WRINKLING CHARACERISTIC OF MEMBRANE INFLATED TRUNCATED CONE

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1 Introduction

Inflatable space structures have received attentions widely, and are applied in several structural designs of aerospace spacecrafts, including inflatable antennae reflectors [1], solar sails, inflatable decoys, inflatable sunshield, and so on.

At present, The study of inflatable beam is mainly focused on the wrinkling load, the collapse load[2],[3] and the displacement on the load response[4] when there is a shearing force, and also focused on the buckling force when there is a axial compressive load[5]. Inflated booms are the key supporting parts of the inflatable structures. There are two types of research methods currently, One is based on membrane theory, and the other is based on thin walled structure theory[6], they have different criteria and physical properties. The membrane theory does not take into account bending rigidity, and such a structure can only bear tension stress. But the thin walled structure can bear shearing force and compression force. The wrinkling criteria of the membrane theory can be divided into the non-negative principal stresses and the non-negative principal strains. Shney [7] et al. have obtained the deflections of inflated beam under uniformly distributed load by non-negative principal stresses. Main [8] et al. have conducted experiments on a inflated cantilever beam and improved Comer's theory by non-negative principal strain. Wood [2] have introduced air pressure into Brazier's model and derived the collapse moment and the wrinkling force with pressure. Inflated truncated cone have more advantages than inflated booms [9], the geometry of an inflatable beam subjected to bending can be optimized by applying tapered beams. Inflated truncated cone is a thin-walled lightweight structure, wrinkles may appear when load exceed the critical value. Wrinkling is one of the main factor which lead boom to failure [10],[11]. Veldman solved the bending problem of inflated truncated cone by using Mohr's circle and non-negative principal stress wrinkling criterion, proposed initial wrinkling location and the maximum tip load, and carried on some experiment [12].

In this paper, the dimensionless initial wrinkling location and the wrinkling region under shear load is analyzed. The experiments corresponding to the wrinkling expand behavior of inflated truncated cone were done.

2 The initial wrinkling location

Fig.1 shows a schematic overview of membrane inflated truncated cone subjected to bending. The cone is clamped at the origin of the Cartesian coordinate system and a shear load *F* that introduces bending are applied at the free end.

Note that from Fig.1, a membrane inflated truncated cone is subject to a tip load F. The radius of the free tip is r_0, the radius of the fixed end is r_1, the half taper angle is α, the radius at coordinate *z* is r, the thickness is t, the length is l, the inflated pressure is p, the angle in section $C-C$ is θ.

The equilibrium equations of the force and moment in section *C-C* are:

$$\begin{cases} p\pi r^2 = \int_A \sigma dA = 2\int_0^{\pi} \sigma r t d\theta \\ M = \int_A \sigma r \cos\theta dA = 2\int_0^{\pi} \sigma r^2 t \cos\theta d\theta \end{cases} \quad (1)$$

When the wrinkle occurs in section *C-C*, the stress parallel to *z*-axis is:

$$\sigma = \sigma_m (1-\cos\theta) \quad (2)$$

Where σ_m is the maximum value of σ.

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Substituting Eq.(2) into Eq.(1) and removing σ_m, we obtain:

$$M_w = \frac{1}{2}\pi p r^3 \tag{3}$$

Assume the radius of free tip r_0 is a constant, since it will be needed to install scientific instrument, the relationship between the radius r and the coordinate z is:

$$r = z\tan\alpha + r_0 \quad (0 \le z \le l) \tag{4}$$

And the total length of the inflated truncated cone l is:

$$l = (r_1 - r_0)\cot\alpha \tag{5}$$

Combine Eq.(3) to Eq.(5), the wrinkling moment for the inflated truncated cone at point *A* can be denoted as:

$$M_w = \frac{\pi p}{2}\left(z\frac{r_1 - r_0}{z_0} + r_0\right)^3 \tag{6}$$

When a shearing force F is carried on the free end, the moment on the section at z is:

$$M = Fz \tag{7}$$

Define wrinkle factor:

$$\lambda = \frac{M}{M_w} = \frac{2Fz}{\pi p\left(z\frac{r_1 - r_0}{z_0} + r_0\right)^3} \tag{8}$$

For the inflated truncated cone at point *A*, if $\lambda < 1$, there is no wrinkle, if $\lambda \ge 1$, wrinkle occurs.

