HYGRO-THERMALLY CURVAURE-STABLE LAMINATES WITH NON-STANDARD PLY ORIENTATIONS.

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1 Abstract

Stacking sequence configurations for hygro-thermally curvature-stable (HTCS) laminates have recently been identified in 9 classes of coupled laminate with standard ply angle orientations +45, −45, 0 and 90°. All arise from the judicious re-alignment of the principal material axis of laminate classes with Bending-Twisting and/or Bending-Extension and Twisting-Shearing coupling; Off-axis material alignment of these parent classes gives rise to more complex combinations of mechanical coupling behavior. However, for standard ply angle orientations +45, −45, 0 and 90°, HTCS solutions were found in only 8-, 12-, 16- and 20-ply laminates. This article considers non-standard ply angle orientations +60, −60, 0 and 90°, which lead to solutions in all ply number groupings for 10 plies and above, thus offering a possibility for tapered warp-free laminate designs.

2 Introduction.

Tailored composite laminates possessing complex mechanical couplings are beginning to find application beyond the aerospace sector, with which they have been traditionally associated, and towards new and emerging applications for which certification is less stringent and design rules have not become entrenched and risk averse. Recent research [1,2] has demonstrated that there is a vast and unexplored laminate design space containing exotic forms of mechanical coupling not previously identified, which includes all interactions between Extension, Shearing, Bending and Twisting, and that a surprisingly broad range of these coupling responses can be achieved without the undesirable warping distortions that result from the high temperature curing process. Such laminate designs may be described as hygro-thermally curvature-stable (HTCS) or warp-free.

The design of aero-elastic compliant rotor blades with tailored Extension-Twisting coupling is a well-known example laminate design concept that requires either specially curved tooling or HTCS properties in order to remain flat after high temperature curing.

Winckler [3] is credited with being the first to discover a solution: an eight-ply HTSC configuration, developed by using the concept of bonding two (or more) symmetric cross-ply $[○/●/●/○]_T$ sub-laminates, where each sub-laminate is counter-rotated by $\pi/8$, giving rise to the laminate: $[22.5/-67.5_2/22.5/-22.5/67.5_2/-22.5]_T$, which possesses Extension-Twisting and Shearing-Bending coupling. Winckler [3] recognized that the symmetric cross-ply sub-laminate represents a hygro-thermally curvature-stable configuration, which remains so after rotation and/or combining with additional sub-laminates through stacking or interlacing.

Chen [4] used an optimisation procedure to maximise the Extension-Twisting coupling of the laminate and investigated several different sub-sequence forms to achieve this. All coupled laminate results were based on 16-ply configurations, optimised for maximum mechanical coupling compliance (b_{16}). The first configuration, based on the most general form: $[\theta_1/\theta_2/\ldots\ldots/\theta_{16}]_T$ gave the following optimum sequence: $[14.62/16.21/-69.56/21.63/-66.34/-59.38/-55.98/-49.52/49.13/56.01/61.46/64.36/-21.3/69.04/-17.01/-14.88]_T$

Cross et al. [5] augmented the theoretical proofs of Chen [4] for the necessary conditions for hygro-thermally curvature-stable coupled laminates, focussing also on maximising the mechanical coupling response, but now with the smallest

possible ply number groupings. A 5-ply anti-symmetric configuration was derived: $[76.3/-33.6/0/33.6/-76.3]_T$. The article also included numerical and experimental validation to assess the robustness of the designs due to ply orientation errors. However, conclusions were drawn entirely on the basis of the anti-symmetric 6-ply solution: $[15/-75/-45]_A$.

A number of subsequent articles have substantially extended this work; the focus, however, remaining almost entirely on maximising the mechanical compliance (b_{16}) using free form rather than standard ply orientations. Only the most recent work [6] has considered combined mechanical coupling, i.e. Extension-Twisting and Bending-Twisting coupling behaviour at the laminate level.

Weaver [7] developed a small number of laminate configurations containing repeating groups of four-ply symmetric sub-sequences with orthogonal orientations which serve to validate a number of the laminate forms proposed by Winckler [3]. The resulting configurations are repeated here for completeness: $[0/90/90/0/\underline{45/-45/-45/45}]_T$, $[90/0/0/90/60/-30/-30/60]_T$, $[0/\underline{45}/90/\underline{-45}/90/\underline{-45}/0/\underline{45}]_T$, $[0/90/\underline{45/-45}/90/0/\underline{-45/45}]_T$, $[90/\underline{45/-45}/0/\underline{-45/45}/0/\underline{-45/45}/90/\underline{45/-45}]_T$, where the repeating 0/90/90/0 sub-laminate is rotated by 45° in the first solution, and by 90° and 60°, respectively, in the second. The concept of sub-laminate 'splicing', proposed by Tsai [8], was also shown to be applicable to coupled hygro-thermally curvature-stable laminates with repeating sub-laminate groupings, whereby an underscore identifies the plies of one sub-laminate which have been 'spliced' or, more appropriately, 'interlaced' with another.

