Crack Detection in Anisotropic Plates through Indentation Test

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SUMMARY
Reliable and robust non-destructive damage detection methods are critical to insure the survivability of civilian, mechanical and aerospace engineering structures, and to assess their remaining useful life. We show that a crack parallel to the major surfaces of an anisotropic plate changes the indentation modulus (i.e., the slope of the indentation load versus the indentation depth curve). Thus an indentation test can be employed to identify a crack parallel to the major surfaces of a plate. We employ the Eshelby-Stroh formalism to study a generalized plane strain problem involving the indentation of a layer by a rigid circular cylinder, and find relations between the indentation modulus and the crack size, the crack position, elastic moduli of the layer, and boundary conditions at its edges.

Keywords: Crack-detection; Indentation; Anisotropic layer

INTRODUCTION
Due to their high specific stiffness and specific strength, composites are being increasingly used in numerous engineering applications. However, mechanical properties of composites may degrade severely with repeated impact and cyclic loading. Failures of structures, particularly aircraft structures, often have tragic consequences. Applications of composites tend to be limited or inhibited by the lack of long-term service experience and the inability to quantify damage and thus determine the remaining useful life of the structure. Damage detection techniques such as thermal deplying and optical microscopy require either the partial or the total destruction of the structural component [1]. Most conventional nondestructive evaluation techniques such as ultrasonic C-scan, X-ray, thermography and eddy current have limited applications as they require a structural component to be taken out of service for a substantial length of time for damage inspection and assessment. Some global damage detection methods [2], based on the assumption that a change in physical properties of a structure causes a change in its modal characteristics, have also been developed. These methods either depend on prior analytical models or prior test data for the detection and the location of
damage or on the output from several sensors bonded to the structure. Technical issues that need to be considered during in situ monitoring of structures include the following: surface-bonded resistive strain gauges are susceptible to electromagnetic and electrical interference in addition to physical damage, and the acoustic emission suffers from low signal-to-noise ratio. No single test can identify all failure modes likely to occur in a composite. Thus two or more techniques should be used simultaneously to ensure the full detection of damage in a composite, and outputs from different techniques should be synthesized to quantitatively and qualitatively ascertain the damage type, its extent and location.

Indentation is an experimental technique to determine mechanical properties of materials. It is commonly believed that the indentation load versus the indentation depth response during unloading corresponds to elastic deformations of the indented material. The indentation modulus, i.e., the slope of the indentation load vs. the indentation depth curve for infinitesimal indentations, is a function of elastic properties of the structure [3]. Therefore, the degradation in elastic properties due to cracks will decrease the indentation modulus, and the change will depend upon the location and the size of cracks. Thus it can be used to identify cracks in a composite.

Here we employ the Eshelby-Stroh formalism to study a generalized plane strain problem involving the indentation by a rigid circular cylinder of a homogeneous anisotropic plate containing a crack parallel to its major surfaces, and quantify changes in the indentation modulus as a function of the location and the length of the crack. This data can be used to identify cracks in a composite structure since its material can be modeled as anisotropic.

**PROBLEM FORMULATION**

Figure 1 depicts a schematic sketch of the generalized plane strain problem involving the indentation of a cracked panel by a rigid smooth circular cylinder studied here. The crack of length $2b$ and thickness $h_b << w_1$ and $w_2$ is in the $x_1x_2$-plane. It is assumed
that the length of the crack, of the cylinder and of the layer in the $x_2$-direction (perpendicular to the plane of the paper) is very large as compared to the length $L$ and the thickness $h$ of the panel. We denote the indentation depth by $u_0$, and the semi-contact width by $c$. Prior to indentation, the centers of the contact width and the crack lengths are, respectively, at $(x_c, h)$ and $(x_b, w_2)$, where $w_1 + w_2 = h$.

