AN ANALYTICAL TOOL TO PREDICT LOAD DISTRIBUTION OF MULTI-BOLT SINGLE-LAP THICK LAMINATE JOINTS

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1 Introduction

With the increasing demand of energy and environmental protection, composite materials have been extensively used in aeronautic structures, especially in the primary load components, due to its weight-saving potential. Therefore, the section of some composite structures could be much thicker than before. Thick-section composites are ones where the effect of geometry, material constituents, lamination scheme, processing and service loading exhibit three dimensional states of stress.

In order to join thick-section composites, it still requires bonding, fastening or hybrid ways, among which the bolted joint is a popular method because of its high reliability, load-carrying capacity and convenience to disassemble [1]. However, stress concentration, composite brittleness and anisotropy are sources of weakness in mechanically fastened joints, what is more, the single-lap joint is more dangerous because of the secondary bending effect induced, and these disadvantages could be more serious when thick-section laminates take part in the joint. Thus, precise prediction of the load distribution in bolted joint composite structures is in bad need for the industry engineers and researchers.

Traditionally, there are 3 methods to study the load distribution in multi-bolt composite joints: experimental test methods, analytical method and finite element analysis method. ASTM committee has developed test procedure ASTM D7248 to assess the load distribution of 2-fastener polymer matrix composite laminates. The load distribution can also be measured by instrumented bolt with rosette strain gauges [2]. However, the test method is expensive and time-consuming, so it's not easy for aircraft designer to adopt. Finite element analysis, which usually requires a 3D nonlinear finite element model since the load is not uniform through the laminate thickness[3,4], often costs dozens of hours to yield the results.

This paper, considering the changes of fasteners flexibility and plates flexibility introduced in thick laminate single-lap joint, will introduce a new analytical tool, which makes the designer in the comfort of predicting the load distribution in multi-bolt single-lap thick laminate joint.

2 Fastener Flexibility in Composite Joint

The flexibility of the mechanical composite joint is composed of the flexibilities of the plates and fasteners, and the co-operations of those flexibilities will influence the load transfer over the fasteners. In addition, the flexibilities of the plates are smaller in thick laminate joint comparing with that in thin laminate joint, which means the fastener flexibility will take more effort in load sharing than that in thin plates joint. Futhermore, the load carried by fastener will not be uniform along the plate thickness in single-lap joint, which will influence the fastener's displacement, that is to say the flexibility of fastener needed to be verified according to the load condition. To be precise, the flexibilities of the fasteners in the thick laminate single-lap joint needs to be more accurately estimated according to the actual.

2.1 Load Condition of Single-lap Bolted Joint

The load condition of single-lap bolted composite joint is shown in Fig.1. it has shown that the fasteners undergo the bearing force (the fastener exert bearing force to the plate inversely)from plates, shearing force and bending moment introduced by eccentric load from the two plates, thus the deformation in the joint includes 6 parts: the shearing deformation of the fastener, the bending deformation of the fastener, the fastener's bearing deformation by "plate 1", the fastener's bearing deformation by "plate 2", the "plate 1" bearing deformation by the fastener, the “plate 2” bearing deformation by the fastener.

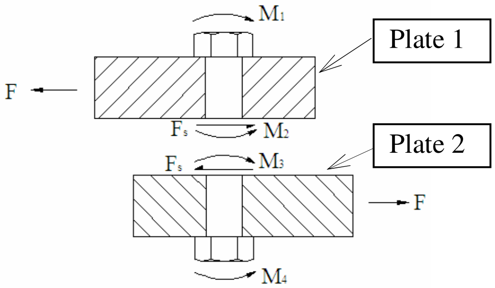
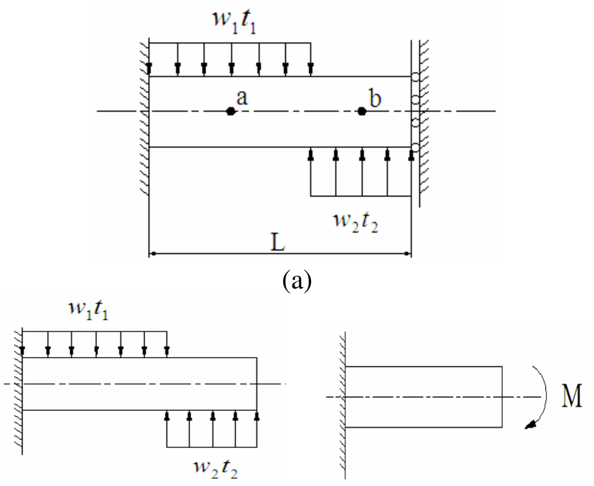


