RANDOM FIELD CHARACTERISATION OF GFRP COMPOSITE PROPERTIES

S Sriramula* and M K Chryssanthopoulos
Faculty of Engineering and Physical Sciences
University of Surrey, Guildford, Surrey, UK, GU2 7XH
*Email: s.sriramula@surrey.ac.uk

SUMMARY

In this paper, spatial variability of geometric and material properties of GFRP composites is studied for the first time based on an extensive experimental study. Random field characteristics are developed considering pultruded structural composite plates. In addition to improving existing stochastic mechanics approaches, such experimental studies are an important ingredient for developing multi-scale stochastic analysis of composite structures.

Keywords: Uncertainty; Composite materials; Random fields; Correlation length; Spatial variation; Experimentation.

INTRODUCTION

The geometric and material properties of FRP composites exhibit an inherent uncertainty due to their complex manufacture and assembly processes. It is widely accepted that these uncertainties are quantified through a probabilistic basis; ultimately, this leads to a rational design of FRP structures. For studying the bulk material response it is suggested to consider these uncertainties in terms of random variable or random process models at a component level. The parameters for these stochastic models are best obtained from appropriate physical experiments on small-scale material coupons. Although extensive studies have been undertaken for random variable characterisation, random field models of composite material properties are so far based primarily on engineering judgement rather than on actual experimental data. The recently evolving multi-scale composite stochastic analysis methods also require experimental data on these macro random field models for validation at different scales.

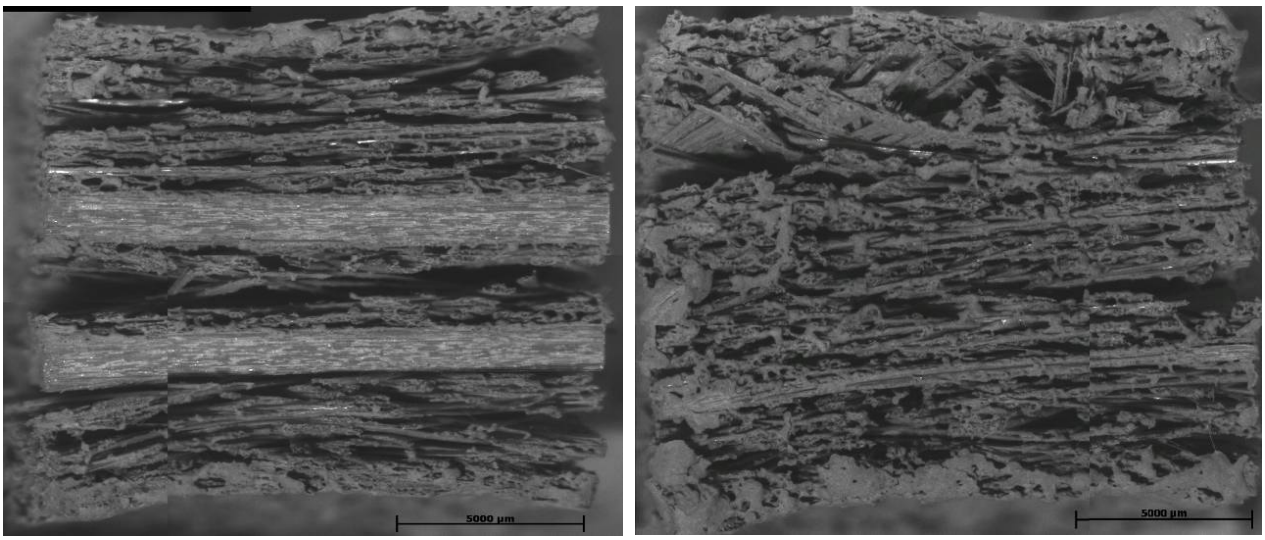
In this paper, this spatial randomness is evaluated experimentally by considering widely used industrial pultruded composites in pristine condition. This is part of an extensive study investigating the stochastic aspects of FRP composites; an earlier experimental and stochastic modelling study on chopped strand weathered GFRP composite panels was presented in [1, 2]. The results from these two studies are expected to contribute towards the quantification of uncertainty in composites based on physical experiments.