In the absence of body forces, equations in rectangular Cartesian coordinates governing deformations of the layer are

$$\sigma_{ij,j} = 0, \quad i = 1,2,3, \quad (1)$$

$$\sigma_{ij} = C_{ijkl} e_{kl}, \quad C_{ijkl} = C_{jikl}, \quad (2)$$

$$e_{kl} = \frac{1}{2} (u_{k,j} + u_{l,k}), \quad (3)$$

where $\sigma_{ij} = \sigma_{ji}$ is the Cauchy stress tensor, $\sigma_{ij,j} = \partial \sigma_{ij} / \partial x_j$, a repeated index implies summation over the range of the index, $e_{kl}$ is the infinitesimal strain tensor, $u_i$ is the displacement of a point in the $x_i$-direction, and $C_{ijkl}$ is an elastic constant of the material of the linear elastic layer. Symmetries indicated in Eq. (2) imply that, for a 3D problem, $C_{ijkl}$ can be written as a symmetric $6 \times 6$ matrix, and $\sigma_{ij}$ and $e_{kl}$ as $6 \times 1$ matrices.

Boundary conditions on the top surface of the layer are:

$$\sigma_{i3} = \sigma_{3i} = 0 \quad \text{on} \quad x_3 = h \quad \text{and} \quad |x_1 - L / 2| > c, \quad (4.1)$$

$$\sigma_{i1} \sin \theta \cos \theta - \sigma_{31} \cos 2\theta - \sigma_{33} \sin \theta \cos \theta = 0, \quad u_3 = R - u_0 - \sqrt{R^2 - (x_1 - L/2)^2}, \quad \text{on} \quad x_3 = h \quad \text{and} \quad |x_1 - L / 2| \leq c. \quad (4.2)$$

Here $\theta = \arcsin((x_1 - L / 2) / R)$. In cylindrical coordinates, the left-hand side of Eq. (4.2) equals the tangential traction $\sigma_{r\theta}$ at a point on the contact surface. Points of the contact surface where $\sigma_{r\theta} \geq 0$ do not touch the indenter. We assume that there is no separation between the indenter and the deformable layer; thus the contact surface is contiguous.

The layer is either taken to be fixed at the edges $x_1 = 0, L$ and traction free at the bottom surface $x_3 = 0$; or (ii) fixed at $x_3 = 0$ and traction free at $x_1 = 0, L$. Boundary conditions at a fixed edge are

$$u_1 = u_3 = 0, \quad (5)$$

and those at a traction free surface are

$$\sigma_{ij} n_j = 0 \quad (6)$$
where \( \mathbf{n} \) is the unit normal vector to the free surface.

We ensure that a crack does not close during the indentation process; thus crack surfaces are taken to be traction free and boundary conditions given by Eq. (6) are applied on the two crack surfaces.

The axial load \( P \) per unit length of the cylinder, or the indentation load, is calculated from

\[
P = -\int_{L/2-x_c}^{L/2} (\sigma_{33} - \sigma_{13} \tan \theta) \, dx_3
\]

(7)

If the bottom surface is fixed, the indentation depth equals the absolute value of the vertical displacement of the centre of the top surface. However, when the bottom surface is traction free then the indentation depth equals the vertical displacement of the centre of the top surface relative to that of the bottom surface.

**ANALYTICAL SOLUTION OF THE PROBLEM**

We assume that the displacement field \( \mathbf{u} \) and hence stresses and strains induced in the layer are functions of \( x_1 \) and \( x_3 \) only, and write a general solution of Eqs. (1) - (3) as follows by using Stroh’s formalism [4]:

\[
\mathbf{u} = \sum_{\alpha=1}^{3} \left[ \mathbf{a}_\alpha f_\alpha (z_\alpha) + \overline{\mathbf{a}}_\alpha f_{\alpha+3} (\overline{z}_\alpha) \right], \quad (8)
\]

\[
\mathbf{\sigma}_1 = -\sum_{\alpha=1}^{3} \left[ p_\alpha \mathbf{b}_\alpha f'_\alpha (z_\alpha) + \overline{p}_\alpha \overline{\mathbf{b}}_\alpha f'_{\alpha+3} (\overline{z}_\alpha) \right], \quad (9)
\]

\[
\mathbf{\sigma}_3 = -\sum_{\alpha=1}^{3} \left[ \mathbf{b}_\alpha f'_\alpha (z_\alpha) + \overline{\mathbf{b}}_\alpha f'_{\alpha+3} (\overline{z}_\alpha) \right], \quad (10)
\]