Fig.1. Load condition in single-lap joint

2.2 Fastener Flexibility over Uniform Bearing Load

1) Force analysis

The Fastener load model over uniform bearing load is shown in Fig.3 (a), which is a statically indeterminate system that can be divided into Fig.3 (b) and Fig.3(c).



(a)

(b) (c)

Fig.3. Fastener over uniform bearing load

The angle of rotation of the beam θ_1 under w_1t_1 , θ_2 under w_2t_2 and θ_3 under the moment M [5],

$$\theta_1 = \frac{w_1 t_1^3}{6EI} \tag{1}$$

$$\theta_2 = \frac{w_2 (L^3 - t_1^3)}{6EI} \tag{2}$$

$$\theta_3 = \frac{ML}{EI} \tag{3}$$

While the rotation angle of the beam on the right equals zero,

$$\theta = \theta_1 - \theta_2 + \theta_3 = 0 \tag{4}$$

Equilibrium equation,

$$P = w_1 t_1 = w_2 t_2 \tag{5}$$

Substituting θ_1 , θ_2 , θ_3 from Equation (1-3) to Equation (4),

$$\frac{w_1 t_1^3}{6EI} - \frac{w_2 (L^3 - t_1^3)}{6EI} + \frac{ML}{EI} = 0 \tag{6}$$

Thus,

$$M = \frac{w_2 (2t_1^2 t_2 + 3t_1 t_2^2 + t_2^3)}{6L} \tag{7}$$

2) Flexibility

As shown in Fig.3, apart from undertaking deformation caused by shear stress and bending moment, the combination of fastener and plate also endure the bearing deformation between the two objects. Hence, the deformation between Point a and Point b of the bolt is the result of the co-operation of the six different constituents mentioned above. The displacements of Point a caused by w_1t_1 、 w_2t_2 、 M,

$$\delta_{a1} = \frac{w_1 (\frac{t_1}{2})^2}{24EI} \left[6t_1^2 - 4t_1 \frac{t_1}{2} + (\frac{t_1}{2})^2 \right] \tag{8}$$

$$\delta_{a2} = -\frac{w_2 (\frac{t_1}{2})^2}{12EI} \left[3t_2 L + 3t_1 t_2 - 2t_2 \frac{t_1}{2} \right] \tag{9}$$

$$\delta_{a3} = -\frac{M (\frac{t_1}{2})^2}{2EI} \tag{10}$$

$$\delta_a = \delta_{a1} + \delta_{a2} + \delta_{a3} \tag{11}$$

Displacements of Point b caused by w_1t_1 、 w_2t_2 、 M,

$$\delta_{b1} = \frac{w_1 t_1^{\ 3}}{24EI} \left[4(t_1 + \frac{t_2}{2}) - t_1 \right] \tag{12}$$

$$\delta_{b2} = -\frac{w_2}{24EI} [(t_1 + \frac{t_2}{2})^4 - 4(t_1 + t_2)(t_1 + \frac{t_2}{2})^3 + 6(t_1 + t_2)^2 (t_1 + \frac{t_2}{2})^2 - 4t_1^3 (t_1 + \frac{t_2}{2}) + t_1^4] \tag{13}$$

$$\delta_{b3} = -\frac{M(t_1+\frac{t_2}{2})^2}{2EI} \tag{14}$$

$$\delta_b = \delta_{b1} + \delta_{b2} + \delta_{b3} \tag{15}$$

The bending flexibility of the bolt,

$$C_{bb} = \frac{\delta}{P} = \frac{\delta_b - \delta_a}{P} = -\frac{9t_1^4 + 57t_1^3t_2 + 96t_1^2t_2^2 + 57t_1t_2^3 + 9t_2^4}{384LEI} \tag{16}$$