The considered structural grade GFRP composites are supplied in a standard size of 1.22×2.44 m^2 with 6.35 mm thickness. These composites consist of unidirectional glass fibre bundles providing longitudinal properties and continuous fibre mat providing

transverse properties in polyester resin with a fire resistant coat. Coupons from these plates are obtained in a pattern suitable to characterise the spatial variability of longitudinal tensile and compressive properties as well as the thickness of the plate. Coupons are obtained in succession starting from the corner of plate representing the longitudinal characteristics. An averaging extensometer is used for tensile strain measurement and is removed from the specimen at around 60% of the ultimate failure strain. The random fields are assumed to be spatially statistically homogeneous so that the primary and higher-order moments are independent of coupon location. The second-order information about spatial variation of considered variables is represented by the covariance and the correlation functions are obtained by suitable normalisations. The considered correlation forms represent the widely used spherical, exponential and variants of Bessel functional classes. The auto and cross correlations among these variables are evaluated in terms of coupon spatial distances for various forms of correlation functions.

MATERIAL DESCRIPTION

The pultruded GFRP composite plates used in this study are manufactured by Strongwell under the trade name EXTREN 525. These structural grade composites come in various shapes and dimensions. For this study, composite plates of 1.22×2.44 m^2 with a thickness of 6.35 mm are chosen as the available area is suitable for evaluating spatial randomness. These plates consist of unidirectional glass fibre bundles and continuous fibre mat in alternate layers in a fire retardant isophthalic polyester resin system with a UV inhibitor. The unidirectional fibre bundles provide longitudinal properties while the fibre mats provide appropriate transverse properties for the composite. Burn off tests performed on these composites have shown that the fibre volume fraction is 34.6%; corresponding microscopic images of the composite shown in Figure 1 also give an indication of the longitudinal and transverse constituents of this composite.



a) Longitudinal view b) Transverse view

Figure 1: Microscopic images of burnt composite portion

EXPERIMENTATION

The objective here is to study the spatial randomness of key material and geometric properties, i.e. longitudinal tensile strength (X_T), longitudinal tensile modulus (E_{11}), longitudinal compressive strength (X_C) and thickness (t) of the composite plates. Although ultimate failure strain in tension is also estimated from the experimental results, it is not examined further in the present study. Coupon dimensions and testing methodology are adopted from ASTM D3039/D 3039M [3] for tensile tests and as per BS EN ISO 14126 [4] for compressive tests. The choice of higher thickness (6.35mm) for the composite plate obviated the use of end tabs in the tensile and compressive tests. This was supported by pilot experiments and from reported experimental studies on same material [5].

For spatial variability modeling, it is very important to obtain the test coupons appropriately in a sequence and the coupon test results need to be marked accordingly. From similar studies on another composite material [1, 2], it was observed that for a sufficiently accurate representation of spatial randomness, coupons should be tested at the closest possible interval.

As the obtained plates have a dimension of 1.22×2.44 m^2 with longitudinal fibres along the longest direction, coupons are obtained to represent the longitudinal properties as shown in Figure 2. The tensile and compressive coupons have dimensions of 250mm×18mm and 125mm×28mm with gauge lengths of 150 mm and 25 mm respectively. Compressive test coupons are obtained immediately next to the tensile specimens; coupon cutting is carried out by means of a water cooled diamond saw. It was possible to obtain 57 coupons for tensile test and 43 coupons for compressive test. The thickness variation in the coupons was studied with a Mitutoyo IP65 digimatic micrometer having a data resolution of 0.001mm.

In the tensile test, a constant, flat rectangular cross section of a coupon is held in an Instron 6025 mechanical testing machine under a monotonically increasing load, applied at a rate of 0.5 mm/min. Strains were measured using an Instron 2650 series averaging extensometer with gauge length up to 50 mm. As shown in Figure 3, a validation test on a coupon with both extensometer and strain gauges also supported the use of the extensometer in subsequent tests. The specimens are loaded to about 50% of the expected failure load, unloaded and then loaded to failure removing the extensometer at around 70% of the expected failure load. The maximum tensile strength (X_T) is determined from maximum load carried by the coupon before failure. The strain data obtained from the averaging extensometer is used to estimate the tensile modulus of composite. Failure strains are obtained by considering linear stress-strain behaviour up to failure; this was confirmed by testing of pilot samples to failure with strain gauges.

Compressive properties are estimated using an Instron INSTRON 8805 high capacity test system with hydraulic grips as it provides an appropriate gripping for compression test [4]. Since the gauge length of coupons is smaller than the extensometer gauge length and it is difficult to control compressive failure, no extensometers were used. Thus, only the maximum compressive strength was recorded in these tests. A typical compression specimen being tested and coupon failure are shown in Figure 4.

