EFFICIENT HIGHER ORDER ZIG-ZAG THEORY FOR COUPLED MAGNETO-ELECTRO-ELASTIC COMPOSITE LAMINATES

J. Lee[1], J.-S. Kim[2], M. Cho[3*]
[1] School of Mechanical and Aerospace Engineering, Seoul National University, Seoul, Korea,
[2] Department of Intelligent Mechanical Engineering, Kumoh National Institute of Technology, Seoul, Korea
[3] Division of WCU Multiscale Mechanical Design, School of Mechanical and Aerospace Engineering, Seoul National University, Seoul, Korea.
* mhcho@snu.ac.kr

Keywords: *Zigzag plate theory, Piezoelectric, Piezomagnetic, Magneto-electro-elastic laminates.*

1 Introduction

Multilayered structures made of piezoelectric and piezomagnetic materials have been used widely due to their special properties of converting the mechanical energy into electrical or magnetic energy and vice versa [1-3]. For the piezoelectric materials, various studies have been carried out to analyze and to design such multilayered smart structures. Furthermore, for the accurate prediction of static and dynamic behaviors, various coupled thermo-electro-elastic analysis containing thermal effects that are significant in multiphysics problems have been carried out. Elasticity solution has been proposed [4] and higher-order zigzag models have been reported [5,6].

In the same context, the analysis of magneto-electro-elastic (MEE) materials has been increasingly demanded recently due to their unique characteristics. For the analysis of the multilayered rectangular MEE plates, Pan [2] obtained the analytical solution under the static sinusoidal load based on the quasi-Stroh formalism and the propagator matrix method. Moreover, Pan and Heyliger [3] solved the cylindrical bending problem of MEE plates with simply-supported edge condition. The vibration analysis of MEE plates has also been carried out using a layerwise-type approximation by Ramirez et al. [7]. , In addition, Annigeri et al. [8] studied the free vibration behavior of MEE beam based on the membrane-type finite element model. However, even though the previous analyses has been reported for engineering applications, one of the major drawbacks of them is that the number of unknowns is dependent upon the number of layers, which means those models are not efficient and thus they have limitations to be applied to the large-scale, sensing and actuating problem (i.e., dynamic analysis of the multilayered, coupled MEE plates). Thus, more efficient theory which also contains the accuracy are required. Among the various studies of the multilayered plate structures, an efficient higher order plate theory (EHOPT) proposed by Cho and Parmerter [9] is the best performer in displacement-based zigzag theories [10] and recommended for the analysis of the MEE plates since numerous studies have been verified the accuracy and efficiency of the EHOPT by analyzing the fully coupled piezoelectric composite plates [7] as well as the conventional composite laminates. This theory reduces the known variables using the top/bottom boundary conditions and the transverse shear stress continuity conditions.

In this study, the multilayered MEE plates are considered to carry out the fully coupled magneto-electro-elastic analysis built upon the EHOPT. The displacement field, electric potential and magnetic potential are assumed as a third order zigzag functions. The number of unknowns is reduced effectively by applying the top/bottom conditions and transverse direction flux continuity conditions. For the practical usage of the present method, finite element discretization based on the beam-type model is applied. To investigate the different responses of the elastic, electric and magnetic quantities, various layups and different loadings are considered by comparing them with the results reported in the literatures.

2 Formulation

A three-dimensional multilayered magneto-electro-elastic plate is shown in Fig. 1. The constitutive equations for a linear, anisotropic and magneto-electro-elastic solid can be written as [1-3]

$$\begin{aligned} \sigma_{ij} &= C_{ijkl}\varepsilon_{kl} - e_{ijk}E_k - q_{ijk}H_k \\ D_i &= e_{ikl}\varepsilon_{kl} + \gamma_{ik}E_k + d_{ik}H_k \\ B_i &= q_{ikl}\varepsilon_{kl} + d_{ik}E_k + \mu_{ik}H_k \end{aligned} \tag{1}$$