The shear flexibility, bearing flexibility of the bolt and the plate flexibility,

$$C_{bs} = \frac{4(t_1+t_2)}{9G_bA_b} \tag{17}$$

$$C_{bbr} = \frac{1}{t_1E_{bbr}} + \frac{1}{t_2E_{bbr}} \tag{18}$$

$$C_{pbr} = \frac{1}{t_1E_{x1}} + \frac{1}{t_2E_{x2}} \tag{19}$$

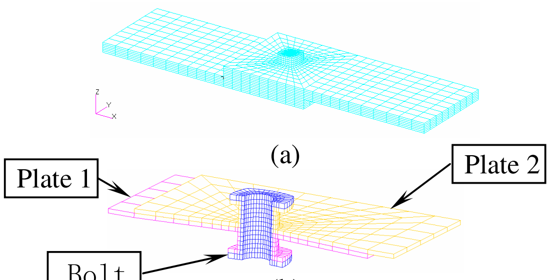
Thus the fastener's flexibility over uniform bearing loads through substitution and calculation,

$$F = \frac{4(t_1+t_2)}{9G_bA_b} + \frac{9t_1^4 + 57t_1^3t_2 + 96t_1^2t_2^2 + 57t_1t_2^3 + 9t_2^4}{384EI(t_1+t_2)} + \frac{1}{t_1E_{bbr}} + \frac{1}{t_2E_{bbr}} + \frac{1}{t_1E_{x1}} + \frac{1}{t_2E_{x2}} \tag{20}$$

2.3 Fastener Flexibility in Single-lap Bolted Joint

The contact force is non-uniform along the plate thickness in the single-lap joints shown in Fig.3, therefore the items in Equation (1) needed to be corrected accordingly. Though they can be obtained through fitting the experimental results, there is no physical meaning on every factor, besides the factors may change with different experimental results, and the experimental cost is high. Several single-joint 3D finite element models, whose modeling method is validate by the test results carried out according to ASTM D 5961 test standard[6], employed the 6 correction factors separately.

The 3D finite element model of single lap bolted composite joint and contact area setting are shown in Fig.4.



(a)

(b)

Fig.4. 3D FE model of single-lap bolted joint and contact bodies

Comparing the bearing deformation of the plates, and the bearing deformation, the shear deformation and bending deformation of the bolt separately, correction factors of the single-lap thick laminate bolted joint could be constructed respectively. The fastener's flexibility in single-lap bolted thick composite joint (named as Liu-Mao in this paper) is shown as:

$$F = \frac{t_1+t_2}{6G_bA_b} + \frac{9t_1^4 + 57t_1^3t_2 + 96t_1^2t_2^2 + 57t_1t_2^3 + 9t_2^4}{38400EI(t_1+t_2)} + \frac{1.15}{t_1E_{bbr}} + \frac{1.15}{t_2E_{bbr}} + \frac{1.15}{t_1E_{x1}} + \frac{1.15}{t_2E_{x2}} \tag{21}$$

3 Analytical Tool of Load Distribution in Multi-bolt Joint

In aircraft structure design, except for seal areas where the fasteners are staggered, joint between two structures can be divided into several "multi-row single-column" substructures. Usually, 3 rows is utmost. Following contents focus on constructing the 2 row and 3 row fasteners joint analytical model.

3.1 Spring Model of Single-lap Joint

The spring model of single lap 2-bolt joint model is shown in Fig.5.

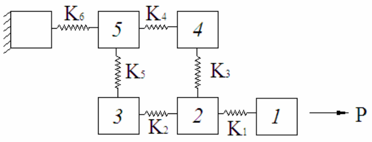


Fig.5. Spring model of single-lap two-bolt joint

K_2 and K_4 are the stiffness of the plate sections between the two bolts, K_1 and K_6 are the stiffness of the plate sections from the bolts to the ends of the plates, which can be calculated by $\frac{EA}{l}$. K_3 and K_5

are the stiffness of the fasteners in the single-lap joint, the reciprocal of the fasteners flexibilities which can be calculated through equation(3).

