OPTIMISATION OF LAMINATED COMPOSITE PLATES CONSIDERING DIFFERENT FAILURE CRITERIA

R.H. Lopez[1], M.A. Luersen[2], J.E.Souza de Cursi[1]
[1] Laboratoire de Mécanique de Rouen, Institut National des Sciences Appliquées (INSA) de Rouen, 76860, Saint-Etienne du Rouvray, France
{rafael.holdorf_lopez, eduardo.souza}@insa-rouen.fr
[2]Mechanical Engineering Department, Universidade Tecnológica Federal do Paraná (UTFPR), Av. Sete de Setembro, 3165, 80230-901, Curitiba, Brazil
luersen@utfpr.edu.br

SUMMARY

The purpose of the present work is to analyse the influence of the failure criterion on the minimum weight and cost of laminated plates subjected to in-plane loads. Three different failure criteria are tested independently: maximum stress, Tsai-Wu and the Puck failure criterion. The optimisation problem is solved by a genetic algorithm.

Keywords: laminated composites, optimisation, Puck failure criterion

INTRODUCTION

A laminated composite is usually tailored according to the designer's needs by choosing the thickness, number and orientation of the plies. To achieve the best results, optimization techniques have been developed and, among them, the genetic algorithm (GA) has been widely used in the design of composite structures [1, 2, 3]. Known advantages of GAs include the following: (i) they do not require gradient information and can be applied to problems where the gradient is hard to obtain or does not exist; (ii) they do not get stuck in local minima; (iii) they can be applied to nonsmooth or discontinuous functions; and (iv) they furnish a set of optimal solutions instead of a single one, thus giving the designer a set of options. In the optimization of laminated composites, the ply thicknesses are often predetermined and the ply orientations are usually restricted to a small set of angles due to manufacturing limitations. This leads to problems of discrete or stacking-sequence optimization. Many objective functions have been used, such as the buckling load [1, 3] (to be maximized), the stiffness in one direction (to be maximized), the strength (to be maximized), as well as the weight or material cost [2, 4, 5] (to be minimized). A common constraint in laminated optimisation problems is the first ply failure, using well knows failure criteria (e.g., Tsai-Hill, Hoffman, Tsai-Wu). In addition to the failure criterion, other restrictions usually involved in the optimal design of laminated composites are laminate symmetry and balance, and a maximum number of contiguous plies (often used to prevent matrix cracking). One of the main criticisms of many studies related to optimal composite design is the use of failure criteria based on the von Mises or Hill yield criteria, which are more suitable for ductile materials [6]. In fact, as the failure behaviour of composite parts is similar to that of brittle material, it would be more appropriate to use criteria suited to materials that exhibit brittle fractures, such as Mohr's criterion. A suitable criterion for composites that takes this fracture behaviour into account is, for instance, the Puck failure criterion (PFC) [6].

This work analyzes the effect of the choice of failure criterion on the minimum weight and cost of laminates. Three different failure criteria are considered: Maximum Stress (MS), Tsai-Wu (TW) and the Puck failure criterion (PFC). Special attention is given to the PFC, since, as previously mentioned, it appears to be better suited to the real behaviour of composite parts. A GA is used to achieve optimization, and constraints related to the first ply failure criterion as well as the symmetry and balance of the laminated plate are taken into account.

FAILURE CRITERIA

Failure analysis of laminated composites is usually based on the stresses in each lamina in the principal material coordinates [7]. The failure criteria can be classified in three classes: limit or non-interactive theories (e.g., maximum stress or maximum strain), interactive theories (e.g., Tsai-Hill, Tsai-Wu or Hoffman) and partially interactive or failure mode-based theories (PFC) [8]. In the present work, one criterion from each class is considered.

