DESIGN FOR PRODUCTION IN FRP BOATS

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SUMMARY

Boatbuilding is an industry with low profit margins and volumes. Boats must therefore be produced with a high quality but low cost. Composites are variable materials and as such it is important to determine how variability can effect the cost. This paper therefore looks at the reliability of production techniques to understand composite costs.

NOMENCLATURE

V_f=	Volume of fibre	$\zeta_{Fi,Ri}$=	Specific gravity of fibre or resin in each ply, i
k=	Deflection failure criteria	w_{max}=	Maximum deflection
w=	Deflection	L =	Length of Panel
B =	Breadth of Panel	P =	Pressure on the panel
E_f =	Young's modulus of fibre	E_m =	Young's modulus of resin
G_f =	Shear modulus of fibre	G_m =	Shear modulus of resin
X_T =	Maximum tensile strength	ϵ_{1T} =	Tensile Strain
∇ =	Gradient	ℓ =	Response function
$\mathcal{S}$ =	Score Function	$H(X_i)$ =	Sample performance
σ^2 =	Variance	μ =	Mean
α =	Shape parameter	λ =	Scale parameter
$a_{l,t}$ =	Crown width	$b_{l,t}$ =	Crown height
$c_{l,t}$ =	Web width	$d_{l,t}$ =	Web height
$e_{l,t}$ =	Base width	T_{Plate} =	Plate thickness

1 INTRODUCTION

Boatbuilding is an industry in which there are tight profit margins and a large competition between boatyards. It is therefore important that boats are produced with the minimum cost and yet still have a high quality either in terms of production or design. Design for production is a process that helps reach this compromise between structural and production engineers.

The boatbuilding industry relies on composites as the main material to be used in construction. These materials have the advantage that they can be produced so that the properties of the material suit the application. This leads to the advantage of having strength across the areas where it is required but also means that as the material is produced it can lead to imperfections on a larger scale than other materials. These imperfections must

Table I: SSA Production Model

Action	Cost(mins)
Fairing Compound	10 minutes/sqm
Smoothing Fairing Compound	60 minutes/sqm
Apply Release Compound	10 minutes/sqm/ply
Cutting cloth	10 minutes/sqm/ply
Laying cloth	5 minutes/sqm/cloth
Cutting and laying core	60 minutes/sqm/core
Apply resin with brush or roll	10 minutes/sqm
Remove the components from the mould	30 minutes/sqm
Quality Inspection	3 minutes/sqm
Trim	15 minutes/m/edge

be incorporated in models for companies to make top quality products within specified budgets. Reliability methods model the manner in which materials can vary statistically dependant upon the manner in which they are made.

In more recent years design for production is being developed through the use of optimisation methods. The methods allow an objective optimisation to be reached between different subsystems, sections that overlap to form the design. For these optimisation results to be usable within a design it is important that the models used are accurately created and portray real life scenarios. Due to the complexities of the design process, optimisation methods cannot be used to create a final design. This therefore means that post-optimisation is important to understand the sensitivity of the overall cost and mass to different input variables. As such this paper develops a cost model, determines the reliability in terms of cost of the material and assesses the sensitivity of different variables to allow analysis in further optimisation.

2 PRODUCTION MODELLING

To determine the cost accurately it is important to model the production route taken by a material. As hand layup is the method of most prevalence within boatbuilding this technique has been modelled. Production modelling was originally performed using a parametric cost model taken from [1] as shown in Table I.

This model has no cost for stiffeners and is for a sandwich plate. This has therefore meant that a stiffener cost model has been attached to the main model replacing the "cutting and laying" core section of the SSA model production model for each longitudinal and transverse section and is shown in Table II.

The time for each action has been transformed into a cost by using a wage of 20 £/hour. To determine the raw material costs for the stiffeners, cost per kg for each material has been used. In this case a price of 20 £/kg for Prime 20LV epoxy, 10 £/kg for carbon fibres, 5 £/kg for vinylester resin and 2 £/kg for E-glass fibres have been used. The mass of the plate has been found using a density where the specific gravities for the fibre and resin

Table II: Stiffener cost model

Action	Cost(mins)
Cutting cloth	10 minutes/sqm/ply
Laying cloth	5 minutes/sqm/cloth
Cutting and laying core	60 minutes/sqm/core
Apply resin with brush or roll	10 minutes/sqm

are taken respectively as ζ_{Fi}=1.8 and ζ_{Ri}=1.38 for the Carbon/Epoxy and ζ_{Fi}=2.56 and ζ_{Ri}=1.44 for the E-glass/Vinylesterthe while the volume is given by the plate geometry.