where σ_{ij}, D_i and B_i are the stress, electric displacement and magnetic induction, respectively. ε_{kl}, E_k and H_k are the strain, electric field and magnetic field, respectively. C_{ijkl}, γ_{ik} and μ_{ik} are the elastic, dielectric and magnetic permeability coefficients, respectively. e_{ijk}, q_{ijk} and d_{ik} are the piezoelectric, piezomagnetic and magnetoelectric coefficients, respectively. The piezoelectric and magnetostrictive layers are made of $BaTiO_3$ and $CoFe_2O_4$, respectively. Corresponding equilibrium equations can be written as

$$\sigma_{ij,j} = 0, \quad D_{i,i} = 0, \quad B_{i,i} = 0 \tag{2}$$

The strain-displacement, electric potential and magnetic potential relationship can be expressed as follows:

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right),\ E_i = -\phi_{,i}\,,\ H_i = -\psi_{,i} \tag{3}$$

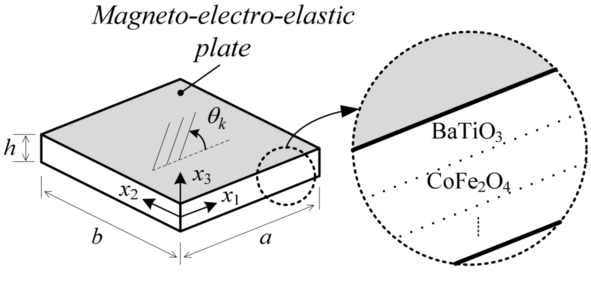

Fig.1. Geometry and coordinates of magneto-electro-elastic plate.

In this study, a fully coupled higher-order zigzag theory is proposed for efficient modeling. The displacement field is assumed by superimposing zigzag linear field to the globally varying field as follows:

$$\begin{aligned} u_1(x,z) &= u_1^0(x) + \chi_1(x)z + \xi_1(x)z^2 \\ &\quad + \eta_1(x)z^3 + \sum_{k=1}^{N-1} S_1^k(x)(z-z_k)\mathcal{H}(z-z_k) \\ u_3(x,z) &= w(x) + r_1(x)z + r_2(x)z^2 \\ &\quad + \sum_{k=1}^{N-1} r^{sk}(x)(z-z_k)\mathcal{H}(z-z_k) \end{aligned} \tag{4}$$

where $\mathcal{H}(z\text{-}z_k)$ is a Heaviside unit step function. u_1^0 and w represent the in-plane displacement and the out-of-plane displacement from the reference plane, respectively. χ_1 are the rotation of the normal about x_1 axis. For the in-plane displacement, a cubic varying field is applied. Whereas, for the out-of-plane displacement, a quadratic field is assumed in order to express the physical behavior of the displacement field accurately. In Eq. (4), the beam-type model which contains x, z and no y axis is considered for the simplicity of the problem.
Similar to the construction of the displacement field, the electric and magnetic potential are obtained by superimposing linear zigzag field onto the global cubic smooth field as follows:

$$\begin{aligned} \phi(x,z) &= \phi_0(x) + \phi_1(x)z + \phi_2(x)z^2 \\ &\quad + \phi_3(x)z^3 + \sum_{k=1}^{N-1} \phi^{sk}(x)(z-z_k)\mathcal{H}(z-z_k) \end{aligned} \tag{5}$$

$$\begin{aligned} \psi(x,z) &= \psi_0(x) + \psi_1(x)z + \psi_2(x)z^2 \\ &\quad + \psi_3(x)z^3 + \sum_{k=1}^{N-1} \psi^{sk}(x)(z-z_k)\mathcal{H}(z-z_k) \end{aligned} \tag{6}$$

