MESHLESS MODELLING OF COMPOSITE LAMINATES

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SUMMARY
For virtual testing of composite components to become a reality, a detailed and accurate simulation of damage and crack propagation is necessary. Unfortunately, such problems easily overwhelm the classical finite element approach, especially when the cracks are modelled using common FE routines (discrete crack model). Meshless methods, especially the element-free Galerkin method (EFG), are attractive for the simplicity of topology regeneration. This paper investigates certain issues of interest in EFG modelling of beam delamination, namely refinement, influence domain and integration schemes.

Keywords: Meshless, element-free Galerkin, crack propagation, model parameters.

INTRODUCTION
Because of the lack of element connectivity, meshless approaches are well suited for problems where the topology evolves during the analysis. An example is the propagation of a crack in an isotropic concrete beam under four point bending load, Carpinteri[1], as shown in Figure 1.

The EFG method, Belytschko et al.[2], has the potential to enhance the simulation of composite failure in a significant way, for two important reasons:

- The ability to predict the crack propagation path as an outcome of the analysis, as opposed to the current use of interface/cohesive elements where the likely path are determined a priori.

- The ease of generating additional nodes in the vicinity of the crack tip, for greater accuracy only where required. The refinement may be advanced as the crack propagates, all without the burden of rebuilding the connectivity network (mesh) required in the finite element (FE) approach.

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Although the element-free Galerkin method has been around for a number of years, it has not yet been used for the simulation of composite beam delamination, even though it is ideal for the implementation of established crack propagation approaches such as the virtual crack closure technique (VCCT), Rybicki and Kanninen [3].

The challenge of implementing this method is to ensure numerical robustness and accuracy. For example, the node-based moving least squares (MLS) interpolation scheme used in the present formulation, uses a number of parameters that cannot always be determined objectively. This paper will investigate two aspects: (1) The determination of the nodal influence domain and the effects of local refinement, and (2) integration schemes for shear locking relief in thin beam modelling.

**THE MOVING LEAST SQUARES INTERPOLATION**

The moving least squares (MLS) interpolation is a data fitting technique proposed by Nayroles et al. [4], that is based on the idea that the closer the supporting nodes are to the target point, the greater the influence they should have on the value of the interpolated function. The derivation of trial functions of any order, based on MLS can be found in Liu [5], which also presents the complete method including the derivation of the Galerkin weak form of the equilibrium equations, integrated via background cells (discussed later in this paper). In this section, the MLS trial functions are derived for a first order, two-dimensional (2D) interpolation.

Denoting the spatial coordinates in two-dimensions by \(x\) and \(y\), a basis of first order (linear) monomials is \(p^T(x,y) = [1 \ x \ y]\). The approximation \(u^h\) to a function \(u\) is then written as:

\[
u^h(x,y) = p^T(x,y)a(x,y),\]

where \(a\) is a vector of \(m(=3)\) coefficients, determined for the optimal fit over a set of \(n\) support nodes taken in a compact neighborhood around the point with coordinates \((x,y)\).
Explicitly, the interpolated function is: 

\[ u^h(x, y) = \begin{bmatrix} a_1(x, y) \\ a_2(x, y) \\ a_3(x, y) \end{bmatrix} \]

such that \( u^h_i = u^h(x_i, y_i) = \begin{bmatrix} a_1(x_i, y_i) \\ a_2(x_i, y_i) \\ a_3(x_i, y_i) \end{bmatrix} \).

The coefficients \( a_i(x, y) \) are obtained by minimizing the weighted least squares norm of the interpolation error:

\[ J(x, y) = \sum_{i=1}^{m} w_i(x, y) (u^h_i - u_i)^2, \]

The weight or kernel \( w_i(x, y) \) is a bell-shaped function of the distance between the interpolation point \((x, y)\) and the support node \((x_i, y_i)\). This results in a system of linear equations:

\[ a(x, y) = A^{-1}(x, y)B(x, y)u \]

where 
\[
A(x, y) = \sum_{i=1}^{m} w_i(x, y)p(x_i, y_i)p^T(x_i, y_i), \\
B(x, y) = \begin{bmatrix} w_1(x, y)p(x, y) & w_2(x, y)p(x, y) & \cdots & w_n(x, y)p(x, y) \end{bmatrix}, \\
u = [u_1 \ \cdots \ u_n]^T
\]

Consequently, the sought trial functions are:

\[ \{\Phi(x, y)\}^T = p^T(x, y)A^{-1}(x, y)B(x, y), \quad u^h(x, y) = \Phi_i(x, y)u_i \]

A few observations are made at this point:

- The kernel \( w_i \) is usually expressed in function of a variable \( d_i \),

\[ d_i = \frac{\|X - X_i\|}{R}, 0 \leq d_i \leq 1, \]
where \( R \) is the radius of the support domain, i.e. the maximum distance between the (integration) point of coordinates \( X = (x, y) \), \( R = \max_i \left( \| X - X_i \| \right) \). This is referred to as radial kernel in multi-dimensional analysis.