Where

\[
(\mathbf{\sigma}_1)_i = \sigma_{i1}, \quad (\mathbf{\sigma}_3)_i = \sigma_{i3}.
\]

(11)

Furthermore, \( f_\alpha (\alpha = 1,2,3) \) are arbitrary analytic functions of \( z_\alpha \), \( \overline{z}_\alpha \) is complex conjugate of \( z_\alpha \), \( f'_\alpha \) denotes the derivative of \( f_\alpha \) with respect to \( z_\alpha \), \( p \) is an eigenvalue, and \( \mathbf{a} \) and \( \mathbf{b} \) are the corresponding eigenvectors of the following eigenvalue problem:

\[
\mathbf{N} \xi = p \xi, \quad (12)
\]

\[
\mathbf{N} = \begin{bmatrix} -\mathbf{T}^{-1} \mathbf{R}^\top & \mathbf{T}^{-1} \\ \mathbf{R} \mathbf{T}^{-1} \mathbf{R}^\top & -\mathbf{Q} - \mathbf{R} \mathbf{T}^{-1} \end{bmatrix}, \quad \xi = \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix},
\]

(13)

\[
Q_{il} = C_{il}, \quad R_{il} = C_{i3l}, \quad T_{il} = C_{i3l}.
\]

(14)
For the strain energy density to be positive definite, \( p \) must be complex [5]. Let 
\( (p_\alpha, a_\alpha), \alpha = 1, 2, \ldots, 6 \) be eigensolutions of Eq. (12) such that

\[
\text{Im}(p_\alpha) > 0, p_{\alpha+3} = \overline{p_\alpha}, a_{\alpha+3} = \overline{a_\alpha}, \alpha = 1, 2, 3,
\]

where \( \overline{p_\alpha} \) is the complex conjugate of \( p_\alpha \). The general solution (9) holds even when the six eigenvalues are not distinct but there exist six linearly independent eigenvectors. Ting [6] has discussed how to modify the general solution when the eigenvalue problem defined by Eq. (12) does not have six linearly independent eigenvectors.

Similar to the technique used to study the indentation of an undamaged plate [7], we separate the entire domain into several regions; the number of subregions depends upon the contact zone and the crack area as depicted in Fig. 2. In order to satisfy boundary conditions and continuity conditions at the interfaces between adjoining subregions, we assume the following series solution for the \( n^{th} \) (\( n = 1, 2, 3 \ldots \)) region.

\[
f_{\alpha}^{(n)}(z_{\alpha}^{(n)}) = d_{\alpha}^{(n)} \exp(\lambda_{\alpha}^{(n)}z_{\alpha}^{(n)}) + e_{\alpha}^{(n)} \exp(\lambda_{\alpha}^{(n)}(p_{\alpha}^{(n)}h - z_{\alpha}^{(n)})) + v_{\alpha}^{(n)} \exp(\eta_{\alpha}^{(n)}z_{\alpha}^{(n)}) + w_{\alpha}^{(n)} \exp(\eta_{\alpha}^{(n)}(p_{\alpha}^{(n)}h - z_{\alpha}^{(n)}))
\]

\[
+ \sum_{k=1}^{\infty} \{ q_{k\alpha}^{(n)} \exp(\lambda_{k\alpha}^{(n)}z_{\alpha}^{(n)}) + r_{k\alpha}^{(n)} \exp(\lambda_{k\alpha}^{(n)}(p_{\alpha}^{(n)}h - z_{\alpha}^{(n)})) \}
\]

\[
+ \sum_{m=1}^{\infty} \{ s_{ma}^{(n)} \exp(\eta_{ma}^{(n)}z_{\alpha}^{(n)}) + t_{ma}^{(n)} \exp(\eta_{ma}^{(n)}(p_{\alpha}^{(n)}h - z_{\alpha}^{(n)})) \}, \quad 0 \leq x_{1}^{(n)} \leq l_{1}^{(n)},
\]

where

\[
z_{\alpha}^{(n)} = x_{1}^{(n)} + p_{\alpha}^{(n)}x_{3}^{(n)}, \quad \lambda_{\alpha}^{(n)} = \frac{\pi i}{2l_{1}^{(n)}}, \quad \lambda_{k\alpha}^{(n)} = \frac{k\pi i}{l_{1}^{(n)}}, \quad \eta_{\alpha}^{(n)} = -\frac{\pi i}{2p_{\alpha}^{(n)}h}, \quad \eta_{ma}^{(n)} = -\frac{m\pi i}{p_{\alpha}^{(n)}h}, \quad i = \sqrt{-1}.
\]