3 RELIABILITY

Reliability methods are used to predict the performance of structures in areas where there are high levels of variability. There are many different methods for the determination of the reliability of a product which fall into two main categories: analytical and simulation. Analytical methods have the advantage that they are computationally inexpensive compared to those carried out with simulation. The main problem can be that these methods can be complicated to solve. There are three levels to analytical reliability.

Level-3 is the full probabilistic method where the model determines the link between the basic design variables affecting the response of the structure and the true nature of the failure domain. Level-2 is a semi-probabilistic method where the failure domain is idealised and is often connected with simplified probability functions of the basic design variables. An example of a Level-2 method is the First Order Reliability Method(FORM) where a first-order Taylor series is used as approximation to the limit sate. This technique can also be undertaken using a second-order Taylor expansion series and this is a Second Order Reliability Method(SORM). Finally the level-1 approach is a deterministic approach using either central or partial safety factors where the models are based upon those of the Level-2 approach. Level-3 methods are rarely used due to the difficulty of modelling fully the entire structure and failure models and are generally used in research, whereas most of the design codes available are using level-1 reliability with some codes moving towards level-2.

The civil industries have developed a number of rules dependant upon reliability. These include the American Institute of Steel Construction(AISC) Load and Resistance Factor Design(LRFD) code for steel building [2], the Ontario Highway Bridge Design Code for bridges [3], EUROCOMP [4], CIRIA [5] and European codes such as CEC. Furthermore certain maritime codes are starting to look at the use of reliability with DNV [6] and IMO [7] starting to develop reliability based sections to their codes.

3.1 Monte Carlo Simulation

The Monte Carlo simulation method has been chosen for the prediction of the reliability of the composites as this technique will allow an easy ability to make changes and allow a large number of variables to affect the prediction of the reliability. The Monte Carlo method has three main steps:

1. Generate a random distribution
2. Structural or production calculations
3. Determine probabilities

The first step is to generate a uniform distribution that can then be mapped to the distribution function using the quantile function. The uniform distribution was found using “Numerical Recipes” [8] to generate a random number and the quantile function dependant upon the distribution. This function will then generate a number of values for each variable and each generation.

Composite structures are by nature variable in their characteristics. For each design there are a number of trade-offs to make in terms of cost, structural integrity, environment and aesthetics. All of these different outputs can be changed through a large number of inputs. It is therefore important to gain an understanding for how much each change will effect the final output and the implications these will have on other outputs. As such a sensitivity index has been used to investigate the impact of each input as shown in eq.1

$$\widehat{\nabla^{(k)}\ell}(\mathbf{u}) = \frac{1}{N}\sum_{i=1}^{N} H(\mathbf{X}_i)\mathcal{S}^{(k)}(\mathbf{u};\mathbf{X}_i) \tag{1}$$

this equation is the gradient of the surface of the output object. This gradient can be found from the score functions of each distribution shown in eq. 2, for the Normal distribution, and eq. 3, for the Weibull distribution.

$$\mathcal{S}(\mathbf{u};x) = (\sigma^{-2}(x-\mu), -\sigma^{-1} + \sigma^{-3}(x-\mu)^2) \tag{2}$$

$$\mathcal{S}(\mathbf{u};x) = (\alpha - 1 + ln(\lambda x)[1-(\lambda x)^{\alpha}], \frac{\alpha}{\lambda}[1-(\lambda x)^{\alpha}]) \tag{3}$$

The sensitivity indices relate to the effect that the input characteristics have upon the output. These values are the gradient and therefore the larger the value the more effect the input has on the output. Having generated a number of input variables these can then be run through the structural or production model. After this processing it will then be possible to determine the reliability of the panel and the sensitivity of the material to each input variable.