The unknown variables in Eqs. (4-6) can be reduced by applying boundary conditions of top/bottom surface and introducing the continuity conditions of transverse stresses, transverse electric and magnetic displacement between each layer. The top/bottom boundary conditions can be expressed as

$$\begin{aligned}
&\sigma_5\big|_{z=0} = \sigma_5\big|_{z=h} = 0\\
&\sigma_3\big|_{z=0} = \overline{p}_0^{(0)},\ \sigma_3\big|_{z=h} = \overline{p}_0^{(h)}\\
&\phi\big|_{z=0} = \overline{\phi}^{(0)}, \quad \phi\big|_{z=h} = \overline{\phi}^{(h)}\\
&\psi\big|_{z=0} = \overline{\psi}^{(0)}, \quad \psi\big|_{z=h} = \overline{\psi}^{(h)}
\end{aligned} \tag{7}$$

where h is a thickness of the multilayered plate. For the simplicity, engineering stress and strain notations are applied. The continuity conditions are applied as follows:

$$\begin{aligned}
&\sigma_3\big|_{z=z_m^-} = \sigma_3\big|_{z=z_m^+},\ \sigma_5\big|_{z=z_m^-} = \sigma_5\big|_{z=z_m^+}\\
&D_3\big|_{z=z_m^-} = D_3\big|_{z=z_m^+},\ B_3\big|_{z=z_m^-} = B_3\big|_{z=z_m^+}
\end{aligned} \tag{8}$$

It must be noted that the stress and displacement fields which obtained from Eqs. (2-8) are statically admissible stress field and kinematically admissible displacement field, respectively, which indicates the fields satisfy proper thickness direction static and kinematic conditions. The equations can only be solved via the eigenvalue problem using the exponential functions. Thus, in this study, by ignoring the in-plane strain contribution to the transverse stress, transverse electric displacement and transverse magnetic induction in the constitutive equations, one can solve the Eqs. (2-8) without using the exponential functions and deriving the eigen-value problem. The reduced constitutive equations of k-th layer are as follows, which only used in applying the continuity conditions.

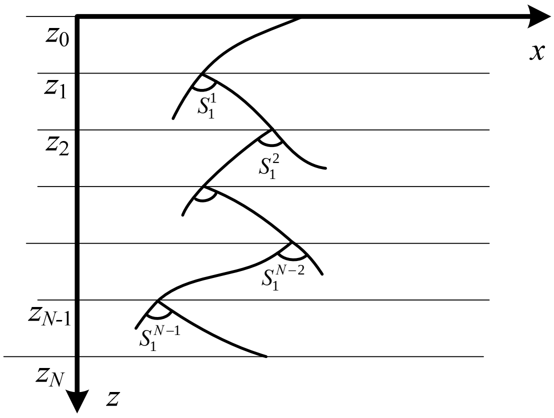

Fig.2. Zigzag field configurations.
(In-plane displacement field)

$$\begin{aligned}
&\sigma_3^{(k)} = C_{33}^{(k)}\varepsilon_3^{(k)} - e_{33}^{(k)}E_3^{(k)} - q_{33}^{(k)}H_3^{(k)}\\
&\sigma_5^{(k)} = C_{55}^{(k)}2\varepsilon_5^{(k)} - e_{15}^{(k)}E_1^{(k)} - q_{15}^{(k)}H_1^{(k)}\\
&D_3^{(k)} = e_{33}^{(k)}\varepsilon_3^{(k)} + \gamma_{33}^{(k)}E_3^{(k)} + d_{33}^{(k)}H_3^{(k)}\\
&B_3^{(k)} = q_{33}^{(k)}\varepsilon_3^{(k)} + d_{33}^{(k)}E_3^{(k)} + \mu_{33}^{(k)}H_3^{(k)}
\end{aligned} \tag{9}$$