The unknowns \( d_{\alpha}^{(n)}, e_{\alpha}^{(n)}, v_{\alpha}^{(n)} \) and \( w_{\alpha}^{(n)} \) are assumed to be real while \( q_{k\alpha}^{(n)}, r_{k\alpha}^{(n)}, s_{ma}^{(n)} \) and \( t_{ma}^{(n)} \) are complex; these will be determined from boundary conditions, and continuity conditions at the interfaces. In Eqs. (16) and (17), \( l_{1}^{(n)} (n = 1, 2, 3) \) is the width of the
nth segment, and \( x_i^{(n)} \) is the \( x_i \) - coordinate of a point in the nth segment measured from the left edge of the segment.

Substituting for \( f_a(z_a) \) from Eq. (16) into Eqs. (8) - (10), we get the following for the displacements \( u^{(n)} \) and stresses \( \sigma_1^{(n)} \) and \( \sigma_3^{(n)} \) in the nth segment:

\[
\begin{align*}
u^{(n)} &= A \left[ \exp(\beta_{op}^{(n)}) \right] d^{(n)} + \exp(\gamma_{op}^{(n)}) e^{(n)} + \sum_{k=1}^{\infty} \left[ \exp(\beta_{k}^{(n)}) \right] q_k^{(n)} + \exp(\gamma_{k}^{(n)}) r_k^{(n)} \right] + \exp(\delta_{op}^{(n)}) v^{(n)} + \exp(\varphi_{op}^{(n)}) w^{(n)} + \sum_{m=1}^{\infty} \left[ \exp(\delta_{m}^{(n)}) \right] s_m^{(n)} + \exp(\varphi_{m}^{(n)}) t_m^{(n)} \right] + \text{conjugate,}
\end{align*}
\]

\[
(18)
\]

\[
\begin{align*}
\sigma_1^{(n)} &= B \left[ -\left( \lambda_{op}^{(n)} \right)^2 \exp(\beta_{op}^{(n)}) \right] d^{(n)} + \left( \lambda_{op}^{(n)} \right)^2 \exp(\gamma_{op}^{(n)}) e^{(n)} - \left( \eta_{op}^{(n)} \right)^2 \exp(\delta_{op}^{(n)}) v^{(n)} + \left( \eta_{op}^{(n)} \right)^2 \exp(\varphi_{op}^{(n)}) w^{(n)} + \sum_{k=1}^{\infty} \left[ -\left( \lambda_{k}^{(n)} \right)^2 \exp(\beta_{k}^{(n)}) \right] q_k^{(n)} + \left( \lambda_{k}^{(n)} \right)^2 \exp(\gamma_{k}^{(n)}) r_k^{(n)} \right] + \sum_{m=1}^{\infty} \left[ -\left( \eta_{m}^{(n)} \right)^2 \exp(\delta_{m}^{(n)}) \right] s_m^{(n)} + \left( \eta_{m}^{(n)} \right)^2 \exp(\varphi_{m}^{(n)}) t_m^{(n)} \right] + \text{conjugate,}
\end{align*}
\]

\[
(19)
\]