Maximum Stress Failure Criterion (MS)

According to the maximum stress theory, failure is predicted when a maximum stress in the principal material coordinates exceeds the respective strength. That is,

$$\begin{array}{ll} \sigma_1 \geq X_T \quad \text{or} \quad \sigma_2 \geq Y_T & \text{(for tensile stresses);} \\ \sigma_1 \leq -X_C \quad \text{or} \quad \sigma_2 \leq -Y_C & \text{(for compressive stresses)} \\ |\tau_{12}| \geq S_{12} & \text{(for shearing stresses)} \end{array} \tag{1}$$

where σ_1 and σ_2 are the normal stresses in the directions 1 and 2, respectively; τ_{12} is the shear stress in the plane 1-2; X_T and X_C are the tensile and compressive strengths parallel to the fibre direction, respectively; Y_T and Y_C are the tensile and compressive strengths normal to the fibre direction, respectively; and S_{12} is the shear strength.

Tsai-Wu Failure Criterion (TW)

The Tsai-Wu criterion, which is intended for use with orthotropic materials, is derived from the von Mises yield criterion. It states that the lamina fails when the following condition is satisfied

$$F_{11}\sigma_1^2 + 2F_{12}\sigma_1\sigma_2 + F_{22}\sigma_2^2 + F_{21}\tau_{12}^2 + F_1\sigma_1 + F_2\sigma_2 \geq 1 \tag{2}$$

where F_i and F_{ij} are parameters that are a function of the strength properties X_T, X_C, Y_T, Y_C and S_{12} (see, for instance, [7]).

Puck Failure Criterion (PFC)

In this section, only the main features of the PFC are presented. The entire derivation can be found in [6]. The PFC follows Mohr's hypothesis that fracture is caused by the stresses that act on the fracture plane. It involves two main failure modes: Fibre Failure (FF) and Inter-Fibre Failure (IFF) [6]. FF is based on the assumption that fibre failure under multiaxial stresses occurs at the same threshold level at which failure occurs for uniaxial stresses. Instead of dealing with the principal material coordinates (axes 1-2-3), IFF equations are derived based on the axes corresponding to the failure plane. These axes are shown in Fig. 1, where θ_{fp} represents the angle at which failure occurs. The PFC therefore provides not only a failure factor, but also the inclination of the plane where failure will probably take place, thus allowing a much better assessment of the consequences of IFF in the laminate.

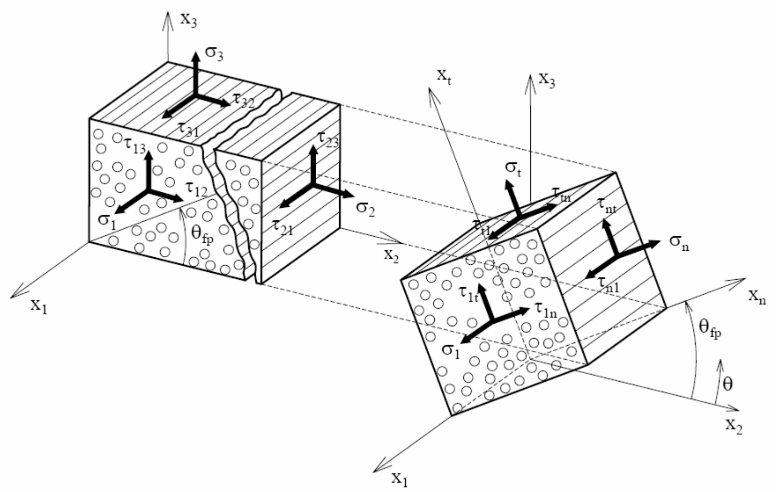

Figure 1: Transformation from the 1-2-3 axes to the failure plane axes (σ_n, τ_{nt}, τ_{n1}).