From the Eqs. (3-9), the layer dependent variables, S_1^k, ϕ^{sk} and ψ^{sk}, are determined in terms of the primary variables of the reference plane as follows:

$$\begin{aligned}
r^{sk} &= \overline{K}_f^{(k,:)}\cdot\overline{f} + \overline{K}_7^{(k)}\phi_2 + \overline{K}_8^{(k)}\phi_3\\
&\quad + \overline{K}_9^{(k)}\psi_2 + \overline{K}_{10}^{(k)}\psi_3\\
\phi^{sk} &= \overline{M}_f^{(k,:)}\cdot\overline{f} + \overline{M}_7^{(k)}\phi_2 + \overline{M}_8^{(k)}\phi_3\\
&\quad + \overline{M}_9^{(k)}\psi_2 + \overline{M}_{10}^{(k)}\psi_3\\
\psi^{sk} &= \overline{P}_f^{(k,:)}\cdot\overline{f} + \overline{P}_7^{(k)}\phi_2 + \overline{P}_8^{(k)}\phi_3\\
&\quad + \overline{P}_9^{(k)}\psi_2 + \overline{P}_{10}^{(k)}\psi_3\\
S_1^k &= \overline{L}_f^{(k,:)}\cdot\overline{f}_{,1} + \overline{L}_7^{(k)}\phi_{2,1} + \overline{L}_8^{(k)}\phi_{3,1}\\
&\quad + \overline{L}_9^{(k)}\psi_{2,1} + \overline{L}_{10}^{(k)}\psi_{3,1} + \overline{L}_{11}^{(k)}\eta_1
\end{aligned} \tag{10}$$

where the coefficients of each primary variable are omitted for the limited space. The vector which contains the top/bottom boundary condition is expressed as,

$$\overline{f} = \left\{\overline{p}_0^{(0)}\ \ \overline{p}_0^{(h)}\ \ \overline{\phi}^{(0)}\ \ \overline{\phi}^{(h)}\ \ \overline{\psi}^{(0)}\ \ \overline{\psi}^{(h)}\right\}^T \tag{11}$$

The number of the primary variables also reduces by applying Eq. (7) to the Eqs. (4-6) and Eq. (10).

3 Virtual Work Principle and Constitutive Equations

The virtual work for a three-dimensional magneto-electro-elastic problem is stated as

$$\begin{aligned}
\delta W = &\int_V \Big(\sigma_1\delta\varepsilon_1 + \sigma_3\delta\varepsilon_3 + \sigma_5\delta 2\varepsilon_5 - D_1\delta E_1\\
&-D_3\delta E_3 - B_1\delta H_1 - B_3\delta H_3\Big)dV\\
&-\int_S \overline{p}_1\delta u_1 dS
\end{aligned} \tag{12}$$

$$
\begin{aligned}
&\left[R_1^{(0)} \quad R_1^{(1)} \quad R_1^{(2)} \quad R_1^{(3)} \quad R_1^{k}\right] = \int_0^h \sigma_1 \left[1 \quad z \quad z^2 \quad z^3 \quad (z-z_k)\mathcal{H}(z-z_k)\right] dz \\
&\left[R_3^{(0)} \quad R_3^{(1)} \quad R_3^{k}\right] = \int_0^h \sigma_3 \left[1 \quad z \quad \mathcal{H}(z-z_k)\right] dz \\
&\left[V_1^{(0)} \quad V_1^{(1)} \quad V_1^{(2)} \quad V_1^{k(0)} \quad V_1^{k(1)}\right] = \int_0^h \sigma_5 \left[1 \quad z \quad z^2 \quad \mathcal{H}(z-z_k) \quad (z-z_k)\mathcal{H}(z-z_k)\right] dz \\
&\left[F_1^{(0)} \quad F_1^{(1)} \quad F_1^{(2)} \quad F_1^{(3)} \quad F_1^{k}\right] = \int_0^h D_1 \left[1 \quad z \quad z^2 \quad z^3 \quad (z-z_k)\mathcal{H}(z-z_k)\right] dz \qquad (13) \\
&\left[F_3^{(0)} \quad F_3^{(1)} \quad F_3^{(2)} \quad F_3^{k}\right] = \int_0^h D_3 \left[1 \quad z \quad z^2 \quad \mathcal{H}(z-z_k)\right] dz \\
&\left[G_1^{(0)} \quad G_1^{(1)} \quad G_1^{(2)} \quad G_1^{(3)} \quad G_1^{k}\right] = \int_0^h B_1 \left[1 \quad z \quad z^2 \quad z^3 \quad (z-z_k)\mathcal{H}(z-z_k)\right] dz \\
&\left[G_3^{(0)} \quad G_3^{(1)} \quad G_3^{(2)} \quad G_3^{k}\right] = \int_0^h B_3 \left[1 \quad z \quad z^2 \quad \mathcal{H}(z-z_k)\right] dz
\end{aligned}
$$