\[
\begin{align*}
\sigma_3^{(n)} &= B \left[ \left( \lambda_{op}^{(n)} \right)^2 \exp(\beta_{op}^{(n)}) \right] d^{(n)} - \left( \lambda_{op}^{(n)} \right)^2 \exp(\gamma_{op}^{(n)}) e^{(n)} + \left( \eta_{op}^{(n)} \right)^2 \exp(\delta_{op}^{(n)}) v^{(n)} - \left( \eta_{op}^{(n)} \right)^2 \exp(\varphi_{op}^{(n)}) w^{(n)} + \sum_{k=1}^{\infty} \left[ \left( \lambda_{k}^{(n)} \right)^2 \exp(\beta_{k}^{(n)}) \right] q_k^{(n)} - \left( \lambda_{k}^{(n)} \right)^2 \exp(\gamma_{k}^{(n)}) r_k^{(n)} \right] + \sum_{m=1}^{\infty} \left[ \left( \eta_{m}^{(n)} \right)^2 \exp(\delta_{m}^{(n)}) \right] s_m^{(n)} - \left( \eta_{m}^{(n)} \right)^2 \exp(\varphi_{m}^{(n)}) t_m^{(n)} \right] + \text{conjugate,}
\end{align*}
\]

\[
(20)
\]

where

\[
A = [a_1, a_2, a_3], \quad B = [b_1, b_2, b_3],
\]

\[
\begin{align*}
\beta_{ka}^{(n)} &= \lambda_{ka}^{(n)} z_a^{(n)}, \quad \gamma_{ka}^{(n)} = \lambda_{ka}^{(n)} (p_a^{(n)} - z_a^{(n)}),
\end{align*}
\]

\[
\begin{align*}
\delta_{ma}^{(n)} &= \eta_{ma}^{(n)} z_a^{(n)}, \quad \varphi_{ma}^{(n)} = \eta_{ma}^{(n)} (t_a^{(n)} - z_a^{(n)}),
\end{align*}
\]

\[
\begin{align*}
\{\varphi, \psi, \chi\} &= \text{diag} [\varphi_1 \varphi_2 \varphi_3, \psi_1 \psi_2 \psi_3, \chi_1 \chi_2 \chi_3],
\end{align*}
\]

\[
\begin{align*}
[d^{(n)}]_a &= d_a^{(n)}, \quad \alpha = 1, 2, 3.
\end{align*}
\]

Substitution from Eqs. (18) - (20) into boundary conditions gives a system of linear algebraic equations whose solution gives unknowns \( d^{(n)} \), \( v^{(n)} \), \( w^{(n)} \), \( q_k^{(n)} \), \( r_k^{(n)} \), \( s_m^{(n)} \) and \( t_m^{(n)} \) (\( n = 1, 2, 3; k = 0, 1, 2, \ldots; m = 0, 1, 2, \ldots \)). In order to maintain approximately the same
period of the largest harmonic on all interfaces and boundaries, we truncate \( k \) to \( K^{(n)} \) and \( m \) to \( M^{(n)} \) for the \( n \)th segment with

\[
K^{(n)} = \text{Ceil}\left(\frac{K^{l(n)}}{L}\right), \quad M^{(n)} = \text{Ceil}\left(\frac{K^{h(n)}}{L}\right)
\]

where \( \text{Ceil}(*) \) gives the smallest integer greater than or equal to \( * \), and \( K \) is the predetermined number of terms.

**VERIFICATION OF THE SOLUTION TECHNIQUE**

During partitioning of the layer into several regions to resolve a crack and the contact region, the body may be divided into several small layers as shown in Fig. 2. However, in order to compute a solution within acceptable errors, \( K^{(n)} \) and \( M^{(n)} \) should be kept large even for these small layers; thus the total number \( K \) of equations may become very large. We have developed a parallelized computer code in Fortran to formulate and solve a large system of simultaneous linear algebraic equations by using the PARDISO package in Intel® Math Kernel Library (MKL).