K_1 and K_6 just influence the stiffness of the whole joint system and have little impact on the fasteners load sharing, therefore, when u_5 is treated as zero, the balance equations can be written as below:

$$\begin{cases} K_2(u_2-u_3)+K_3(u_2-u_4)=P \\ K_2(u_3-u_2)+K_5u_3=0 \\ K_3(u_4-u_2)+K_4u_4=0 \end{cases} \tag{22}$$

Calculate u_2, u_3 and u_4:

$$u_2=\frac{P}{(K_2+K_3-\frac{K_3^2}{K_3+K_4}-\frac{K_2^2}{K_2+K_5})} \tag{23}$$

$$u_3=\frac{K_2P}{(K_2+K_5)(K_2+K_3-\frac{K_3^2}{K_3+K_4}-\frac{K_2^2}{K_2+K_5})} \tag{24}$$

$$u_4=\frac{K_3P}{(K_3+K_4)(K_2+K_3-\frac{K_3^2}{K_3+K_4}-\frac{K_2^2}{K_2+K_5})} \tag{25}$$

The loads carried by bolt 1 and bolt 2 are:

$$P_1=K_5(u_3-u_5) \tag{26}$$

$$P_2=K_3(u_2-u_4) \tag{27}$$

The displacement of the whole joint under load P,

$$u_1=\frac{P}{(K_2+K_3-\frac{K_3^2}{K_3+K_4}-\frac{K_2^2}{K_2+K_5})}+\frac{P}{K_1}+\frac{P}{K_6} \tag{28}$$

The load distribution of 3-bolt single-lap joint can be obtain through the same method with that of 2-bolt joint.

3.2 Model Validation

Load distribution tests following ASTM D 7248 standard (shown in Fig.7) were used to validate the load distribution calculation method based on the spring model mentioned above.

There are 3 groups of specimens, whose geometry configuration of the joints are shown in table.1, where t1, t2, W, e, p, D stand for the two plate thicknesses, widths, edge distance, row distance and hole diameter respectively. Table.3 listed the equivalent engineering constants of the laminate plate. The detail test method, parameters and results can also refer to reference 7.

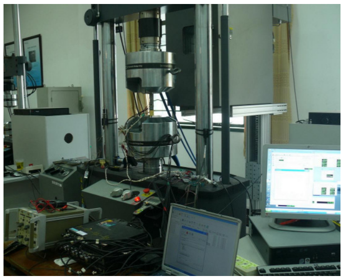


Fig.7 Load distribution test

Table.1 geometry configuration

Test No.	W (mm)	e (mm)	P (mm)	t1 (mm)	t2 (mm)	D (mm)
1	36.47	15.17	29.83	3.81	3.81	5
2	36	20	25	2.5	7.5	5
3	60	36	60	10	32.57	12

Table.2 mechanical properties

Test No.	E_1(GPa)	E_2(GPa)	G_{12}(GPa)	υ_{12}
1	131	8.8	5.2	0.3
2	131	8.11	3.66	0.34
3	135	9.4	5	0.28

Table.3 Lay-ups and equivalent constants

No.	lay-up	E_x(GPa)	E_y(GPa)	G_{xy}(GPa)	υ_{xy}
1	$[45/0/-45/90/0]_{3s}$	63.2	40.7	13.5	0.364
2	$[45/0/-45/0/90/0/45/0/-45/0]_{s}$	73.5	29.7	15.2	0.44
3	$[45/0/-45/0/90/0/45/0/-45/0]_{4s}$	79.8	32.9	17	0.41

The load distribution of different bolts is calculated using fastener flexibility equation from ASTM D 7248[8](Equation 29), Hart-Smith[9] (Equation 30) and this paper (Equation 21). The fastener flexibility equations are shown as below. The results of the load distribution are shown in Table.4, from which it shows that the result calculated through equation presented in this paper is more accurate than the other two.

$$C_F=\frac{8(2t_S+t_P)(1+\nu_F)}{3\pi E_F d^2}+\frac{64(8t_S^3+16t_S^2t_P+t_P^3)}{192\pi E_F d^4} \tag{29}$$
$$+\frac{2t_S+t_P}{t_St_PE_F}+\frac{1}{t_SE_{xS}}+\frac{2}{t_PE_{xP}}$$