Cross et al. [5] provide an important clue to discovering an entire range of non-standard ply orientations that are the basis for this study. Listed are HTCS configurations developed in the absence of repeating cross-ply sub-laminates assumed by others [3,7]. Instead, solutions contain combinations of π/3 extensionally isotropic and cross-ply sub-sequences; the result being a transformation from uncoupled isotropic properties to coupled, but warp free solutions. This transformation can be understood from the well-known fact that the addition of cross-plies to an otherwise uncoupled laminate renders the laminate coupled in Extension-Bending and Shearing-Twisting; which is a parent class for the majority of the forgoing solutions.

The concept can be seen clearly from one of two new 13-ply laminate solutions described later in the paper: $[+/-/\bigcirc/-/\bigcirc/+/\bigcirc/+/-/\bigcirc/\bullet/\bullet/\bigcirc]_T$, where +, –, ○ and ● become +60, -60, 0 and 90° (or +30, -30, 90 and 0°), i.e. $[60/-60/0/-60/0/60/0/60/-60/0/90/90/0]_T$. Here, the first nine plies of the stacking sequence represent a quasi-isotropic laminate: $[60/-60/0/-60/0/60/0/60/-60]_T$, with Bending-Twisting coupling, but the addition of the cross-ply sub-laminate $[0/90/90/0]_T$ to the outer surface, results in a hygro-thermally curvature stable laminate, possessing Extension-Bending, Shearing-Twisting and Bending-Twisting coupling when axis aligned.

By contrast, the single 8-ply laminate solution: $[+/\bigcirc/-/\bigcirc/-/\bullet/+/\bigcirc]_T$, i.e. $[60/0/-60/0/-60/90/60/0]_T$, which possesses Extension-Bending, Shearing-Twisting and Bending-Twisting coupling, illustrates an example where the cross-ply and π/3 extensionally isotropic sub-sequences are interlaced rather than added. Several 8-ply solutions were presented by Cross et al. [5], but in fact correspond to the above sequence with off-axis rotation, reversal of the stacking sequence and sign switching.

3 Development of hygro-thermally curvature-stable (HTCS) laminates designs.

The necessary conditions for hygro-thermally curvature-stable behaviour can be found in numerous articles [2,4,5,7,9], but are summarized here in terms of the well-known lamination parameters and the equivalent form of the extensional and coupling stiffness matrices, which vary with material axis alignment, β, as follows:

$\underline{\beta = m\pi/2 \text{ and } \pi/8 + m\pi/2 \ (m = 0, 1, 2, 3)}$

$$\xi_1 = \xi_2 = \xi_3 = 0; \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{11} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \qquad (1)$$

$\underline{\beta = m\pi/2 \ (m = 0, 1, 2, 3)}$

$$\xi_5 = \xi_7 = \xi_8 = 0; \begin{bmatrix} B_{11} & -B_{11} & 0 \\ -B_{11} & B_{11} & 0 \\ 0 & 0 & -B_{11} \end{bmatrix} \qquad (2)$$

$\underline{\beta = \pi/8 + m\pi/2 \ (m = 0, 1, 2, 3)}$

$$\xi_5 = \xi_6 = \xi_7 = 0; \begin{bmatrix} 0 & 0 & B_{16} \\ 0 & 0 & -B_{16} \\ B_{16} & -B_{16} & 0 \end{bmatrix} \quad (3)$$

$\underline{\beta \neq m\pi/2,\ \pi/8 + m\pi/2}$

$$\xi_1 = \xi_3 = 0; \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{11} & -A_{16} \\ A_{16} & -A_{16} & A_{66} \end{bmatrix} \quad (4)$$

and

$$\xi_5 = \xi_7 = 0; \begin{bmatrix} B_{11} & -B_{11} & B_{16} \\ -B_{11} & B_{11} & -B_{16} \\ B_{16} & -B_{16} & -B_{11} \end{bmatrix} \quad (5)$$

Definitive listings of coupled laminates have recently been derived for all classes of coupled laminate described above. These definitive listings are presented in symbolic form, together with non-dimensional parameters; making each configuration independent of both material properties and fibre orientations. The following sections describe the significance of the non-dimensional parameters and how they may be used to: develop stiffness relations for a given configuration; assess the thermal response of a particular configuration and; demonstrate the necessary conditions for hygro-thermally curvature-stable response.