For the analysis of slender beams in two dimensions (e.g. plane strain), the aspect ratio between longitudinal and transverse scales makes the use of the norm of the total distance unfeasible. Instead, the distances along each direction are considered separately, referred to as tensor kernel.

\[
d_i = \frac{(x-x_i)(y-y_i)}{R_x R_y}, \quad 0 \leq d_i \leq 1 \quad \text{and} \quad R_x = \max_i (x-x_i), \quad R_y = \max_i (y-y_i)
\]

- The trial functions are calculated for each (integration) point of the domain, as the coefficients \( a_i \) vary with the position coordinates \((x, y)\), hence the 'moving' aspect of the interpolation.

- The trial functions calculation involves the inverse of the matrix \( A \); a 'good' support domain is essential to ensure a good conditioning of that matrix.

- Although a linear basis of monomials is used, the trials functions are not linear and in fact, are not polynomials, because of the variation of coefficients \( a \).

- If a complete basis of \( m = 4 \) monomials is used, \( p^T = [1 \ x \ y \ xy] \), it can be shown that the interpolation over the domain enclosed by the four support nodes shown in Figure 2 are identical to the isoparametric shape functions of an element made of these four nodes. This can be utilized near boundaries, in lieu of meshless/FE transition elements, as is often done at boundaries.

![Figure 2: Nodes distributed at corners of integration cell](image)

**NODAL INFLUENCE AND REFINEMENT**

The support domain, that is the set of nodes that support the MLS interpolation, is found by using nodal influence domains: First, each node has an influence over a certain neighborhood that is calculated or assigned. Then for a given integration point, all the nodes for which the integration point falls within their influence domain are selected to
constitute the support domain of the integration point. This is illustrated in Figure 3 for radial kernels.

![Figure 3: Nodal domain of influence and support nodes selection](image)

In this figure, the domain of influence includes the node of interest as well as the three nodes closest to it; the circular domain of influence is shown with the corresponding node at its centre. Those domains of influence that enclose the integration point, marked by a plus sign, are represented by a dashed line. The set of support nodes for the integration point is hence the nodes with those domains of influence, and the radius of the support domain $R$ is also shown. As mentioned, tensor kernels are required for modelling slender structures, hence distinct radii of influence are required, for each dimension. Moreover, it was found that using two radii per direction as illustrated in Figure 4, a total of four parameters representing the distances from the node to the right, left, top and bottom edges, of its domain of influence yields even better results.

![Figure 4: Specification of nodal influence domain. Left: with two radii. Right: with four radii.](image)
Several methods can be used to determine the nodal domain of influence:
(1) fixed radius where a fixed size is assigned to all nodes;
(2) third closest node, illustrated in Figure 3;
(3) minimum polygon where the domain of influence is made of the minimum
set of nodes forming a convex enclosure around the given node.

When the nodes are uniformly spaced, all three methods yield the same resulting
support set. However, when local refinement is utilized and there is a spatial variation of
nodal density, the last method is better suited to reproduce that variation, especially with
laminated structures for which the two dimensions (longitudinal and transverse) need to
be considered distinctly.

In order to appreciate the effect of refinement, the interpolation of a field variable is
performed using various refinement patterns. The chosen variable is the transverse
normal stress $\sigma_{22}$ in a double cantilever beam (DCB) specimen [6], and nodal values
are obtained from a simple stress analysis with ABAQUS/Standard, and shown in
Figure 5.

$$RMS = \frac{\sum_{i=1}^{N} [S_{MLS}(i) - S(i)]^2}{N},$$
where $S$ is the value of the stress $\sigma_{22}$ and $N$ the number of points where the error is evaluated, in a region of size 10mm (5mm before and 5mm after) around the crack tip.