3.2 Structural Reliability

For the validation shown below, the modelling was undertaken based upon the stress limit state and the deflection limit state. It is possible to see the variables that would need varying from Eq. 4 and Eq. 5. The total runs of the random number generator is therefore the number of runs N multiplied by the number of variables, 9.

$$\begin{aligned} \text{Stress} \; = \; & X_t(E_f, E_m, V_f, \epsilon_f^*) \\ & -\sigma_{max}(L, B, P, E_f, E_m, G_F, G_m, V_f) \end{aligned} \tag{4}$$

$$
\begin{aligned}
\text{Deflection} &= k \times w_{max} \\
&\quad -w(L, B, P, E_f, E_m, G_F, G_m, V_f)
\end{aligned} \tag{5}
$$

Having run the variables through the structural model it is then possible to determine whether for a given set of variables, the limit state has failed (i.e. less than or equal to zero). For the validation of the Monte Carlo methods this was carried out for the stress limit state, Eq. 4, and the deflection limit state, Eq. 5. In the results tables shown for the deflection limit state it is assumed that the value of k=2. This means that the deflection of the panel must reach higher than 2 times the deflection produced using the mean value for each input. For the stress limit state the tensile strength is calculated for each plate. This is done using Eq. 6 and a varying value of failure strain where the average tensile strength was 1470MPa for the Carbon/Epoxy case and 887.5MPa for the E-glass/vinylester case.

$$
X_t = (E_f V_f + E_m V_m)\epsilon_{1T} \tag{6}
$$

These criteria determine the failure within the plate. The total number of failed panels can then be assessed. The code is run for a number of panels, N. The number of panels that fail are then compared to the total which gives directly the probability of failure and hence the reliability index.

3.3 Production Reliability

The production reliability was determined in much the same manner as for the structural models. The production is very dependant upon the shape of the panel and the proportion of resin and fibre. The cost of the panel has been compared to that of the average panel created using the mean of each input value. If the panel is of a higher cost than this average panel, it is determined that the panel has failed. From this information it is then possible to define a cost limit state shown in eq.7.

$$
\begin{aligned}
\text{Cost} &= C_{Average} \\
&\quad -C_{max}(L, B, P, V_f, a_{l,t}, b_{l,t}, c_{l,t}, d_{l,t}, T_{plate})
\end{aligned} \tag{7}
$$

where the cost limit state is dependant upon $C_{Average}$ which is the average panel.

4 RESULTS

4.1 Reliability Validation

Validation of the Monte Carlo simulation that is being used for the reliability studies was determined by comparison with work previously carried out on a composite grillage plate. The plate has been modelled previously and compares to Clarkson [9], the results of which are recorded in Sobey et al. [10]. To determine the reliability of the plate it is assumed to

Table III: Panel Material Properties - Validation

	Carbon/Epoxy		E-glass/Vinylester		
Material	Mean	CoV	Mean	CoV	Distribution
Length	3810mm	3	3810mm	3	Normal
Breadth	3810mm	3	3810mm	3	Normal
Pressure	137kPa	15	137kPa	15	Weibull
E_f	826GPa	5	71GPa	5	Normal
E_m	3GPa	3	3.4GPa	3	Normal
G_f	413GPa	3	35.5GPa	3	Normal
G_m	1.09GPa	3	1.13GPa	3	Normal
V_f	0.6	3	0.5	10	Normal
ϵ_f	0.3	3	2.5	3	Normal

have characteristics as shown in Table III. The mean and Coefficient of Variation (CoV) for each variable have been taken from [11] to allow a direct comparison between the work.

From these properties it is then possible to run a Monte Carlo simulation to approximate the probability of failure and to determine the number of generations that need to be run for an accurate comparison with other reliability methods. This simulation has been tested for convergence as shown in Table IV.

Table IV: Convergence of Monte Carlo Simulation

Runs	Failures	Probability of Failure
10^6	1	1×10^{-6}
10^7	18	1.8×10^{-6}
10^8	146	1.46×10^{-6}
10^9	1490	1.49×10^{-6}

It is then possible to compare the reliability index for the panel with those created using different methods from the previous work and shown in Table. V .