IFF is subdivided into three failure modes, as described in [6], which are referred to as A, B and C. Mode A occurs when the lamina is subjected to tensile transverse stress, whereas modes B and C correspond to compressive transverse stress. The classification is based on the idea that a tensile stress σ_n>0 promotes fracture, while a compressive stress σ_n <0 impedes shear fracture. For σ_n <0, the shear stresses τ_{nt} and τ_{n1} (or just one of them) have to face an additional fracture resistance, which increases with $|\sigma_n|$, analogously to an internal friction [6]. The distinction between modes B and C is based on their failure angles, which are 0º for mode B and a different value for mode C. In addition, failure mode C is considered more severe, since it produces oblique cracks and may lead to serious delamination. The equations for the PFC are summarized in Table 1, where we also introduce *a weakening factor* f_w, which decreases the strength of the laminate due to high stress in the fibre direction. According to [7], f_w is given by

$$f_w = \left(0.9 f_{E(FF)}\right)^n \qquad (3)$$

where $f_{E(FF)}$ is the failure effort for FF in the lamina, and n depends on the matrix of the laminate (for instance, n = 6 for epoxy). We refer henceforth to this situation as PFC_fw, while we denote the situation where f_w = 0 by PFC.

Table 1: Equations for the PFC [6].

Type of failure	Failure Mode	Failure Condition ($f_{E(FF)}$ *or* $f_{E(IFF)}$)	Condition for validity
Fibre Failure (FF)	Tensile	$\frac{S}{\varepsilon_{1T}} = 1$	if $S \geq 0$
$S = \varepsilon_1 + \frac{v_{f12}}{E_{f1}} m_{\sigma 1} \sigma_2$	Compressive	$-\frac{S}{\varepsilon_{1C}} + (10\gamma_{21})^2 = 1$	if $S < 0$
Inter Fibre Failure (IFF)	Mode A	$\sqrt{\left(\frac{\tau_{21}}{S_{21}}\right)^2 + \left(1 - p_{\perp\parallel}^{(+)} \frac{Y_T}{S_{21}}\right)^2 \left(\frac{\sigma_2}{Y_T}\right)^2} + p_{\perp\parallel}^{(+)} \frac{\sigma_2}{S_{21}} + f_w = 1$	$\sigma_2 \geq 0$
	Mode B	$\frac{1}{S_{21}}\left(\sqrt{\tau_{21}^2 + \left(p_{\perp\parallel}^{(-)}\sigma_2\right)^2} + p_{\perp\parallel}^{(-)}\sigma_2\right) + f_w = 1$	$\sigma_2 < 0$ and $0 \leq \left\lvert\frac{\sigma_2}{\tau_{21}}\right\rvert \leq \frac{R_{\perp}^{A}}{\lvert\tau_{21}\rvert}$
	Mode C	$\left[\left(\frac{\tau_{21}}{2\left(1 + p_{\perp\perp}^{(-)} S_{21}\right)}\right)^2 + \left(\frac{\sigma_2}{Y_C}\right)^2\right] \frac{Y_C}{(-\sigma_2)} + f_w = 1$	$\sigma_2 < 0$ and $0 \leq \left\lvert\frac{\tau_{21}}{\sigma_2}\right\rvert \leq \frac{\lvert\tau_{21}\rvert}{R_{\perp}^{A}}$
Definitions	$p_{\perp\parallel}^{(+)} = -\left(\frac{d\tau_{21}}{d\sigma_2}\right)_{\sigma_2=0}$ of (σ_2, τ_{21}) curve, $\sigma_2 \geq 0$	$p_{\perp\parallel}^{(-)} = -\left(\frac{d\tau_{21}}{d\sigma_2}\right)_{\sigma_2=0}$ of (σ_2, τ_{21}) curve, $\sigma_2 \leq 0$	
Parameter relationships	$R_{\perp\perp}^{A} = \frac{Y_C}{2\left(1 + p_{\perp\perp}^{(-)}\right)} = \frac{S_{21}}{2p_{\perp\parallel}^{(-)}}\left(\sqrt{1 + 2p_{\perp\parallel}^{(-)}\frac{Y_C}{S_{21}}} - 1\right)$	$p_{\perp\perp}^{(-)} = p_{\perp\parallel}^{(-)} \frac{R_{\perp\perp}^{A}}{S_{21}}$	$\tau_{21c} = S_{21}\left(\sqrt{1 + 2p_{\perp\perp}^{(-)}}\right)$