Wrinkles appear where λ achieve 1 firstly on the taper, λ reaches the maximum value at the position. So the relation between the wrinkle factor λ and the initial wrinkling location z_w are:

$$\begin{cases} \dfrac{\partial\lambda}{\partial z} = 0 \ \ and \ \ \dfrac{\partial^2\lambda}{\partial z^2} < 0 \quad (0 \le z \le l) & (a) \\ \dfrac{\partial\lambda}{\partial z} > 0, \ then \ z_w = l & (b) \\ \dfrac{\partial\lambda}{\partial z} < 0, \ then \ z_w = 0 & (c) \end{cases} \tag{9}$$

The inflated truncated cone will collapse at the position where λ reaches the maximum value.

From Eq.(9), we obtain the dimensionless initial wrinkling location vs. the taper ratio is:

$$\frac{z_w}{z_0} = \begin{cases} 1 & (1 \le \dfrac{r_1}{r_0} < \dfrac{3}{2}) \\ \dfrac{1}{2(\dfrac{r_1}{r_0} - 1)} & (\dfrac{r_1}{r_0} \ge \dfrac{3}{2}) \end{cases} \tag{10}$$

It can be known that the initial wrinkling location is determined for a particular inflated truncated cone from Eq.(10). λ achieve the maximum value at the location where inflated truncated cone collapses. So inflated truncated cone collapses at initial wrinkling location.

3 The wrinkling region

When moment exceed the wrinkling moment, wrinkles will expand to form a wrinkling region. In the region, we have:

$$\lambda \ge 1 \ (0 \le z \le l) \tag{11}$$

The experiments are done to found that the wrinkles will expand until the inflated truncated cone collapses after wrinkles occur on the inflated truncated cone. The region is named wrinkled region.

Substitute Eq.(8) in Eq.(11), the issue of solve the wrinkled region can be translated into solve the following inequality:

$$\pi p\left(z\tan\alpha + r_0\right)^3 - 2Fz \le 0 \tag{12}$$

Substitution of Eq. (8) in Eq. (11) and re-writing gives an expression for the wrinkling condition in z-direction:

$$\pi p\left(\frac{r_1 - r_0}{z_0}z + r_0\right)^3 - 2Fz \le 0 \tag{13}$$

If inequality (13) is a equality, the Boundary value can be derived by Shengjin's formula, the roots are as follows:

$$z_1 = \frac{-b - 2\sqrt{A}\cos\frac{\theta}{3}}{3a} \tag{14}$$

$$z_2 = \frac{-b + \sqrt{A}\left(\cos\frac{\theta}{3} + \sqrt{3}\sin\frac{\theta}{3}\right)}{3\pi p \tan^3\alpha}, \tag{15}$$

$$z_3 = \frac{-b + \sqrt{A}\left(\cos\frac{\theta}{3} - \sqrt{3}\sin\frac{\theta}{3}\right)}{3a} \tag{16}$$

And $\theta = \arccos T$, $T = \frac{2Ab - 3aB}{2A^{\frac{3}{2}}}$, ($A>0$, $-1<T<1$).

Namely,

$$T = \frac{9}{2} r_0 \sqrt{\frac{\pi p \tan\alpha}{6F}}$$

In the article, $T > 0$, $\theta \subset \left(0, \frac{\pi}{2}\right)$, so $z_1 < 0$, $z_2 > z_3$. If the roots distributed over the inflated truncated cone, they fulfill $0 \le z \le l$, so the root z_1 can be ignored.

For inflated truncated cone, if $z_2 > l$, the wrinkling region is $z_{wa} \subset (z_3, l)$; if $z_2 < l$, the wrinkling region is $z_{wa} \subset (z_3, z_2)$.

Especially, experiments of several inflated truncated cones are carried on, we can get The wrinkling region under experiment and theory, as is shown in Table 2. Fig. 2 shows the experiment setup.

4 The experiment

The inflated truncated cones are made of polyimide. Fig.2 shows the wrinkling test of inflated truncated cones. The collapse location and the wrinkling region boundary are obtained from the experiment.