From these results it is possible to see that a good degree of accuracy between the methods between the Monte Carlo simulation and the previous methods givinf 5.5% of the probability of failure and 1.43% of the reliability index for the FORM results. Compared to the SORM results the Monte Carlo simulation produced results 39.7% of the probability of failure giving 1.48% of the reliability index. This shows the method could be used for the analysis of the structurally optimised plate.

4.2 Structural Reliability

Having validated the Monte Carlo methods it was then possible to determine the sensitivity of the output to each of the inputs. In terms of the structural model these results are

Table V: Comparison of FORM/SORM and Monte Carlo Reliability

		Probability of Failure P_f (10^{-6})	
Method	Reliability Index,β	Deflection Limit State	Stress Limit State
FORM [11]	4.6927	1.384	0
SORM [11]	4.7446	1.045	0
Monte Carlo	4.97	1.49	0

shown in fig. 1 for the case of the Carbon/Epoxy panel

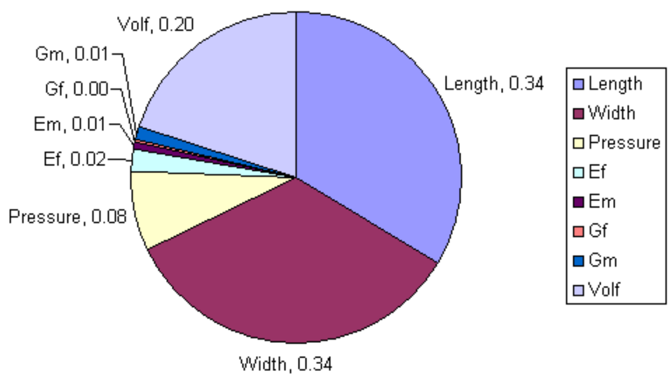

Fig.1: Sensitivity of deflection to inputs - Carbon/Epoxy

and fig. 2 for the E-glass/Vinylester test case.

These results show that the stress limit state is not affected by the material propeties. This is because the stresses are in the region of 170MPa for the average panel with a failure of 1470MPa in the case of the Carbon/Epoxy and 887.5MPa E-glass/Vinylester. The results for the stress have therefore been discounted. For the results shown in Table 1 and 2 each of the gradients has been normalised using the average value for the characteristic. Using these normalised values it is possible to compare these values to each other in terms of effect on the deflection. The sections of this fig. were then compared to that of Das [11] and showed a good correlation between the importance of each. The main difference between the two result was that they were less sensitive to the pressure.

4.3 Production Reliability

As shown in section 3.3 the sensitivity and reliability of the panel to different inputs can be predicted. The modelling has been carried out using normal distributions assuming that production engineers are as likely to make a mistake in one direction as another. It was assumed that the thickness of the stiffeners, being dependant on the number of plies, varied very little. For the cost it is also important that any reduction in cost does

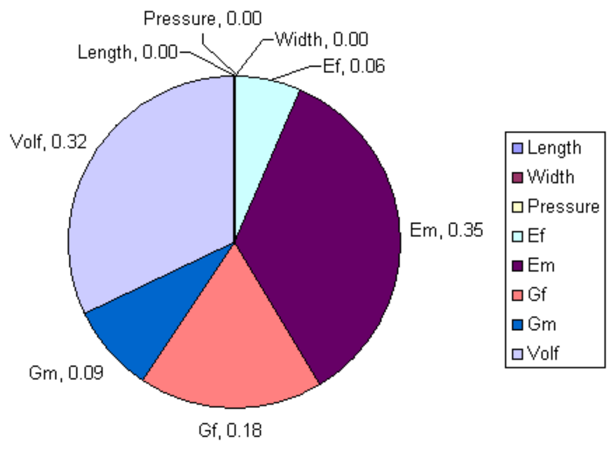

Fig.2: Sensitivity of deflection to inputs - E-glass/Vinylester

not lead to failure therefore for every failed panel the cost of the grillage was assumed to be £1,000,000. The properties for these results are shown in Table VI where it has been assumed that the Carbon/Epoxy is made using pre-preg where as the E-glass/Vinylester was made using hand layup.