GENETIC ALGORITHM

Genetic algorithms loosely parallel biological evolution and were originally inspired by Darwin's theory of natural selection. The specific mechanics of genetic algorithms often use the language of microbiology, and their implementation frequently mimics genetic operations. A GA generally involves genetic operators (such as crossover and mutation) and selection operators intended to improve an initial random population. Selection usually involves a fitness function characterizing the quality of an individual in terms of the objective function and the other elements of the actual populations. Thus, a GA usually starts with the generation of a random initial population and iterates by generating a sequence of populations from the initial one. At each step the genetic operators are applied to generate new individuals. The fitness of each available individual is computed and the whole population is ranked according to increasing fitness. A subpopulation is then selected to form a new population. In this work, tournament selection is applied, and the process is repeated until a stopping condition is satisfied.

The classical binary representation is ***not*** used; instead, the allowed angle values represent the genes of the chromosomes (i.e., $[0_2 \quad \pm 45 \quad 90_2]$). In our numerical examples, we consider the optimization of a hybrid laminated composite (see Problem 2 in Section Numerical Results). In this case, each individual in the population is represented by two chromosomes: the first describes the angle of orientation of the layer, and the second the layer material. When genetic operators are applied, they work simultaneously on both chromosomes of each individual. The genetic operators employed in this work are crossover, mutation, gene swap, stack-deletion and stack-addition. Crossover is the basic genetic operator. It involves combining the information from two parents to create one or two new individuals. The X1-thin crossover operator was used [5]. The mutation operator, which randomly changes the value of a gene in the chromosome, must be applied in the GA to guarantee gene diversity so that the algorithm does not get stuck in local minima. The gene-swap operator selects two genes randomly from a laminate and then swaps them. It was introduced by Le Riche and Haftka [5]. In the example of the hybrid laminated composite referred to above, the crossover points of the two chromosomes for each individual are the same. In addition, when the gene-swap is applied, both orientation and material are swapped. We introduce two supplementary operators. The first one adds and the second deletes a lamina of the composite part under design. Both operators always act on the lamina closest to the mid-surface of the laminate, since it has the weakest effect on the bending properties of the structure. This feature may be important when buckling is involved, since buckling is highly dependent on the bending properties of the laminate. It is more convenient to delete the lamina with the weakest influence on the bending properties, since it is observed in practice that the algorithm rapidly converges to the best design for the most external laminae.

In GAs, the most common ways of handling constraints are data structure, repair strategies and penalty functions [9]. The symmetry and balance of the laminate were handled by using the data structure strategy, which consists of coding only half of the laminate and considering that each stack of the laminate is formed by two laminae with the same orientation but opposite signs (for instance, $\pm 45^0$). A double-multiplicative dynamic penalty approach [9] was used to take into account the failure criteria. This approach leads to a penalty term being added to the objective function. The main advantage of this approach is that the penalization parameters do not need to be tuned.

NUMERICAL RESULTS

Problem 1 – Weight minimization

Let us consider the minimal weight design of a laminated composite plate under the constraints of laminate symmetry and balance as well as the first ply failure criteria. The allowable orientation angle values are 0_2, ±45 and 90_2 degrees. Thus, the optimization problem can be stated as

$$\begin{array}{ll} \text{Find:} & \{\theta_k,\ n\},\ \theta_k \in \{0_2,\ \pm 45,\ 90_2\},\ k = 1 \text{ to } n \\ \text{Minimize:} & \text{Weight} \\ \text{Subject to:} & \text{First fibre failure constraint: MS, TW or PFC} \end{array} \quad (4)$$

where θ_k is the orientation of each stack of the laminate and n the total number of stacks. As already mentioned, each stack is composed of two layers to guarantee balance.