The wrinkling behavior of the inflated truncated cone depends on the inflated pressure p, the tip load F and the cross-sectional radius r. In the wrinkling test, three different radii at fixed end (3cm, 4cm and 6cm) are selected, the radii at free end are constants (2cm). The inflated truncated cones are inflated by 10KPa pressure during the test, it is fixed on a wooden platform, as shown in Fig.2, and the displacement is measured by total station. The wrinkling region boundary is shown in Fig.3, the calculated results agree with the experiment results well.

The initial wrinkling location or collapse location of several inflated truncated cones under theory and experiment given in Tab.1 makes a contrastive analysis, and the is not more than 7%. If the location is in the fixed end or close to the fixed end, the relative error is bigger because of the boundary regional effect.

Fig.3 shows comparisons between theoretical modeling (th.) and experimental results (exp.). Three inflated truncated cones were loaded a shearing force pressurized at 10KPa separately, they were stable before the shearing force reached to 0.5N. But the inflated truncated cone with 3cm of fixed radius wrinkled from the fixed end with load higher than 0.5N. As the shearing force increasing to 0.8N, the inflated truncated cone with 4cm of fixed radius wrinkled from the middle position and expanded quickly, and wrinkles would expand to the fix end when load was higher than 1N. When the force reached to 1.7N, the inflated truncated cone with 3cm of fixed radius wrinkled from the position near the free end and expanded slowly. The experimental results validate the theoretical results very well. Fig.4 shows the wrinkled region of inflated truncated cone in the experiment.

5 Conclusion

A new factor called wrinkle factor is introduced to define the wrinkling location and collapse load of a membrane inflated truncated cone under bending. A wrinkling prediction and analysis model are developed to describe wrinkled region emergence and extension. Comparisons with experimental results agree well. The model and results will be useful in design, analysis and optimization of inflated truncated cone.

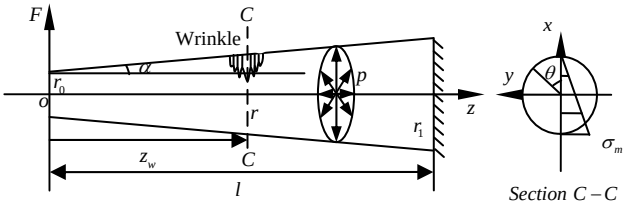

Fig. 1 Stress analysis of inflated truncated cone

Fig. 2 Inflated truncated cone under shear load

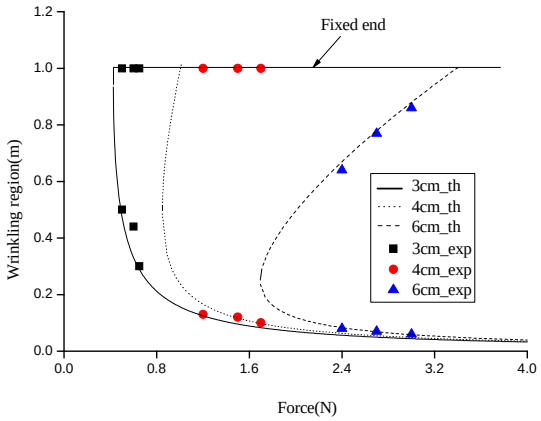

Fig. 3 Theoretical and experimental values of wrinkling region with different fixed radius (10KPa)

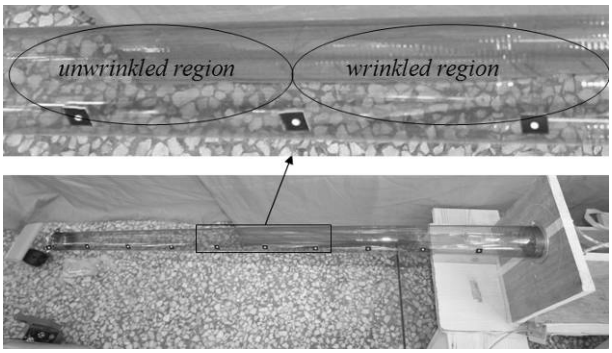

Fig. 4 Wrinkled region of inflated truncated cone

Table 1 Initial wrinkling location under theory and experiment

Sample	1	2	3	4
r_1 (cm)	2	3	4	6
Theoretical z_w (m)	1	1	0.5	0.25
experimental z_w (m)	0.93	0.945	0.503	0.25
Relative error	7%	5.5%	0.6%	0%

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