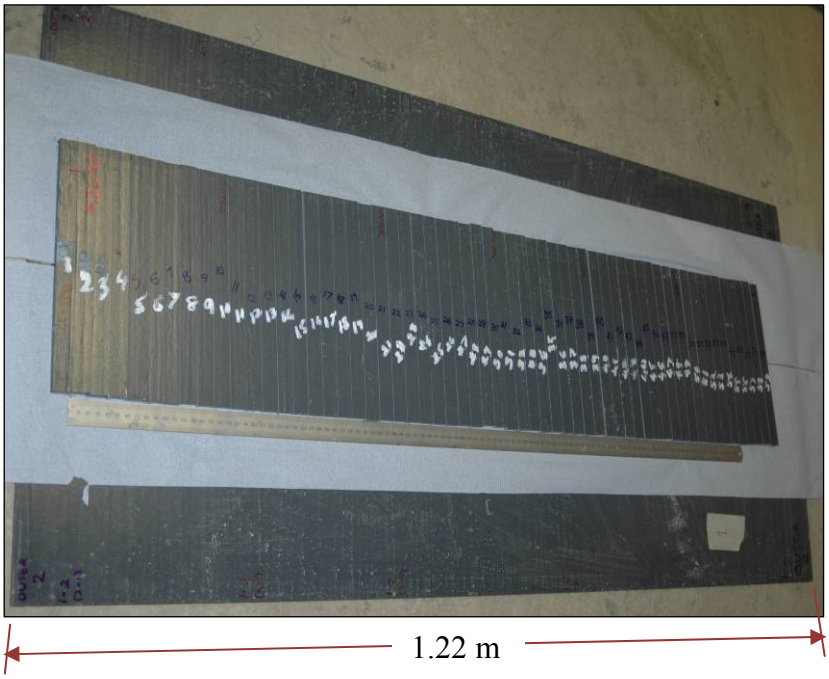


Figure 2: Coupon cutting sequence for longitudinal properties

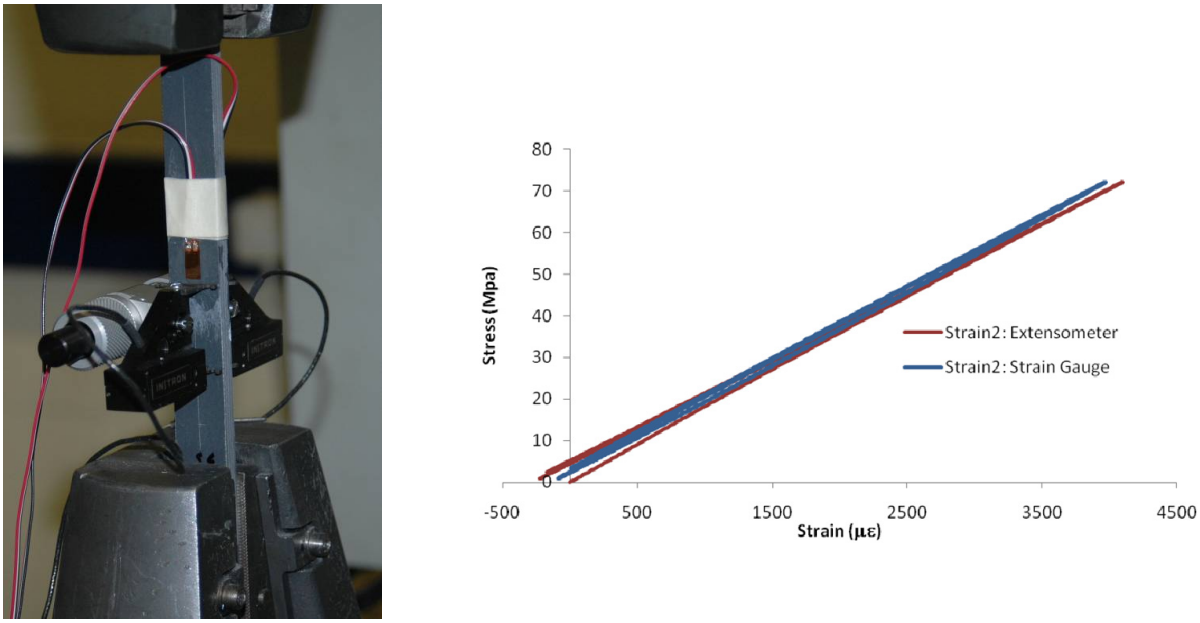


Figure 3: Typical tensile test including extensometer benchmarking

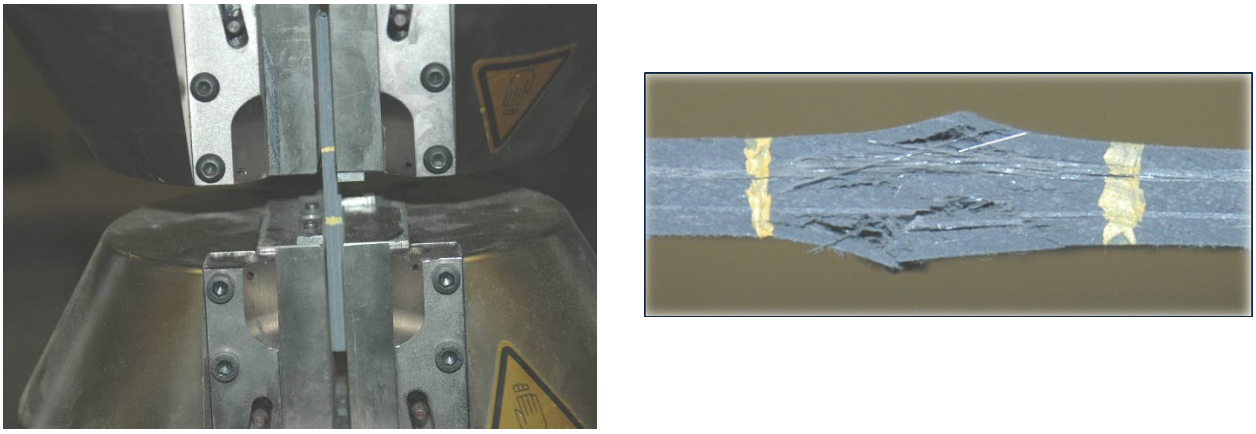
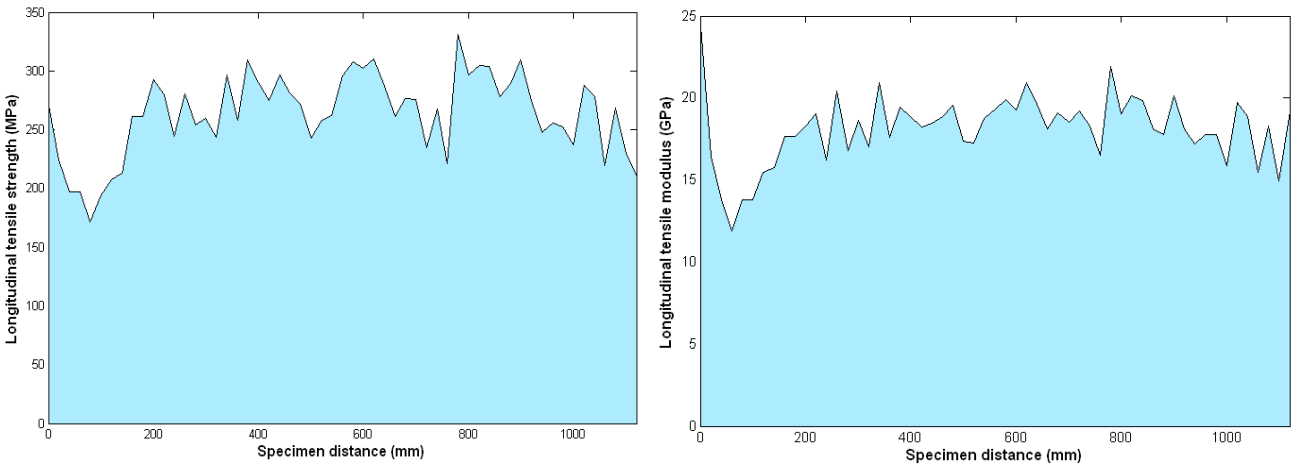


Figure 4: Typical compressive test and coupon failure

Experimental spatial randomness

Experimental results indicate significant spatial variation in the mechanical properties and thickness, with a typical spatial variability plot of X_T and E_{11} shown in Figure 5 highlighting this aspect. This could be visualised in different ways. For example, in another dimension, considering the E_{11} of the coupons at reference locations 2, 29 and 56, i.e., approximately at the start, centre and end of the panel, the relative variation of longitudinal tensile modulus of other coupons is shown in Figure 6. These figures indicate how X_T and E_{11} vary with respect to a particular location, and can be used in conjunction with the modelling technique described in the following section to determine spatial property characteristics. Similar variations are observed for other property variables, although with different peak (maximum and minimum) value locations.



