ACCURATE MODELING OF ELECTROMAGNETIC WAVE PROPAGATION PHENOMENA IN MULTILAYER MAGNETOELECTRIC THIN FILMS

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1 Introduction

The magnetoelectric (ME) effect is defined as the induced electric polarization of a material in an applied magnetic field, or the induced magnetization of the material in an applied electric field [1-4]. The ME effect occurs naturally in single phase crystals such as antiferromagnetic chromium oxide (Cr_2O_3), and can also be artificially realized through layered piezoelectric and piezomagnetic/magnetostrictive thin films. Here, we are interested solely in the artificial ME effect realized via a product property on the mechanical interaction between layered piezoelectric and piezomagnetic thin films.

There have been several experimental and theoretical studies on ME materials for a half century; however, there remains a lack of accurate theoretical models to accurately describe the media [2]. The lack of accurate theoretical models creates limitations understanding the physics of layered thin film-based ME toward device applications. This factor, along with the small magnitudes of the ME coefficients achieved, are reasons why few ME applications currently exist.

We have obtained accurate, robust theoretical models that closely approximates the effective material parameters for the longitudinal, transverse and in-plane ME configurations [1]. A figurative description of all three ME configurations with appropriate DC bias electric and magnetic fields are shown in Fig.1 The theoretical models characterize the ME thin film in terms of its constitutive equations for the magnetic flux density and electric displacement field, in which the electric and magnetic fields are coupled. We show here the results of the theoretical models obtained in comparison to other theoretical and experimental results found in the literature.

Recently, there have been a number of proposals on the use of ME substrates in microwave device applications [5]. The device proposals make use of the ferromagnetic resonance effect. In that instance, use of ME materials allows electric and magnetic field tuning of the device. Here, we investigate the propagation characteristics of plane electromagnetic (EM) waves in bulk ME samples, with the intent to apply the phenomena to device applications. The study uses results from the theoretical model of the ME film. A magnetoelectric wave equation is derived using Maxwell's equations, and analytical solutions are constructed to gain insight on the propagation, polarization, and mode characteristics. The solutions show many interesting phenomena for EM wave propagation in bulk multilayer ME thin films. The results create a need for future works on the application of ME thin films to planar waveguides.

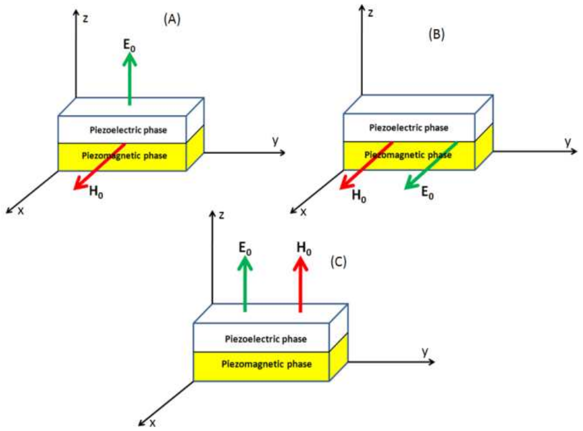

Fig.1. Magnetoelectric configurations. (A) Transverse; (B) In-plane ; (C) Longitudinal.

2 Theoretical Model for the ME effect

For ME laminate composites, there are three major orientations in terms of biasing and poling directions. These are longitudinal, transverse, and in-plane. Proper modeling of the media for each magnetoelectric configuration results in similar general constitutive equations of the form

$$\mathbf{D} = \bar{\bar{\alpha}}^{H}\mathbf{H} + \bar{\bar{\varepsilon}}\mathbf{E} \qquad (1)$$

$$\mathbf{B} = \bar{\bar{\alpha}}^{E}\mathbf{E} + \bar{\bar{\mu}}\mathbf{H} \qquad (2)$$