3.1 Non-dimensional parameters for coupled laminates

The calculation of non-dimensional coupling stiffness parameters is readily demonstrated for the 8-ply laminate $[+/\bigcirc/-/\bigcirc/-/\bullet/+/\bigcirc]_T$, where elements of coupling stiffness matrix,

$$B_{ij} = \Sigma Q'_{ij,k}(z_k^2 - z_{k-1}^2)/2 \quad (6)$$

may be written in sequence order for the (k = 1 to) 8 individual plies, where z, representing the distance from the laminate mid-plane, is expressed here in terms of the uniform ply thickness t:

$$B_{ij} = \{Q'_{ij+}((-3t)^2 - (-4t)^2) + Q'_{ij\bigcirc}((-2t)^2 - (-3t)^2) + Q'_{ij-}((-t)^2 - (-2t)^2) + Q'_{ij\bigcirc}((0)^2 - (-t)^2) + Q'_{ij-}((t)^2 - (0)^2) + Q'_{ij\bullet}((2t)^2 - (t)^2) + Q'_{ij+}((3t)^2 - (2t)^2) + Q'_{ij\bigcirc}((4t)^2 - (3t)^2))\}/2 \quad (7)$$

and subscripts i, j = 1, 2, 6.

The coupling stiffness contribution from the angle plies is therefore:

$$B_{ij+} = -2t^2/2 \times Q'_{ij+} \quad (8)$$

$$B_{ij-} = -2t^2/2 \times Q'_{ij-} \quad (9)$$

and from the cross-plies:

$$B_{ij\bigcirc} = t^2/2 \times Q'_{ij\bigcirc} \quad (10)$$

$$B_{ij\bullet} = 3t^2/2 \times Q'_{ij\bullet} \quad (11)$$

These coupling stiffness terms may also be written in alternative form:

$$B_{ij+} = \chi_+ t^2/4 \times Q'_{ij+} \quad (12)$$

$$B_{ij-} = \chi_- t^2/4 \times Q'_{ij-} \quad (13)$$

$$B_{ij\bigcirc} = \chi_\bigcirc t^2/4 \times Q'_{ij\bigcirc} \quad (14)$$

and

$$B_{ij\bullet} = \chi_\bullet t^2/4 \times Q'_{ij\bullet} \quad (15)$$

respectively, where $\chi_+ = \chi_- = -4$, $\chi_\bigcirc = 2$ and $\chi_\bullet = 6$. Similar non-dimensional parameters can be developed for the Extensional and Bending Stiffnesses. These non-dimensional parameters, together with the transformed reduced stiffness, Q'_{ij}, for each ply orientation and constant ply thickness, t, facilitate simple calculation of the elements of the extensional, coupling and bending stiffness matrices from:

$$A_{ij} = \{n_+Q'_{ij+} + n_-Q'_{ij-} + n_\bigcirc Q'_{ij\bigcirc} + n_\bullet Q'_{ij\bullet}\}t$$

$$B_{ij} = \{\chi_+Q'_{ij+} + \chi_-Q'_{ij-} + \chi_\bigcirc Q'_{ij\bigcirc} + \chi_\bullet Q'_{ij\bullet}\}t^2/4$$

$$D_{ij} = \{\zeta_+Q'_{ij+} + \zeta_-Q'_{ij-} + \zeta_\bigcirc Q'_{ij\bigcirc} + \zeta_\bullet Q'_{ij\bullet}\}t^3/12 \quad (16)$$