a) X_T b) E_{11}

Figure 5: Spatial variation of X_T and E_{11} along the considered strip

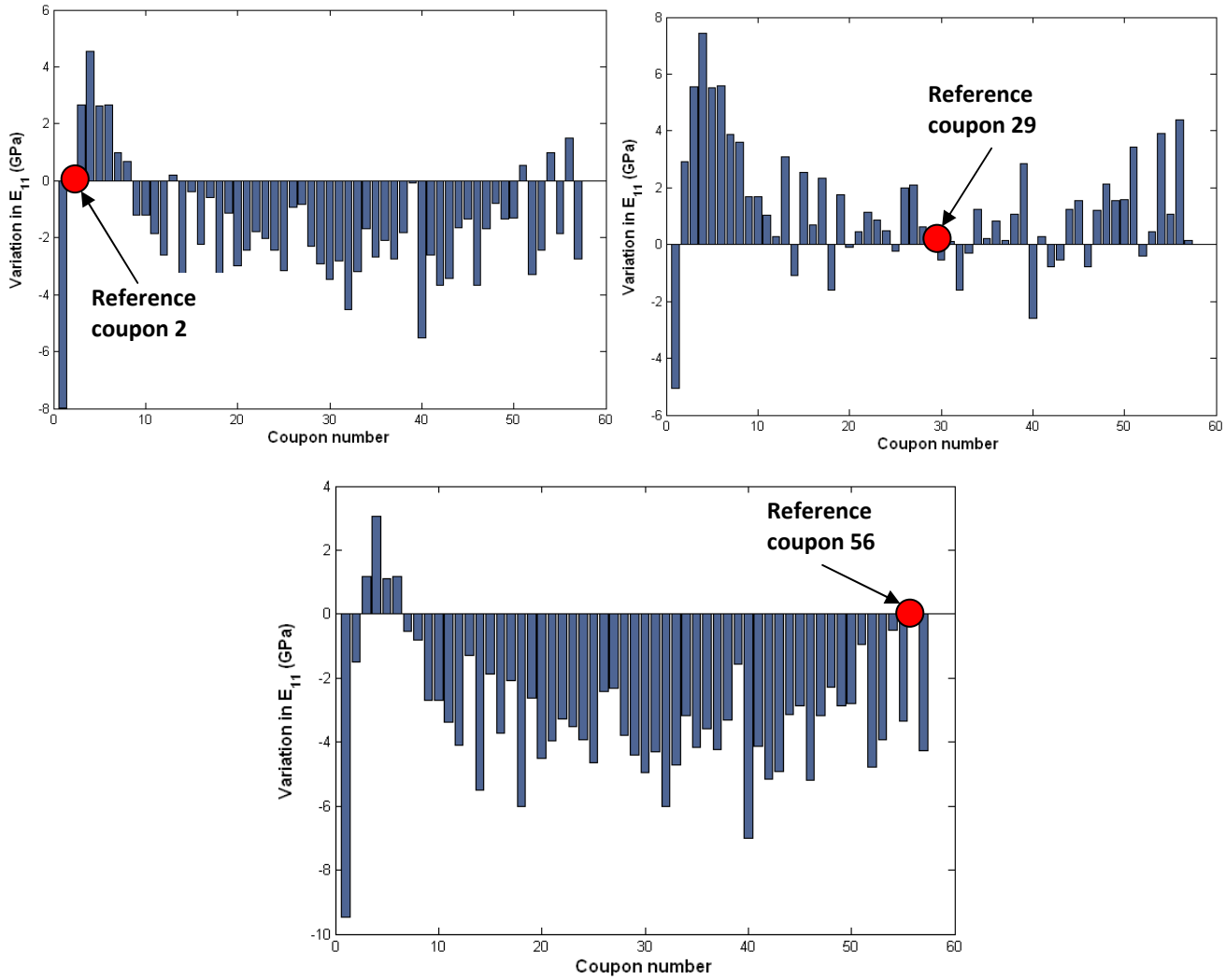


Figure 6: Variation of E_{11} with reference to coupons 2, 29 and 56

SPATIAL VARIABILITY MODELLING

Modelling considerations

Stochastic analysis of composite structures incorporating spatial variation has potential advantages over simpler approaches [6, 7]. For example, recently Carbillet *et al*. [8] observed that including the spatial variability associated with the tensile modulus in the reliability analysis of a damaged composite increased the probability of failure fourfold compared to a traditional random variable based analysis. Including the spatial variability of other variables could further affect any reliability estimates. However, studies considering these effects are limited in number, and are largely based on engineering judgment rather than real test data. The need to develop validated models which capture spatial randomness has been expressed by diverse research groups; see for example the conclusion of an NSF-supported workshop on multi-scale material modeling [9].

The spatial variability of mechanical and geometric properties of the test series is herein modelled by assuming that the underlying random field is statistically homogeneous.

This implies that the mean, covariance, correlation properties and any higher order moments are independent of the coupon location reference. There exist many approaches to characterise the spatial variability, such as those based on variograms, correlograms, periodograms and wavelet functions [10, 11]. Even though some of these approaches are interrelated, the present study aims to estimate the correlation functions based on 'finite scale' first; other required functions, such as the spectral density, can then be evaluated numerically.

Correlation functions

The correlation functions are estimated from the test data and a suitable functional form that produces minimum error is selected. For a discrete set of *n* data points in the vector *x,* the auto-correlation function is given by [12]

$$R_{xx}(\tau_j) = \frac{1}{\sigma_x^2}\left(\sum_{i=1}^{n-j+1}(x_i - \mu_x)(x_{i+j-1} - \mu_x)\right) \quad j = 0,1,...,n \tag{1}$$

where μ_x and σ_x are the mean and standard deviations values and τ_j is the corresponding lag. A wide range of functions satisfying the required mathematical conditions are available [10, 13], variants of these functions also satisfy the model requirements. Some suitable functions for the present study are given below:

i) Spherical correlation functions