where $\bar{p}_1$ denotes the in-plane traction which did not consider in Eq. (7). The stress resultants can be obtained by integrating the stresses in the through-the-thickness direction as shown in Eq. (13). The constitutive equations of the laminate can be derived using the Eqs. (3-6) and Eqs. (11-12) as follows:

$$
\begin{Bmatrix} \mathbf{R}_1 \\ \mathbf{R}_3 \\ \mathbf{F}_3 \\ \mathbf{G}_3 \end{Bmatrix} = \begin{bmatrix} \mathcal{A}_1 & \mathcal{A}_2 & -\mathcal{B}_1 & -\mathcal{D}_1 \\ \mathcal{A}_3 & \mathcal{A}_4 & -\mathcal{B}_2 & -\mathcal{D}_2 \\ \mathcal{B}_3 & \mathcal{B}_4 & \mathcal{E}_3 & \mathcal{F}_1 \\ \mathcal{D}_3 & \mathcal{D}_4 & \mathcal{F}_2 & \mathcal{G}_1 \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}_1 \\ \boldsymbol{\varepsilon}_3 \\ \mathbf{E}_3 \\ \mathbf{H}_3 \end{Bmatrix} \qquad (14)
$$

$$
\begin{Bmatrix} \mathbf{V} \\ \mathbf{F}_1 \\ \mathbf{G}_1 \end{Bmatrix} = \begin{bmatrix} \mathcal{A}_5 & -\mathcal{B}_5 & -\mathcal{D}_5 \\ \mathcal{B}_6 & \mathcal{E}_4 & \mathcal{F}_3 \\ \mathcal{D}_6 & \mathcal{F}_4 & \mathcal{G}_2 \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}_5 \\ \mathbf{E}_1 \\ \mathbf{H}_1 \end{Bmatrix} \qquad (15)
$$

where the resultants in the left-hand side are the column vectors in Eq. (13). The stiffness matrices are calculated by integrating the material constants in Eq. (12) along the thickness direction. The strains, electric field and magnetic field in the right-hand side are the column vectors which are defined similar to those in Eq. (13). The sizes of the vectors and matrix in Eq. (14) and (15) are (16×1), (16×16) and (15×1), (15×15), respectively.

4 Finite Element Model

To assess the validity of the present method, a finite element is developed for one-dimensional problems. The primary displacement, magnetic and electric potential unknowns are expressed in terms of nodal values and interpolation functions as follows:

$$
\begin{aligned}
&\left[u_1^0 \quad \eta_1\right] = \sum_{m=1}^{n} \mathbf{N}_m \left[u_1^0 \quad \eta_1\right]_m, \\
&\left[w \quad \phi_2 \quad \phi_3 \quad \psi_2 \quad \psi_3\right] \qquad (16) \\
&\quad = \sum_{m=1}^{n} \begin{pmatrix} \mathbf{Q}_m \left[w \quad \phi_2 \quad \phi_3 \quad \psi_2 \quad \psi_3\right]_m + \\ \mathbf{Y}_m \left[w_{,1} \quad \phi_{2,1} \quad \phi_{3,1} \quad \psi_{2,1} \quad \psi_{3,1}\right]_m \end{pmatrix}
\end{aligned}
$$