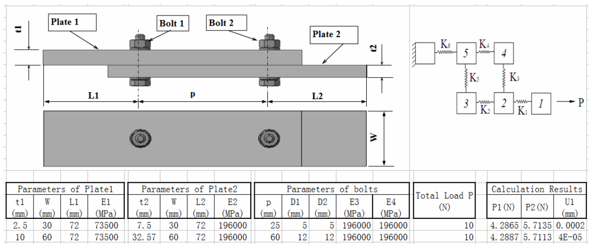
$$\frac{1}{K}=\frac{\delta}{P}=\frac{2(t_1+t_2)}{3G_bA_b} \tag{30}$$
$$+(\frac{t_1+t_2}{t_1t_2E_{bbr}}+\frac{1}{t_1\left(\sqrt{E_LE_T}\right)_1}+\frac{1}{t_2\left(\sqrt{E_LE_T}\right)_2})(1+3\beta)$$

Table.4 load distribution results (in percentage)

Test No.	ASTM D 7248	Hart-smith	Liu-Mao	Test
1	52.72	53.76	54.65	56.23
Error (%)	6.2	4.4	2.8	—
2	54.93	54.16	56.92	57.22
Error (%)	4.0	5.3	0.5	—
3	51.51	53.42	56.68	57.83
Error (%)	11.0	7.6	2.0	—

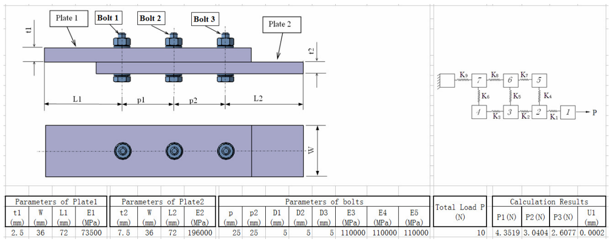
3.3 Load Distribution Analytical Tool in Excel

The analytical tools, compiled in Excel, are shown in Figure.8 and Figure.9, in which the parameters are thickness of plate1 (t1) and plate2 (t2), distances from the bolts to the plate ends (L1 and L2), elastic modulus of the plates and bolts (E1, E2, E3 and E4), plate width (W), row distance (p), and the total load (P). Upon entering all the parameters, the calculation will be accomplished automatically. Moreover, the analytical tools could be used in the calculation with bolts that have different diameters and elastic modulus, hence a great convenience for the engineers at the beginning of structure design.



Parameters of Plate1				Parameters of Plate2				Parameters of bolts					Total Load P (N)	Calculation Results		
t1 (mm)	W (mm)	L1 (mm)	E1 (MPa)	t2 (mm)	W (mm)	L2 (mm)	E2 (MPa)	p (mm)	D1 (mm)	D2 (mm)	E3 (MPa)	E4 (MPa)		P1 (N)	P2 (N)	U1 (mm)
2.5	30	72	73500	7.5	30	72	196000	25	5	5	196000	196000	10	4.2865	5.7135	0.0002
10	60	72	73500	32.57	60	72	196000	60	12	12	196000	196000	10	4.2887	5.7113	4E-05

Fig.8 the analytical tool for 2 fasteners



Parameters of Plate1				Parameters of Plate2				Parameters of bolts								Total Load P (N)	Calculation Results			
t1 (mm)	W (mm)	L1 (mm)	E1 (MPa)	t2 (mm)	W (mm)	L2 (mm)	E2 (MPa)	p (mm)	p2 (mm)	D1 (mm)	D2 (mm)	D3 (mm)	E3 (MPa)	E4 (MPa)	E5 (MPa)		P1 (N)	P2 (N)	P3 (N)	U1 (mm)
2.5	36	72	73500	7.5	36	72	196000	25	25	5	5	5	110000	110000	110000	10	4.3519	3.0404	2.6077	0.0002

Fig.9 the analytical tool for 3 fasteners

4 Influences of Different Factors on Load Distribution

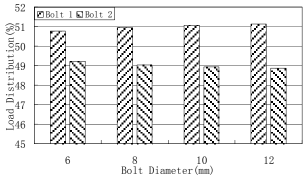
4.1 Ratio of Bolt Diameter to Plate Thickness

For the single-lap joint, one of the plate is laminate, with stacking sequence being [0°/±45°/90°]16S, ply thickness being 0.125mm, and the material properties is shown in Table.5; the other is 8mm thick 30CrMnSi plate(E=196GPa,υ=0.3).