$$R(\tau) = \left(1 - \frac{|\tau|}{b}\right) \tag{2a}$$

$$R(\tau) = \left(1 - \frac{3|\tau|}{2b} + \frac{1}{2}\left(\frac{|\tau|}{b}\right)^3\right) \tag{2b}$$

ii) Exponential class

$$R(\tau) = \exp\left(-\frac{|\tau|}{b}\right) \tag{3a}$$

$$R(\tau) = \exp\left(-\left(\frac{|\tau|}{b}\right)^2\right) \tag{3b}$$

iii) Extended exponential class

$$R(\tau) = \left(1 - \frac{|\tau|}{b}\right)\exp\left(-\frac{|\tau|}{b}\right) \tag{4}$$

where *b* is the spatial correlation length. In case of oscillating correlations, other variants of Bessel functions are useful. The squared exponential model in Eq (3b) is widely referred as the Gaussian correlation function. In the theoretical studies performed using random process models the most commonly used class is, in fact, the exponential [14, 15].

Cross correlations between the variables can be obtained by a similar definition but with the functional forms being scaled by a power series to take care of the asymmetric patterns.

Correlation models of the experimental data

The centre line of the first coupon in the sequence is considered as the reference origin for modelling the process. To represent the lags, the actual spacing between coupon centres is considered. The auto and cross correlation estimates are obtained as discussed earlier and the best fitting functions are arrived at by using an unconstrained nonlinear minimization of the SSR (sum of squared residuals) with respect to the correlation length.

The spatial correlations are functions of the spatial correlation length '*b*'. If *b* is large, the corresponding random field is smoothly varying whereas the converse implies a highly correlated field. This parameter is critical from a stochastic analysis point of view, as it significantly influences the computational requirements in, for example, a stochastic finite element analysis (SFEA). A typical auto-correlation plot for the compression strength is shown in Figure 7; the correlation function oscillates at larger specimen spacing as the correlation dies down in the finite scale. The exponential correlation function in Eq. (3a) is found to give the lowest error for this variable with a *b* value of 54.88 mm. The obtained correlation length values for different functions are given in Table 1. It is clear that the correlation length is dependent on the functional form assumed and, therefore, any comparison need to be consistent with regard to the latter. It would appear that the exponential class may be adopted for such studies and this will enable meaningful comparison to be undertaken for different composite material systems.