where n is the number of nodes in a typical finite element. $\mathbf{N}_\mathrm{m}$ is a Lagrangian interpolation function and $\mathbf{Q}_\mathrm{m}$, $\mathbf{Y}_\mathrm{m}$ are Hermite interpolation functions.
The components of the field variables and the nodal displacement vectors are given by

$$
\begin{aligned}
&\boldsymbol{\varepsilon}_1 = \left\{\varepsilon_1^{(0)} \ \varepsilon_1^{(1)} \ \varepsilon_1^{(2)} \ \varepsilon_1^{(3)} \ \varepsilon_1^{1} \ \cdots \ \varepsilon_1^{N-1}\right\}^T \\
&\boldsymbol{\varepsilon}_3 = \left\{\varepsilon_3^{(0)} \ \varepsilon_3^{(1)} \ \varepsilon_3^{1} \ \cdots \ \varepsilon_3^{N-1}\right\}^T \\
&\boldsymbol{\varepsilon}_5 = \left\{\varepsilon_5^{(0)} \ \varepsilon_5^{(1)} \ \varepsilon_5^{(2)} \ \varepsilon_5^{1(0)} \ \cdots \ \varepsilon_5^{N-1(0)} \right. \\
&\qquad\qquad \left. \varepsilon_5^{1(1)} \ \cdots \ \varepsilon_5^{N-1(1)}\right\}^T \qquad (17) \\
&\mathbf{E}_1 = \left\{E_1^{(0)} \ E_1^{(1)} \ E_1^{(2)} \ E_1^{(3)} \ E_1^{1} \cdots E_1^{N-1}\right\}^T \\
&\mathbf{E}_3 = \left\{E_3^{(0)} \ E_3^{(1)} \ E_3^{(2)} \ E_3^{1} \ \cdots E_3^{N-1}\right\}^T \\
&\mathbf{H}_1 = \left\{H_1^{(0)} \ H_1^{(1)} \ H_1^{(2)} \ H_1^{(3)} \ H_1^{1} \cdots H_1^{N-1}\right\}^T \\
&\mathbf{H}_3 = \left\{H_3^{(0)} \ H_3^{(1)} \ H_3^{(2)} \ H_3^{1} \cdots H_3^{N-1}\right\}^T
\end{aligned}
$$

#

$$\begin{aligned}
&\{u_n\} = \left\{u_1^0 \quad w \quad w_{,1} \quad \phi_2 \quad \phi_{2,1} \quad \phi_3 \quad \phi_{3,1} \quad \psi_2 \quad \psi_{2,1} \quad \psi_3 \quad \psi_{3,1} \quad \eta_1\right\}_n^T \\
&\hat{\boldsymbol{\varepsilon}} = \left\{\boldsymbol{\varepsilon}_1 \quad \boldsymbol{\varepsilon}_3 \quad \boldsymbol{\varepsilon}_5 \quad \mathbf{E}_1 \quad \mathbf{E}_3 \quad \mathbf{H}_1 \quad \mathbf{H}_3\right\}^T \\
&[\mathbf{B}] = \left[\mathbf{B}^{\boldsymbol{\varepsilon}1} \quad \mathbf{B}^{\boldsymbol{\varepsilon}3} \quad \mathbf{B}^{\boldsymbol{\varepsilon}5} \quad \mathbf{B}^{\mathbf{E}1} \quad \mathbf{B}^{\mathbf{E}3} \quad \mathbf{B}^{\mathbf{H}1} \quad \mathbf{B}^{\mathbf{H}3}\right]^T \{u_n\} \\
&\left[\mathbf{B}^{\langle\ \rangle}\right] = \left[\mathbf{B}_1^{\langle\ \rangle}, \cdots, \mathbf{B}_n^{\langle\ \rangle}\right], \quad \hat{\boldsymbol{\varepsilon}} = [\mathbf{B}]\{u_n\}
\end{aligned} \tag{18}$$