Table 1: Root mean square of the MLS interpolation error

<table>
<thead>
<tr>
<th>PATTERN</th>
<th>RMS</th>
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<tbody>
<tr>
<td><img src="a" alt="Diagram" /></td>
<td>120 MPa</td>
</tr>
<tr>
<td><img src="b" alt="Diagram" /></td>
<td>4.9 MPa</td>
</tr>
<tr>
<td><img src="c" alt="Diagram" /></td>
<td>5.7 MPa</td>
</tr>
</tbody>
</table>
Table 1 shows that the interpolation is improved as the number of nodes placed in the region around the crack tip is increased. However, if the last pattern (c) is selected, and the refinement is made too dense: instead of 5 subdivisions, 40 subdivisions are used in the cells near the crack tip, as per Figure 6, then it becomes impossible to calculate the MLS shape functions, as the relative closeness of two nodes in the support set causes the matrix A to become singular. This does not occur when the refinement, as dense as desired, is placed throughout the entire thickness of the beam.

Figure 6: Support domain outside refinement zone for very dense refinement

It could be argued that this formulation is identical to the FE method, especially since cells are subdivided to match the nodal discretization. In fact, a 'unit cell' stiffness matrix could be constructed and subsequently mapped onto each cell of the domain since it is guaranteed that all nodes will be located at (or near) the cell's corners. However, the advantage of using the EFG approach is, as mentioned, the possibility to alter the topology during the course of a single analysis.

**SHEAR LOCKING AND INTEGRATION SCHEME**

Using the refinement scheme proposed in the previous section, attempts to model the delamination of a double cantilever beam were met with mixed success, as the number of nodes required along the beam was substantial, Figure 7.
This is the typical shear locking problem, common when structures with high aspect ratio, linear elements are used. In FE, a simple solution is the use a one-point reduced integration scheme\(^2\), but is not directly applicable to EFG as it is desirable not to assimilate integration cells to finite elements.

Although MLS shape functions are not identical to polynomials of the order of the basis functions used, the discrepancy is not significant when a very small number of nodes are used in the support set \((m \approx n)\). Therefore the shear locking problem should be expected here as well as in FE.

An option is to use selective integration whereby the bending terms of the stiffness matrix are integrated with full integration, over the integration cells, as usual; and the shear terms of the stiffness matrix are computed using nodal integration. This was proposed for volumetric locking by Dolbow and Belytschko[7] and was successfully applied here for the analysis of a few test cases, described next.

**Case 1: Clamped beam**

A beam clamped at one end and subjected to a transverse displacement at the other end was analyzed using both full and the suggested mixed/nodal integration schemes and results reported, along with theoretical results, in Table 2.

\(^2\) Although the procedure may result in an ill-conditioned discrete system, involving rigid body modes.
When full integration is used, convergence is attained only when the nodal spacing is nearly identical in both directions. In contrast, the mixed nodal/Gauss integration scheme is similar to reduced integration in FE and converges with the coarsest discretization along the beam.

**Case 2: Double cantilever beam**

The double cantilever beam (DCB) test is one of the simplest delamination tests. Test setup and geometry, shown in Figure 8, corresponds to the NAFEMS benchmark[6].

![Double cantilever beam: geometry and test setup](image)

<table>
<thead>
<tr>
<th>E=125GPa</th>
<th>v=0.29</th>
<th>G_{c1}=G_{c2}=280N/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>L=100mm</td>
<td>a=30mm</td>
<td>h=3mm</td>
</tr>
</tbody>
</table>

B is the width (through the page) of the DCB. The propagation of the crack under quasi-static displacement loading is simulated using the proposed refinement and integration schemes for EFG, and using VCCT for crack advancement.
The discretizations corresponding to initial and final phases of propagation are shown in Figure 9, and it can be seen that the total number of nodes utilized here is significantly less than what was initially required under the full integration scheme, Figure 7. The solution of the simulation in terms of force versus applied displacement is provided in Figure 10 which shows that the 10mm nodal spacing yields the converged solution.
CONCLUDING REMARKS

In this paper, a meshless method was investigated for simulating delamination in composite laminates. The main motivation for the approach was the potential for modelling cracks with complex propagation paths, through the simplicity with which nodes can be created, removed or translated within the domain. On the other hand, EFG is known to be vulnerable to a number of numerical parameters, including the relative refinement size and the selection of the nodal domain of influence.

The outcome of this investigation was that crack tip refinement enhanced the solution, provided that the refinement was done through the entire thickness of the laminate. Additionally, a total of four radii of influence, two for each direction, were required to specify the nodal domain of influence for a two-dimensional problem.

With respect to the integration scheme, a mixed nodal and Gaussian quadrature relieved the shear locking problem for beam modelling, since MLS shape functions were nearly linear in the suggested configuration. This was hence similar to selective reduced integration in the FE approach.

The double cantilever beam benchmark for delamination was then simulated successfully using a rather coarse mesh size. This result sustains the idea that the element-free Galerkin method has the potential to play an important role in the simulation of complex damage mechanisms in composite laminates.

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References