Similar modeling is performed on significant cross correlation estimates and a typical plot is shown in Figure 8. The suitable correlation functions are obtained by modifying the standard forms. Unlike auto-correlation function, the maximum value of cross correlation function may not be one and its sign depends on the linear correlation between the variables being positive or negative. Also, the cross correlation plots are asymmetric about the centre point and the obtained models should reflect this for variable pairs. With the knowledge of correlation functions, the spectral density of the required random processes can be evaluated by considering the corresponding Fourier transform [10].

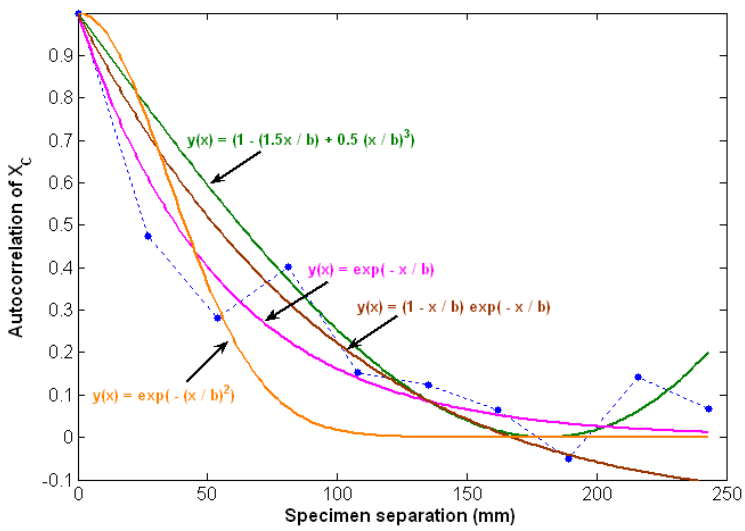


Figure 7: Auto correlation estimates of X_C

Table 1. Estimated correlation lengths for X_C

Functional form	Correlation length, b (mm)
Eq. 2b	180.33
Eq. 3a	54.88
Eq. 3b	49.66
Eq. 4	167.64

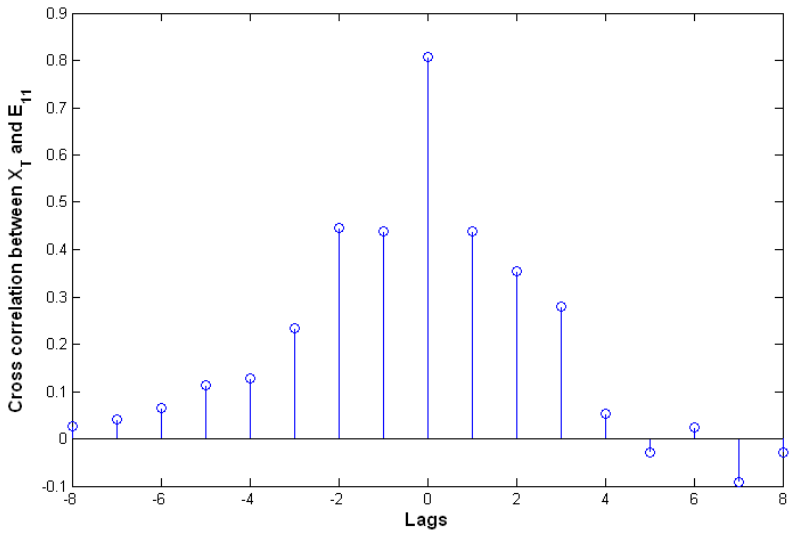


Figure 8: Cross correlation of X_T and E_{11}

CONCLUDING REMARKS

Spatial variability of FRP composites is relatively unexplored compared to traditional random variable based stochastic analysis. The spatial characteristics of GFRP composites are investigated by an extensive experimental analysis of the tensile and compressive properties and thickness of pultruded composite plates. An appropriate cutting plan is devised to extract coupons from the composite plate to model the spatial randomness. Longitudinal tensile and compressive tests are carried out on the obtained coupons and typical results of the auto and cross correlation function patterns of these variables are discussed. The auto-correlation functional forms are evaluated for the minimal error and the correlation lengths are obtained. For this class of composites a correlation length of about 50 mm in an exponential auto-correlation function seems to be a good approximation. Care should be taken while evaluating the cross correlations to consider the asymmetry and the possible negative dependencies. Combined with a previous study on aged and weathered panels, this study is expected to provide spatial variability modeling options for different levels of physical variations in composite properties. It is hoped that these studies aid in formulating appropriate SFEA and multi-scale methods for composite structures.

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