where ***D***, ***B***, ***E***, and ***H*** are, respectively, the electric displacement field, magnetic flux, electric and magnetic field. Also, $\bar{\bar{\varepsilon}}, \bar{\bar{\mu}}, \bar{\bar{\alpha}}^{H}$ and $\bar{\bar{\alpha}}^{E}$ are the effective permittivity, permeability, magnetic magnetoelectric and electric magnetoelectric susceptibilities, respectively. The magnetoelectric susceptibilities define the amount of ME coupling realized in the thin film. Experimental measurements of the ME effect are most times presented in terms of the ME voltage coefficient α', which is related to the ME susceptibility by $\alpha' = \varepsilon_0\varepsilon_r\alpha$. In previous research, general relationships have been obtained in the form of equation (1) and (2). However, great deviations existed between the theoretical models obtained in those works and experimental data available in the literature [2]. We have used a different approach, where we apply fundamental electro-magnetic wave boundary conditions [1, 6] on the fields within the composite structure. We introduce new theoretical models for the longitudinal ME effect in a piezoelectric-magnetostrictive bilayer that better approximates the experimental results. The model is obtained by solving the constitutive equations of each layer for the all fields present, and then applying a field averaging method [7] along with boundary conditions on the components of the fields at the composite interface, to obtain homogenized material properties. The homogenized layer is characterized in terms of its effective permeability, effective permittivity and the effective ME susceptibility tensor with constitutive equations of the form shown in (1) and (2).

We present results of the theoretical model in Figs. 2 and 3. The effects of an imperfect interface coupling, *k*, between the films are shown in Fig. 2, and a comparison of the current model to others in the literature is presented in Fig. 3. The improved accuracy of the model when compared to measured result is observed.

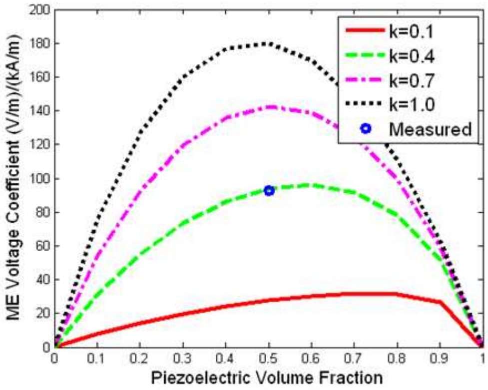

Fig.2. ME voltage coefficient for a cobalt ferrite/PZT bilayer vs volume fraction of piezoelectric film. Curves for various values of *k* are shown as well.

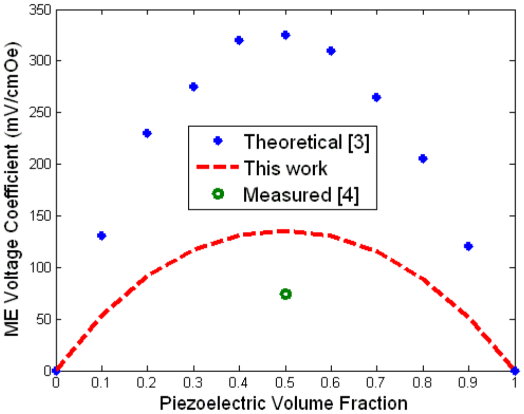

Fig.3. Comparison of the proposed model to other theoretical model, and experimental result in the literature. Here, k=1

3 Electromagnetic Wave Propagation

The magnetoelectric wave equation for the ME media is derived using Maxwell's equations and Eq. (1) and (2). The wave equation is expressed as

$$-\left\{\bar{k}\times\bar{\bar{\mu}}^{-1}\left[\left(\bar{k}\times\mathbf{E}\right)-\omega\bar{\bar{\alpha}}^{E}\mathbf{E}\right]\right\} = \omega^{2}\bar{\bar{\varepsilon}}\mathbf{E}+\bar{\bar{\alpha}}^{H}\bar{\bar{\mu}}^{-1}\left[\omega\left(\bar{k}\times\mathbf{E}\right)-\omega^{2}\bar{\bar{\alpha}}^{E}\mathbf{E}\right]. \qquad (3)$$

For non-trivial solutions, the determinant of Eq. (3) equals zero. Hence, we obtain the wave numbers for the propagating fields. Based upon the set-up of the ME thin film (X-Y plane, see Fig. 1), we can only consider propagation along the *xy*-plane. Complete solutions for propagation along the *x* and *y* directions for all three ME configurations are obtained. For each case analyzed, we obtained two distinct wave numbers; each comprises a forward and backward propagating wave. We obtained ordinary and extraordinary waves for the transverse ME configuration with cases of birefringence. We also obtained information on the polarization of the propagating waves, and observed only elliptical and linear polarized waves in the thin films. No circularly polarized waves were observed for cases studied here. Introduction of the ME parameter also results in a change to the mode of propagation from transverse electromagnetic (TEM) to transverse electric (TE).