Different loading conditions were considered, and the optimization problem was solved for three different first ply failure criteria. We considered both the PFC_fw and PFC approach, the latter of which does not take into account the influence of the stress σ_1 in IFF. As previously observed, PFC corresponds to $f_w = 0$, while PFC_fw involves Eq. (3).

Let us consider a carbon-epoxy square laminated plate subjected to in-plane loads per unit length N_x, N_y and N_{xy}, as shown in Fig. 2. Each layer is 0.1 mm thick, and the length and width of the plate are 1.0 m. The elastic material properties of the layers are E_1 = 116600 MPa, E_2 = 7673 MPa, G_{12} = 4173 MPa, Poisson's ratio ν_{12} = 0.27 and mass density ρ = 1605 kg/m^3, and the failure properties of the lamina are given in Table 2. The plate is analysed using classical lamination theory [7].

The optimization results for uniaxial loading ($N_{xx} \neq 0$, $N_{yy} = N_{xy} = 0$) are given in Table 3. It can be noted that the PFC provides not only the failure factor (f_E), but also the most probable failure mode (in brackets after the failure effort). The amount of stacks shown in all the tables correspond to the symmetric part of the laminate, and the total number is twice this value. The last column of the table (%) gives the relative weight difference between the laminate optimized using the PFC and the laminate optimized using the other failure criteria.

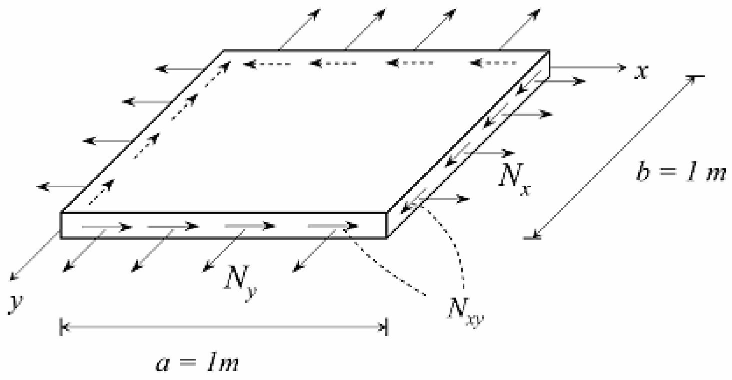

Figure 2: Laminated composite plate subjected to in-plane loads.

Table 2: Strength properties of carbon-epoxy (CE) lamina.

X_T	2062 MPa	Y_C	240 MPa	ε_{1T}	0.0175	$m_{\sigma f}$	1.1
X_C	1701 MPa	S_{21}	105 MPa	ε_{1C}	0.014	$p_{\perp\parallel}^{(+)}$	0.3
Y_T	70 MPa	E_{f1}	230 GPa	ν_{f12}	0.23	$p_{\perp\parallel}^{(-)}$	0.25

Table 3: Optimal design for different failure criteria and uniaxial loading.

Loading (N/mm)			Failure Criteria (FC) and weight			Stacking and weight difference			
Nx	Ny	Nxy	FC	Weight (N)	f_E	0_2	±45	90_2	%[a]
-10000	0	0	PFC	102	0.96 (FF)[b]	16			-
			PFC_fw	102	0.96 (FF)	16			0
			TW	95	0.96	15			-6.9
			MS	95	0.98	15			-6.9
10000	0	0	PFC	83	0.94 (FF)	13			-
			PFC_fw	83	0.94 (FF)	13			0
			TW	83	0.86	13			0
			MS	83	0.93	13			0

[a] percentage difference in relation to the weight obtained using the PFC.
[b] for PFC and PFC_fw, the letters in brackets indicate the failure mode that had the highest coefficient.