where (k), the superscript with brackets, is the coefficient of z^k. N denotes the number of layers. [**B**] represents the strain-displacement matrix, and the details of the matrix are omitted for the limited space. The stiffness matrix can be constructed from the virtual work principle as follows:

$$\begin{aligned}
\delta W &= \int_x \boldsymbol{\varepsilon}^T \mathbf{C} \delta \boldsymbol{\varepsilon} dx \\
&= \Big(\int_x \{u_n\}^T [\mathbf{B}]^T \mathbf{C} [\mathbf{B}] dx \\
&\qquad - \int_x \boldsymbol{\varepsilon}_f \mathbf{C} [\mathbf{B}] dx \Big) \delta \{u_n\} \\
&= \big([\mathbf{K}]\{u_n\} - \{f\}\big) \delta \{u_n\} = 0
\end{aligned} \tag{19}$$

For the multilayered MEE laminates, the top/bottom boundary conditions are the external forces in the finite element problem. Finally, nodal unknowns can be obtained by solving the Eq. (19).

5 Conclusions

A fundamental higher order zigzag theory of multi-layered MEE plate is presented in this paper. Starting from the fully-coupled three-dimensional field equations, the displacements and potentials are assumed as cubic and quadratic zigzag functions. The top/bottom boundary conditions and the continuity conditions are employed in order to reduce the layer-dependent variables. For the application of the present method, a finite element is developed for one-dimensional problems. The final finite element governing equations contain the fully coupled magneto-electro-elastic behaviors. To examine the coupling effects of MEE plate, the electrical and magnetic loadings as well as the mechanical loading are considered for various layup configurations. The present method can be used as an efficient tool for the analysis of such smart composite structures.

Acknowledgements

This work was supported by Defense Acquisition Program Administraion and Agency for Defense Development under the contract UD100048JD.

References

[1] C.W. Nan, "Magnetoelectric effect in composite of piezoelectric and piezomagnetic phases". *Physical Review B*, Vol. 50, No. 9, pp 6082-6088, 1994.

[2] E. Pan "Exact solutions for simply supported and multilayered magneto-electro-elastic plates". *Journal of Applied Mechanics*, Vol. 68, pp 608-618, 2001.

[3] E. Pan and P.R. Heyliger, "Exact solutions for magneto-electro-elastic laminates in cylindrical bending". *International Journal of Solids and Structures*, Vol. 40, pp 6859-6876, 2003.

[4] K. Xu, A.K. Noor and Y.Y. Tang, "Three-dimensional solutions for coupled thermoelectro-elastic response of multilayered plates". *Computer Methods in Applied Mechanics and Engineering*, Vol. 126, No. 3-4, pp 355-371, 1995.

[5] M. Cho and J. Oh, "Higher order zig-zag theory for fully coupled thermo-electric-mechanical smart composite plates". *International Journal of Solids and Structures*, Vol. 41, No. 5-6, pp 1331-1356, 2004.

[6] J. Oh and M. Cho, "A finite element based on cubic zig-zag plate theory for the prediction of thermo-electric-mechanical behaviors". *International Journal of Solids and Structures*, Vol. 41, No. 5-6, pp 1357-1375, 2004.

[7] F. Ramirez, P.R. Heyliger and E. Pan, "Free vibration response of two-dimensional magneto-electro-elastic laminated plates". *Journal of Sound and Vibration*, Vol. 292, No. 3-5, pp 626-644, 2006.

[8] A.R. Annigeri, N. Ganesan and S. Swarnamani, "Free vibration behavior of multiphase and layered magneto-electro-elastic beam", Journal of Sound and Vibration, Vol. 299, No. 1-2, pp 44-63, 2007.

[9] M. Cho and R.R. Parmerter, "Efficient higher order composite plate theory for general lamination configurations". *AIAA Journal*, Vol. 31, No. 7, pp 1299-1306, 1993.

[10] E. Carrera, "Historical review of zig-zag theories for multilayered plates and shells", Applied Mechanics Review (ASME), Vol. 56, No. 3, pp 287-308.