We obtain a simpler form of the vector wave equation for the ME media using an identity $\bar{\bar{g}}$, such that [6]

$$\bar{\bar{g}} \cdot \mathbf{E} = \bar{k} \times \mathbf{E}. \qquad (4)$$

Applying vector manipulations to (3), we can obtain a simpler expression for the ME vector wave equation as

$$\left[\left(\bar{\bar{g}} + \omega\bar{\bar{\alpha}}^{H}\right) \cdot \bar{\bar{\mu}}^{-1} \cdot \left(\bar{\bar{g}} - \omega\bar{\bar{\alpha}}^{E}\right) + \omega^{2}\bar{\bar{\varepsilon}}\right] \cdot \mathbf{E} = 0. \qquad (5)$$

For non-trivial solutions, the determinant of the matrix acting on the electric field, **E**, must equal zero. Hence,

$$\left|\left(\bar{\bar{g}} + \omega\bar{\bar{\alpha}}^{H}\right) \cdot \bar{\bar{\mu}}^{-1} \cdot \left(\bar{\bar{g}} - \omega\bar{\bar{\alpha}}^{E}\right) + \omega^{2}\bar{\bar{\varepsilon}}\right| = 0. \qquad (6)$$

Equation (6) is in terms of the operator $\bar{\bar{g}}$, rather than the wave number. To obtain solutions in terms of the wave number, we use Eq. (4) to express the operator $\bar{\bar{g}}$ in matrix form as

$$\bar{\bar{g}} = \begin{bmatrix} 0 & -k_z & k_y \\ k_z & 0 & -k_x \\ -k_y & k_x & 0 \end{bmatrix}. \qquad (7)$$

In Eq. (7), k_x, k_y, and k_z, respectively are the *x*, *y*, and *z* directed wave numbers, and represent propagation in the *x*, *y* and *z* directions. Since we only consider propagation along the sample plane, $k_z = 0$ for all cases.

Using the outlined procedure, analytical solutions for plane wave propagation in bulk magnetoelectric composites have been obtained. The solutions provide insights to the method of propagation in ME materials. For the longitudinal magnetoelectric configuration, we obtained linearly polarized waves in the media. The waves are tilted at an angle defined by the inverse tangent of the ratio for the field magnitudes. We observed TEM propagation for the longitudinal configuration, with no changes to the propagation characteristics with the ME components turned off.

The transverse ME propagation is more interesting as we obtained ordinary and extraordinary waves with propagation perpendicular to the dc magnetic field bias. The propagation direction induces a birefringent characteristic in the thin film. For propagation along the dc magnetic field bias, we obtained elliptically polarized waves, with changes to the mode of propagation as the ME effect is turned on/off. The propagation mode changes from TE to TEM as the ME components are set to zero. This is very interesting as the ME effect had not been identified as having the ability to change the mode of propagation in composites.

Results from the plane wave equations for the in-plane magnetoelectric configuration shows that the wave characteristics are dependent on only the transverse material parameters when the propagation is along the dc magnetic field bias. This implies that the ME coupling tensors do not affect the wave number of polarization of the propagating waves. Generally, we obtain elliptically polarized waves when the propagation direction is along the dc magnetic field bias, and linearly polarized waves when perpendicular to the bias.

4 Conclusion

The magnetoelectric effect in composite bilayers has been investigated. Accurate theoretical models representing the effect in piezomagnetic and piezoelectric layers have been obtained. These models give a better approximation of the magneto-electric effect as obtained in experimental data. Further investigations of the electromagnetic wave propagation in layered magnetoelectric materials have been obtained. Future studies are required to make use of the great potential held within the ME composite material. Most important is a better understanding of the electromagnetic phenomenon in ME composite materials towards device applications.

An in-depth understanding of EM wave propagation in ME composites will be invaluable towards finding more ME devices with practical applications. Tatarenko et al. have recently introduced applications of ME composites in devices such as microwave attenuators, bandpass filters, and phase shifters [5]. These applications involve the propagation of electromagnetic waves through ME composites in planar waveguide structures. The ME composite for such devices comprised of transition metal-doped ferrites as the magnetostrictive phase, and lead zirconate titanate (PZT) as the piezoelectric phase. The induced ME effect offered both electric and magnetic field control of such tunable devices.

This paper reflects that more research on the ME effect in composite materials is required to unlock the potentials that lie within the composite layers.

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