It can be seen that for tensile uniaxial loading the three failure criteria led to the same results. However, for the compressive case, the TW and MS criteria generated lighter structures. In addition, TW results in a lower failure effort. The failure mode identified using the PFC was FF, which means that rupture of the laminate would probably occur in the direction of the fibre.

The results for positive-positive biaxial loading and different shear load values are given in Table 4. The weight of the structure predicted by the PFC, PFC_fw and MS are the same for all the shear loading conditions. Furthermore, except for the highest shear load, these criteria generated lighter structures than those obtained with the TW criterion. For the PFC, the most probable failure mode is IFF mode A, which corresponds to the failure plane $\theta_{fp} = 0^o$ (see Fig. 2). It can be seen from these results that for $N_{xy} = 0$, PFC and MS produced the same optimal weight but different stacking sequences. The stacking sequence obtained using the MS criterion was therefore tested with the PFC and PFC_fw criteria and was found to lead to a failure (mode A) for both PFC and PFC_fw. This means that for the same weight and very similar failure efforts, the MS criterion yielded a structure that does not satisfy the PFC restrictions on failure. This analysis can be extended relatively straightforwardly to the other shear loads considered in Table 4, as the stacks are all oriented at ±45.

Table 4: Optimal design for different failure criteria and biaxial (+,+) loading.

Loading (N/mm)			Failure Criteria (FC) and weight			Stacking and weight difference			
Nx	Ny	Nxy	FC	Weight (N)	f_E	0_2	±45	90_2	%
3000	3000	0	PFC	108	0.96 (A)	6	5	6	-
			PFC_fw	108	0.96 (A)	2	13	2	0
			TW	114	0.97	6	6	6	5.6
			MS	108	0.96	5	7	5	0
3000	3000	500	PFC	114	1.00 (A)		18		-
			PFC_fw	114	1.00 (A)		18		0
			TW	121	0.97		19		6.1
			MS	114	1.00		18		0
3000	3000	1000	PFC	127	0.98 (A)		20		-
			PFC_fw	127	0.98 (A)		20		0
			TW	127	0.99		20		0
			MS	127	0.98		20		0

Table 5 shows the results for optimum weight for a negative-negative biaxial load condition and different shear loads. In this situation, the result furnished by each failure criterion is quite different. The TW and MS criteria led to structures about 25% lighter than those obtained using the PFC. Thus, the failure criterion has a significant influence on the optimal weight of the structure in this situation. Here again, the failure mode determined by the PFC was IF. It is interesting to note that the case where the shear load is equal to zero led to different stacking sequences for each criterion. For this condition (when $N_{xy} = 0$), TW and MS stacking sequences were tested according to the PFC, and the analysis showed that the design based on the TW criterion would fail in mode C, while that based on the MS criterion would not fail.

The results for positive-negative biaxial loading are shown in Table 6. In this situation, the results obtained using the different criteria differed significantly. Compared with PFC, TW overestimates the optimal weight by about 50% and the MS criterion underestimates the weight of the plate by about 30%. In this situation, the optimal weight is heavily dependent on the failure criteria and loading condition. The most probable failure mode predicted by the PFC depends on the shear load and the value of f_w.

Table 5: Optimal design for different failure criteria for biaxial (-,-) loading.

Loading (N/mm)			Failure Criteria (FC) and weight			Stacking and weight difference			
Nx	Ny	Nxy	FC	Weight (N)	f_E	0_2	±45	90_2	%
-3000	-3000	0	PFC	57	0.93 (FF)	3	3	3	-
			PFC_fw	57	0.93 (FF)		9		0
			TW	51	0.80	4		4	-10.8
			MS	57	0.91	4	1	4	0
-3000	-3000	500	PFC	76	0.95 (FF)		12		-
			PFC_fw	76	0.95 (FF)		12		0
			TW	57	0.73		9		-24.7
			MS	64	0.96		10		-16.3
-3000	-3000	1000	PFC	95	0.95 (FF)		15		-
			PFC_fw	95	0.95 (FF)		15		0
			TW	64	0.75		10		-33.1
			MS	70	1.00		11		-26.4

Table 6: Optimal design for different failure criteria for biaxial (+,-) loading.