To verify the computer code, we compare our results with the analytical solution of Hwu and Fan [8] for the indentation of an undamaged orthotropic half-space by the smooth rigid parabolic indenter, \( x_3 = \frac{(x_1 - L/2)^2}{2R} \), where \( R \) is the radius of curvature of the indenter at the point \((L/2, 0)\). Values assigned to various material and geometric parameters are listed below.

\[
\begin{align*}
E_1 &= 25.0 \text{ GPa}, \quad E_2 = E_3 = 1.0 \text{ GPa}, \quad G_{23} = 0.2 \text{ GPa}, \\
G_{12} &= G_{31} = 0.5 \text{ GPa}, \quad \nu_{12} = \nu_{23} = \nu_{13} = 0.25, \\
L &= 1.0 \text{ m}, \quad h = 0.4 \text{ m}, \quad R = 1.0 \text{ m}, \quad 2c = 0.04 \text{ m}, \quad 2b = 0.2 \text{ m}, \quad h_6 = 0.002 \text{ m}.
\end{align*}
\]

Relations \( E_{ij}/\nu_{ij} = E_{ji}/\nu_{ji} \) (no sum on \( i \) and \( j \)) and values of parameters listed in Eq. (23) give \( \nu_{23} = \nu_{31} = 0.01 \) and \( \nu_{12} = 0.25 \). We have compared in Fig. 3 the pressure distribution on the contact surface obtained from the analytical solution of Hwu and Fan [8] with that computed by using the present method in which the entire laminate is divided into 15 layers and \( K \) is set equal to 1000 in the series solution represented by Eqs. (18) - (20). It is clear that the two pressure distributions agree well with each other, and the maximum error in the pressure computed over the region \((x_1 - L/2) < 0.9c\) is 5.2%.

Suo [9] has shown that the stress field near tip of a crack in an infinite homogeneous anisotropic plate has \( r^{-1/2} \) type singularity, where \( r \) is the distance from the crack tip along the crack interface. Results plotted in Fig. 4 give the \( r^{-\lambda} \) singularity for a crack at the interface between two homogeneous anisotropic layers perfectly bonded except for the crack surface type of the stress field near the crack tip in an anisotropic plate indented by a cylinder. However, \( \lambda \) varies from 0.48 to 0.64 when the crack length \( 2b \) changes from 0.06 m to 0.14 m. Possible sources of deviations from the theoretical
value of 0.5 include our considering a rectangular crack rather than an elliptic one with sharp corners, the plates are finite size, materials of the two plates are different, and the loading is not close to that considered in the analytical solution. The usual elliptical crack could not be studied since our problem formulation requires that all bounding surfaces of different regions be parallel to the coordinate planes. One could

Fig. 3 For a homogeneous orthotropic layer, comparison of the presently computed pressure distribution on the contact surface with that of Hwu and Fan (1998).

Fig. 4 Variation with the distance from the crack tip of the transverse normal stress for a crack between two anisotropic layers; the plot is on log-log scale.
approximate an elliptical surface by the sum of numerous small size horizontal surfaces but that will increase immensely the computational effort.

**PARAMETRIC STUDY**

We have investigated the effect on the indentation modulus of the crack size, the crack position, material properties and boundary conditions. Unless otherwise noted, we have taken $L = 1.0\, \text{m}$, $h = 0.2\, \text{m}$, $R = 1.0\, \text{m}$, $h_b = 2\, \text{mm}$, the center of the indenter at the point $(L/2, R+h)$, the center of the crack at the point $(x_b, w_l)$, and values of material parameters listed in Eq. (23).

**Crack Size**

With the center of the crack at the point $(L/2, h/2)$ and for crack lengths varying from 0 (no damage) to 0.2m, we have plotted in Fig. 5 the indentation load vs. the indentation depth curves. It is clear that the indentation modulus decreases noticeably with an increase in the crack length. For indentation depth of 2 mm, and crack lengths of 0.1m, 0.15m, and 0.2m, the indentation load drops, respectively, by 75%, 60%, and 50% of the indentation load for the undamaged layer. Thus results of the indentation test can be used to detect damage in a laminated composite. When the crack length decreases to 0.05m or 5% of the length of the layer, the indentation modulus equals 90% of that for the undamaged layer.

![Fig. 5 Effect of the crack length, 2b, on the indentation load vs. the indentation depth curves for a composite layer with points on the bottom surface held stationary.](image)
Crack Position

For a crack of length $2b = 0.15\,\text{m}$, Fig. 6 exhibits the indentation load vs. the indentation depth curves for four locations of the crack below the indenter. As the crack location moves towards the indenter, the indentation load for the same indentation depth drops rapidly.

