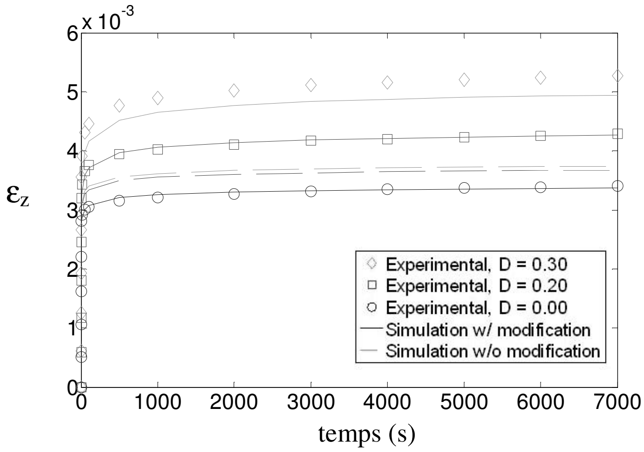
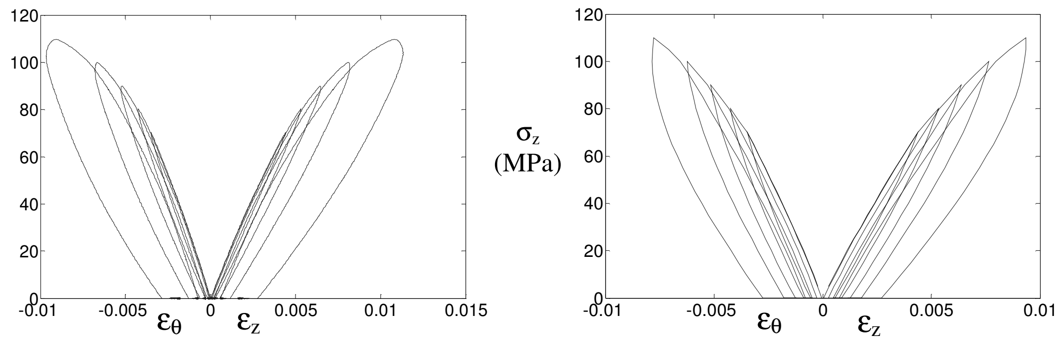


Figure 7 – Comparison of model with [±45°] creep tests at 50 MPa

Figure 8 shows a comparison of experimental data with the model for a full series of progressive loadings and unloadings. Overall, the model gives an accurate prediction of damage progression and accumulation of irreversible strains. The largest source of error seems to be the evolution of viscoelastic behaviour with the accumulation of damage, as previously discussed.



(a) (b)

Figure 8a) Results of quasi-static progressive loading test [±45°] and b) simulation

CONCLUSION

A series of quasi-static and viscoelastic tests have been formed to characterize the axial, transverse, and shear behaviour of a carbon fibre / epoxy resin material. Viscoelastic, plasticity, and viscoplasticity phenomenon are observed in addition to damage progression. The LMARC model is presented, and the necessary parameters are determined from the experimental data. Overall, the model is in good agreement with the data and is found to be capable of simulating the wide variety of behaviours observed for carbon/epoxy materials. A number of changes can be made to improve the model. Adding a dissipation potential to include time-dependent damage would increase the accuracy of the creep simulations, and would also permit modelling fatigue damage evolution. A relation between the time dependent damage and the viscoelastic parameters would further improve the accuracy of the model.

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