3.2 Lamination parameters for coupled laminates

Lamination parameters, originally conceived by Tsai and Hahn [10], offer an alternative set of non-dimensional expressions when ply angles are a design constraint. For optimum design of angle-ply and cross-ply laminates, lamination parameters offer a convenient tool, since they allow the stiffness terms to be expressed as linear design variables. The optimized lamination parameters may then be matched against a corresponding set of stacking sequences with given laminate thickness H (= $n \times t$). The lamination parameters are related to the non-dimensional parameters by the following expressions:

$$\xi_1 = \xi_1^A = \{n_+\cos(2\theta_+) + n_-\cos(2\theta_-) + n_\bigcirc\cos(2\theta_\bigcirc) + n_\bullet\cos(2\theta_\bullet)\}/n$$

$$\xi_2 = \xi_2^A = \{n_+\cos(4\theta_+) + n_-\cos(4\theta_-) + n_\bigcirc\cos(4\theta_\bigcirc) + n_\bullet\cos(4\theta_\bullet)\}/n$$

$$\xi_3 = \xi_3^A = \{n_+\sin(2\theta_+) + n_-\sin(2\theta_-) + n_\bigcirc\sin(2\theta_\bigcirc) + n_\bullet\sin(2\theta_\bullet)\}/n$$

$$\xi_4 = \xi_4^A = \{n_+\sin(4\theta_+) + n_-\sin(4\theta_-) + n_\bigcirc\sin(4\theta_\bigcirc) + n_\bullet\sin(4\theta_\bullet)\}/n \quad (17)$$

$$\xi_5 = \xi_1^B = \{ \chi_+\cos(2\theta_+) + \chi_-\cos(2\theta_-) + \chi_O\cos(2\theta_O) + \chi_\bullet\cos(2\theta_\bullet)\}/n^2$$
$$\xi_6 = \xi_2^B = \{ \chi_+\cos(4\theta_+) + \chi_-\cos(4\theta_-) + \chi_O\cos(4\theta_O) + \chi_\bullet\cos(4\theta_\bullet)\}/n^2$$
$$\xi_7 = \xi_3^B = \{ \chi_+\sin(2\theta_+) + \chi_-\sin(2\theta_-) + \chi_O\sin(2\theta_O) + \chi_\bullet\sin(2\theta_\bullet)\}/n^2$$
$$\xi_8 = \xi_4^B = \{ \chi_+\sin(4\theta_+) + \chi_-\sin(4\theta_-) + \chi_O\sin(4\theta_O) + \chi_\bullet\sin(4\theta_\bullet)\}/n^2 \quad (18)$$
$$\xi_9 = \xi_1^D = \{\zeta_+\cos(2\theta_+) + \zeta_-\cos(2\theta_-) + \zeta_O\cos(2\theta_O) + \zeta_\bullet\cos(2\theta_\bullet)\}/n^3$$
$$\xi_{10} = \xi_2^D = \{\zeta_+\cos(4\theta_+) + \zeta_-\cos(4\theta_-) + \zeta_O\cos(4\theta_O) + \zeta_\bullet\cos(4\theta_\bullet)\}/n^3$$
$$\xi_{11} = \xi_3^D = \{\zeta_+\sin(2\theta_+) + \zeta_-\sin(2\theta_-) + \zeta_O\sin(2\theta_O) + \zeta_\bullet\sin(2\theta_\bullet)\}/n^3$$
$$\xi_{12} = \xi_4^D = \{\zeta_+\sin(4\theta_+) + \zeta_-\sin(4\theta_-) + \zeta_O\sin(4\theta_O) + \zeta_\bullet\sin(4\theta_\bullet)\}/n^3 \quad (19)$$

Elements of the thermal force and moment resultants are related to the lamination parameters [10], laminate invariants and thermal coefficients by:

$$N_x^{Thermal} = \Delta T\{U_1^{Thermal} + \xi_1 U_2^{Thermal}\} \times H/2$$
$$N_y^{Thermal} = \Delta T\{U_1^{Thermal} - \xi_1 U_2^{Thermal}\} \times H/2$$
$$N_{xy}^{Thermal} = \Delta T\{\xi_3 U_2^{Thermal}\} \times H/2 \quad (20)$$
$$M_x^{Thermal} = \Delta T\{\xi_5 U_2^{Thermal}\} \times H^2/8$$
$$M_y^{Thermal} = \Delta T\{-\xi_5 U_2^{Thermal}\} \times H^2/8$$
$$M_{xy}^{Thermal} = \Delta T\{\xi_7 U_2^{Thermal}\} \times H^2/8 \quad (21)$$

where the laminate invariants are calculated from the reduced stiffness terms, Q_{ij}:

$$U_1 = \{3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}\}/8$$
$$U_2 = \{Q_{11} - Q_{22}\}/2$$
$$U_3 = \{Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}\}/8$$
$$U_4 = \{Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}\}/8$$
$$U_5 = \{Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}\}/8 \quad (22)$$

and thermal coefficients (Chen, 2003):

$$U_1^{Thermal} = \alpha_{11}Q_{11} + (\alpha_{11} + \alpha_{22})Q_{12} + \alpha_{22}Q_{22}$$
$$U_2^{Thermal} = \alpha_{11}Q_{11} + (\alpha_{22} - \alpha_{11})Q_{12} + \alpha_{22}Q_{22} \quad (23)$$