Loading (N/mm)			Failure Criteria (FC) and weight			Stacking and weight difference			
Nx	Ny	Nxy	FC	Weight (N)	lambda	0_2	±45	90_2	%
3000	-3000	0	PFC	64	1.00 (A)	5		5	-
			PFC_fw	76	0.86 (FF)	7		5	19.2
			TW	95	0.91	9		6	49.0
			MS	64	1.00	5		5	0
3000	-3000	500	PFC	89	0.94 (FF)	7	2	5	-
			PFC_fw	95	0.96 (A)	7	3	5	7.1
			TW	114	0.97	10	2	6	28.6
			MS	70	0.99	6	1	5	-21.4
3000	-3000	1000	PFC	108	0.95 (FF)	7	6	3	-
			PFC_fw	121	0.97 (A)	6	7	6	11.8
			TW	134	1.00	10	5	6	23.6
			MS	76	0.98	5	2	5	-29.4

Problem 2 – Material cost minimization of a hybrid laminate

In this problem, the material cost minimization of a hybrid laminated composite plate is described. Two types of layers are considered: carbon-epoxy (CE) and glass-epoxy (GE). The former is lighter and stronger, while the latter has a cost advantage as the price per square meter of this laminate is about 8 times less. As in Problem 1, the laminate is subjected to symmetry and balance constraints as well as the first ply failure criteria. A maximum weight constraint is also used in this problem. Thus, the optimization problem reads as follows

Find: $\{\theta_k,\ \mathrm{mat}_k,\ n\},\ \theta_k \in \{0_2,\ \pm 45,\ 90_2\},\ \mathrm{mat}_k \in \{\mathrm{GE},\ \mathrm{CE}\},\ k = 1 \text{ to } n$

Minimize: Material cost (5)

Subject to: First fibre failure constraint and a maximum weight of 70 N

In this problem, each CE and GE layer is assumed to cost 1 and 8 monetary units (m.u.), respectively. The properties of the CE layer and the plate dimensions are the same as those in Example 1. The elastic material properties of the GE layers are E_1 = 37600 MPa, E_2 = 9584 MPa, G_{12} = 4081 MPa, Poisson's ratio ν_{12} = 0.26 and mass density ρ = 1903 kg/m^3, and the failure properties of the lamina are shown in Table 7. The in-plane applied loads are fixed values (N_x = 2000 N/mm and N_y = -2000 N/mm), and the optimization results are shown in Table 8. The underlined figures for the orientation correspond to GE stacks, and the remaining figures to CE stacks.

Table 7: Strength properties of glass-epoxy (GE) lamina.

X_T	1134 MPa	Y_C	150 MPa	ε_{1T}	0.0302	$m_{\sigma f}$	1.3
X_C	1031 MPa	S_{21}	75 MPa	ε_{1C}	0.0295	$p_{\perp\parallel}^{(+)}$	0.3
Y_T	54 MPa	E_{f1}	72 GPa	ν_{f12}	0.22	$p_{\perp\parallel}^{(-)}$	0.25

It is interesting to note that the optimum obtained followed the same pattern in every case. All layers with an orientation of 0º are made of CE, while those with an orientation of 90º are made of GE. In addition, the GE laminae were the closest to failure. The cheapest structure (i.e, with the lowest material cost) was obtained using the PFC, while the TW criterion resulted in a material cost over 30% higher and yielded the heaviest structure. Table 8 also shows the maximum failure factor for the CE and GE laminae. The TW criterion yielded the largest gap between the maximum failure efforts for the two different materials at the optimum. Again, the PFC provides not only the failure effort, but also the expected failure mode of the structure. For example, the PFC predicts that the most probable failure mode is FF, while PFC_fw predicts IFF (mode A). As in weight minimization, each failure criterion yielded a different optimum. This reinforces the idea that the failure criterion significantly modifies the optimal design.