For the crack center located at $x_b = 0.5\,\text{m}, 0.4\,\text{m}$ and $0.3\,\text{m}$, results plotted in Fig. 7 show that for the two later locations of the crack center, the indentation load vs. the indentation depth curves are essentially the same as that for an undamaged layer. From plots of Figs. 6 and 7 one can conclude that the indentation test is effective in detecting cracks located close to the indenter.

Fig. 6 For different vertical distances of the crack below the indenter, the indentation load vs. the indentation depth curves for a laminated composite with points on the bottom surface of the composite restrained from moving.

For the crack center located at $x_b = 0.5\,\text{m}, 0.4\,\text{m}$ and $0.3\,\text{m}$, results plotted in Fig. 7 show that for the two later locations of the crack center, the indentation load vs. the indentation depth curves are essentially the same as that for an undamaged layer. From plots of Figs. 6 and 7 one can conclude that the indentation test is effective in detecting cracks located close to the indenter.

Fig. 7 For different crack locations on the interface between the two layers, the indentation load vs. the indentation depth curves for a composite layer with points on the bottom surface having null displacements.
Material Properties

Batra and Jiang [7] have shown that the two material parameters of the composite layer significantly influencing the indentation modulus are Young’s modulus $E_3$ in the direction of indentation and the shear modulus $G_{13}$ in the plane of deformation. For the crack length $2b = 0.1$ m, the center of the crack located at $(L/2, h/2)$, and different values of $E_3$ and $G_{13}$ we have plotted in Figs. 8 and 9 the indentation load vs. the indentation depth. These results evince that the difference in the indentation load for a given

![Fig. 8](image1.png)

**Fig. 8** For three values of Young’s modulus in the transverse direction, the indentation load vs. the indentation depth curves of a laminated composite with points on the bottom surface restrained from moving.

![Fig. 9](image2.png)

**Fig. 9** For three values of the in-plane shear modulus, the indentation load vs. the indentation depth curves of a laminated composite with points on the bottom surface restrained from moving.
indentation depth increases rapidly with an increase in the value of $E_3$ and a decrease in the value of $G_{13}$. Thus the indentation test can be readily used to find an inter-laminar crack in a composite whose Young’s modulus in the indentation direction is very large and the shear modulus $G_{13}$ is very small.

**Boundary Conditions**

For a composite layer with fixed edges and traction free bottom surface, we have plotted in Fig. 10 for three locations of the crack, the indentation load vs. the indentation depth curves. These results are qualitatively similar to those for a layer with edges clamped. Results for different crack positions, crack sizes and material properties are not exhibited since they are also similar to those for a layer with clamped edges.

![Graph](image)

**DISCUSSION**

We note that the indentation test is a well developed technique for determining mechanical properties of a material. By periodically finding the indentation modulus, the initiation and the growth of a crack in a layer can be quantified. The use of sensitive instruments capable of measuring loads as small as a nano-Newton and indentations of about 0.1 nm, the accuracy and the reliability of the method can be enhanced. Furthermore, elastic deformations used to find the indentation modulus do not introduce any additional damage to the structure.
As stated by Rytter [10], the damage identification involves determining the existence of damage, its geometric location, its severity or magnitude, and prediction of the remaining service life of the structure. The present work addresses the first three issues at least partially but sheds no light on the durability of the structures. By conducting several numerical simulations, one can establish a functional relation between the change in the indentation modulus and other variables such as the crack length, the crack location, and values of material and geometric parameters.

CONCLUSIONS

We have employed the Eshelby-Stroh formalism to study a generalized plane strain problem involving the indentation of a cracked-layer by a rigid circular cylinder and compared the indentation load vs. the indentation depth curves with the corresponding ones for the undamaged layer. Since the slope of the indentation load vs. the indentation depth curve for a layer with a crack deviates noticeably from that of the undamaged one, the traditional indentation test can be used to detect cracks. The change in the indentation modulus increases with an increase in the crack length, a decrease in the vertical distance between the crack and the indenter, and an increase in Young’s modulus of the layer material in the indentation direction. The change in the indentation modulus decreases as either the crack moves away from the indenter or the in-plane shear modulus increases.

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