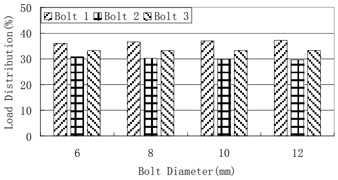
Table.5 Material properties of T300/QY8911

E_{11}(GPa)	E_{22}(GPa)	G_{12}(Gpa)	υ_{12}
135	8.8	4.47	0.33

The hole diameters distinguished by D=6mm, 8mm, 10mm and 12mm.The load distribution got from the analytical tool is plotted in Fig.11.



(a) 2 bolts



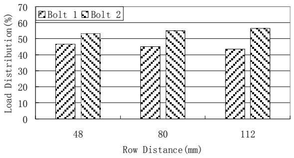
(b) 3 bolts

Fig.11 Influence of *t*/*d*

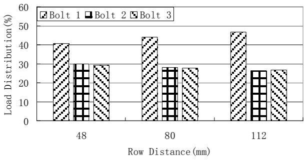
Both the diagrams (a) and (b) indicate that the most loaded bolt endures much more load, with the increasing of the bolt diameter. The change of bolt diameter causes the proportion of the stiffness between bolt and plates which result in the uneven load distribution should be the explanation.

4.2 Ratio of Row Distance to Plate Thickness

Use the same plates depicted in section 4.1, but the row distance is 3t, 5t and 7t, respectively.



(a) 2 bolts



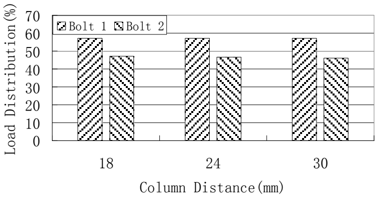
(b) 3 bolts

Fig.12 Influence of *p*/*t*

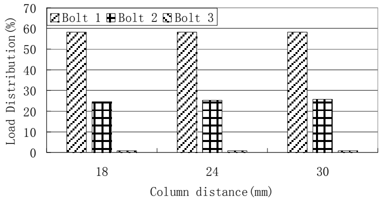
The results in Fig.12 (a) imply the distribution becomes more uneven among fasteners with the increase of row distance.

4.3 Ratio of Plate Width to Plate Thickness

With all the other parameters identical with section 4.1, the variable s, column distance varies from 3D to 5D. Results in the two-fastener and three- fastener situations are plotted in Fig.13.



(a) 2 bolts



(b) 3 bolts

Fig.13 Influence of *W*/*t*

As shown in Fig.13, the load distributions in different bolts remain nearly the same with the variation of column distance. These indicate that the column distance is not as significant as other parameters discussed above in the influence over load distribution.

Conclusions

(1)The calculation tool getting load distributions of thick-laminate multi-bolt single-lap joint was validated to be accurate and efficient, especially when dealing with the thick plates.

(2)Considering that in most of the situations, the joint area shall be divided into several regions containing the original number of rows but only one column of fasteners, accuracy of the solution could be guaranteed. Thus, it is a very useful tool for the designers to commence the evaluation of parameters at the very beginning.

(3)The increasing of bolt diameters and row distances has impact on the load distribution, which develops more uneven among fasteners. Due to the simplified method which divides the multi-row, multi-column joint area into multi-row, single-column sub areas, the influence of column distance over load distribution is equivalent of the influence of plate width of the sub area over load distribution, which has little effect on the load distribution.

(4) The stiffness ratio has great influence on the load distribution, which differs more seriously between fasteners when raising the stiffness ratio. Besides, the distribution becomes more uneven when the ratio of fasteners distance to the plate thickness comes larger.

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