Table VI: Panel Material Properties - Cost

	Carbon/Epoxy		E-glass/Vinylester		
Material	Mean	CoV	Mean	CoV	Distribution
Length	3810mm	3	3810mm	3	Normal
Breadth	3810mm	3	3810mm	3	Normal
V_f	0.6	1	0.5	10	Normal
Crown Height	18.288	10	18.288	10	Normal
Crown Width	127	10	127	10	Normal
Web Height	254	1	254	1	Normal
Web width	9.144	1	9.144	1	Normal
Plate Thickness	18.288	1	18.288	1	Normal

The results for the Carbon/Epoxy sensitivities are shown in fig. 3 and the E-glass/Vinylester case is shown in fig. 4.

From these figure it is possible to see that the pressure and volume fraction played the largest part in the cost. This is because the collapse of the structure was the most important point to avoid. Furthermore as the cost of the materials was very different between the resin and the fibre a large change in this value led to a large change in the cost. The use of analogous production models will also affect the sensitivity of each input to the reliability. The sensitivity results can therefore be more accurately obtained by using production models that better represent the actual processes in a yard.

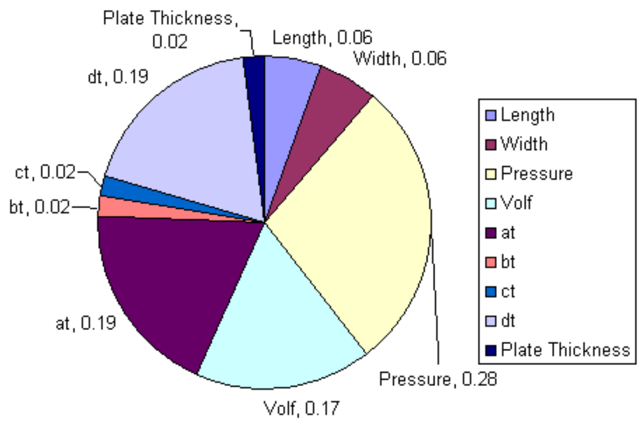

Fig.3: Carbon/Epoxy sensitivity of cost to inputs - Carbon/Epoxy

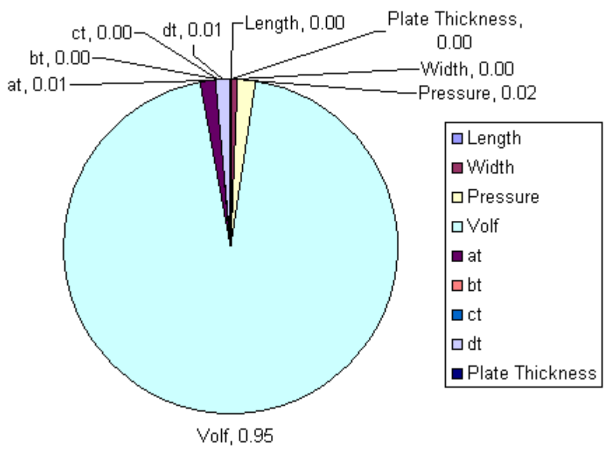

Fig.4: E-glass/Vinylester sensitivity of cost to inputs - E-glass/Vinylester

5 CONCLUSIONS

The paper has developed a method to determine the importance of individual characteristics on the cost of composites. The development of these methods will allow accurate optimisation of composite panels and furthermore give a better understanding to designers about the manner in which characteristics effect the cost of a product. The method for this work has been compared to that of previous work using stress and deflection analysis as the comparison. It has therefore been possible to predict the reliability for the panels in terms of structural integrity and also in terms of the cost to produce the panels.

For future development of the work it will be important to develop the production model to better represent real boat yards and the manner in which production occurs. This will allow for an effective understanding of how the panel characteristics effect the panel production and the importance of accuracy within certain areas of this process. Furthermore a greater understanding of the variability of each characteristic and the manner in which this variability occurs will be important.

For future development of the reliability methods it will be important to take the gradients developed and transfer these into the optimisation methods. This can be done by creating a response surface of the reliability of the plate. Developing this response surface will allow effective optimisation of panels through reliability. Further to this the understanding of this surface will allow designers an insight into the manner in which changes to their subsystems effect the rest of the design. The cost and the deflection should also be combined to determine an overall sensitivity of the numbers and reliability of the panels to ensure that low cost but reliable structures are designed and produced.

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