Table 8: Optimal design of the laminate for different failure criteria.

Cost and weight			Failure Criteria (f_E)		Stacking and cost difference	
Failure Criterion	Cost (m.u.)	Weight (N)	CE	GE	Stacking sequence	%
PFC	144	55.57	0.81 (C)	0.95 (FF)	$\left[(0_2)_4\left(\underline{90_2}\right)_4\right]_S$	-
PFC_fw	148	63.11	0.69 (C)	0.94 (A)	$\left[(0_2)_4\left(\underline{90_2}\right)_5\right]_S$	2.7
TW	208	68.23	0.27	0.99	$\left[(0_2)_6\left(\underline{90_2}\right)_4\right]_S$	30.1
MS	148	63.11	0.66	0.84	$\left[(0_2)_4\left(\underline{90_2}\right)_5\right]_S$	2.7

CONCLUSIONS

In this paper, the effect of the failure criterion on the minimum weight and material cost of laminated plates was investigated. A genetic algorithm was developed and employed as an optimization tool because of its ability to deal with non-convex, multimodal and discrete optimization problems, of which the design of laminated composites is an example. The maximum stress, Tsai-Wu and Puck failure criteria (PFC) were tested for different loading conditions. Special attention was accorded to the PFC, since it appears to lead to the best description of the real behaviour of laminated composite structures. Two versions of PFC were considered: one involving a weakening factor and another that did not.

The results of this study show that the optimal weight of a laminated composite depends on the failure criterion as well as the load conditions (especially for the positive-compressive load condition) and that there is no direct connection between optimal weight and failure criterion: in other words, there was no criterion that was always the most or the least conservative. Our findings underline the dependence of the optimal design on the failure criterion chosen. Thus, the use of criteria that closely reflect the actual behaviour of the laminated composites under study is critical. In addition, it was observed that the optimal design obtained using a given failure criterion is not necessarily safe when a different criterion is considered as was the case of the positive-positive loading condition when $N_{xy}=0$. Although the optimal design and failure effort yielded by the MS and PFC criteria had the same weight, the MS design failed when tested by the PFC due to different stacking sequences. Thus, when optimizing laminated composite structures, the choice of a failure criterion corresponding to the real behaviour of the structure is crucial for both economy and safety.

References

1. Le Riche R, Haftka R. Optimization of laminate stacking sequence for buckling load maximization by genetic algorithm. AIAA Journal 1993;31:951-956.
2. Nagendra S, Jestin D, Gurdal Z, Haftka R, Watson L. Improved genetic algorithm for the design of stiffened composite panels. Computers & Structures 1994;58:543-555.
3. Liu B, Haftka R, Akgun M, Todoroki A. Permutation genetic algorithm for stacking sequence design of composite laminates. Computer Methods in Applied Mechanics and Engineering 2000;186:357-372.
4. Seresta O, Gürdal Z, Adams DB, Watson LT. Optimal design of composite wing structures with blended laminates. Composites Part B: engineering 2007; 38:469-480.
5. Le Riche R, Haftka R. Improved genetic algorithm for minimum thickness composite laminate design. Composites Engineering 1995;5:143-161.
6. Puck A, Schürmann H. Failure analysis of FRP laminates by means of physically based phenomenological models. Composites Science and Technology 1998;58:1045-1067.
7. Jones RM. Mechanics of Composite Materials, 2nd ed., Taylor and Francis, Philadelphia, USA, 1999.
8. Daniel IM. Failure of Composite Materials. Blackwell Publishing Ltd Strain 2007; 43:4–12.
9. Puzzi S, Carpinteri A. A double-multiplicative dynamic penalty approach for constrained evolutionary. Structural and Multidisciplinary Optimization 2008;35:431-445.