Finally, the reduced stiffness terms are calculated from the material properties:

$$Q_{11} = E_1/(1 - \nu_{12}\nu_{21})$$
$$Q_{12} = \nu_{12}E_2/(1 - \nu_{12}\nu_{21})$$
$$Q_{22} = E_2/(1 - \nu_{12}\nu_{21})$$
$$Q_{66} = G_{12} \quad (24)$$

It is recognized that behaviour due to changes in temperature and moisture content are synonymous in the context of hygro-thermally curvature-stable design, where the associated thermal and moisture expansion coefficients are interchangeable. However, discussion is restricted to the thermal loading condition in this article.

4 Results and Discussion.

In addition to the two new 13-ply laminate solutions previously mentioned, which in fact differ only by a change in the sign of the last four plies: $[+/-/O/-/O/+/O/+/-/\bullet/O/O/\bullet]_T$, the number of solutions from this Extension-Bending and Shearing-Twisting coupled parent class increases to 19, 76, 89, 177, etc., for 15-, 16-, 17- and 18-ply laminates, respectively. Examples for each ply number grouping, are given below, noting that they represent the minimum and maximum coupling stiffness, B_{16} (and D_{26}), when the off-axis material alignment, $\beta = \pi/8$:

$[+/-/\bullet/O/O/O/-/O/\bullet/+/+/O/-/O/\bullet]_T$

:

$[+/-/-/O/+/+/-/O/O/O/O/O/\bullet/\bullet/\bullet]_T$

$[+/O/-/O/-/-/+/+/O/+/O/-/O/\bullet/\bullet/O]_T$

:

$[+/-/\bullet/O/-/O/O/+/+/O/-/O/O/+/-/-/+]_T$

$[+/\bullet/-/O/O/-/O/O/\bullet/+/O/+/-/O/+/-]_T$

:

$[+/O/-/O/-/-/+/+/O/+/O/-/O/\bullet/\bullet/O]_T$

$[+/-/\bullet/O/-/O/O/+/+/O/-/O/O/+/-/-/+]_T$

:

$[+/-/O/-/O/+/O/+/-/O/O/\bullet/\bullet/\bullet/\bullet/O/O]_T$

$[+/\bullet/-/O/O/\bullet/-/O/O/O/+/O/\bullet/+/-/O/+/-]_T$

:

$[+/-/O/+/-/-/O/O/O/+/O/+/\bullet/-/\bullet/O/\bullet/O]_T$

Similarly for the parent class with Extension-Bending, Shearing-Twisting and Bending-Twisting coupling the number of solutions increases from the previously identified [5] single 8-ply laminate solution: $[+/O/-/O/-/\bullet/+/O]_T$, to 8, 14, 40, 135, 494, 1,188, etc. for 10-, 11-, 12-, 13-, 14-, 15-ply laminates, respectively. Examples for each ply number grouping, using the same criteria described above, are given below:

$[+/-/O/\bullet/O/O/-/+/O/\bullet]_T$

:

$[+/O/-/-/O/+/O/\bullet/\bullet/O]_T$

$[+/-/\bullet/O/O/O/-/O/+/+/-]_T$

:

[+/O/–/O/+/–/–/O/●/+/O]$_T$

[+/–/●/O/O/●/O/O/O/–/+/●]$_T$

:

[+/–/O/–/+/O/O/O/●/●/●/O]$_T$

[+/●/–/O/O/●/O/O/O/–/+/–/+]$_T$

:

[+/O/–/O/–/+/–/+/O/O/●/●/O]$_T$

[+/–/●/–/O/O/O/O/+/O/+/+/–/–]$_T$

:

[+/–/–/+/O/O/O/O/●/O/●/●/●/O]$_T$

[+/●/●/O/–/O/O/O/O/●/O/–/–/+/+]$_T$

:

[+/+/–/–/–/O/O/O/+/O/O/O/●/●/●]$_T$

[+/●/●/–/O/O/O/O/–/O/O/+/–/+/+/–]$_T$

:

[+/–/O/–/+/O/O/O/●/O/●/●/●/●/O/O]$_T$

[+/●/–/●/O/O/O/●/O/O/O/O/●/–/+/–/+]$_T$

:

[+/–/+/–/–/O/+/O/O/O/O/O/●/●/O/●/●]$_T$

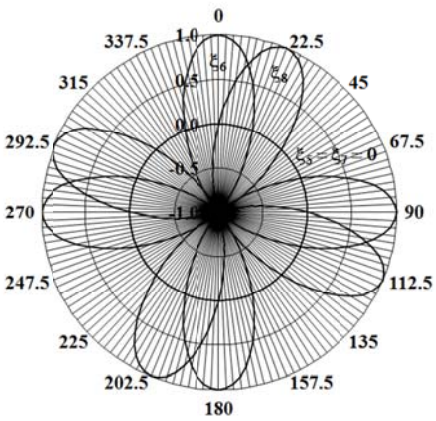

Figure 1 - Polar plot of Lamination parameters ξ_5 - ξ_8, corresponding to off-axis material alignment, β, for the 16-ply HTCS laminate with Isotropic extensional and bending stiffness: [+/–$_2$/+/–/+$_2$/–/O/●$_2$/O/●/O$_2$/●]$_T$, after Ref. [11], with standard ply orientations ±45, 0 and 90° in place of symbols +, –, O and ●, respectively.

Comparison of the magnitudes of the maximum couplings achievable by these non-standard ply orientation, where stacking sequence symbols +, –, O and ● become +60, -60, 0 and 90° (or +30, -30, 90 and 0°), in contrast to standard +45, -45, 0 and 90° ply orientations, as in previous studies [2,11], reveals a general reduction.

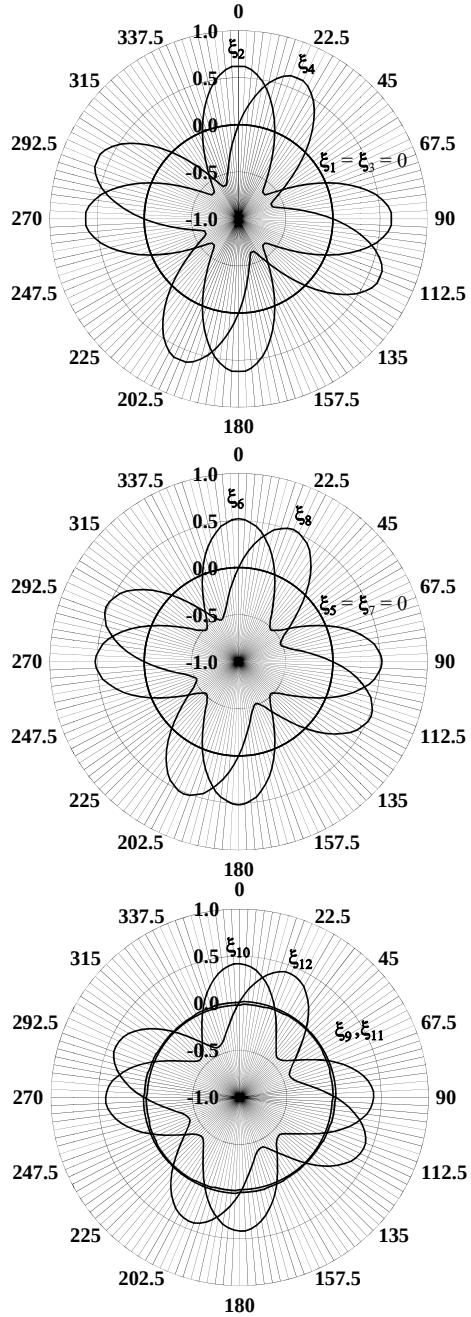

Figure 2 - Polar plots of Lamination parameters (top) ξ_1 - ξ_4; (middle) ξ_5 - ξ_8; (bottom) ξ_9 - ξ_{12} corresponding to off-axis material alignment, β, for the 16-ply hygro-thermally curvature-stable laminate stacking sequence [+/–/O/–/+/O$_3$/●/O/●$_4$/O$_2$]$_T$, with non-standard ply orientations ±60, 0 and 90° in place of symbols +, –, O and ●, respectively.

This comparison is demonstrated by polar plots of the Lamination parameters, illustrated in Figs 1 and 2. Here, the coupling magnitude achievable from the

non-standard 16 ply solutions may be compared with that of a laminate developed elsewhere [11], with standard ply orientations ±45, 0 and 90° and with Isotropic Extensional and Bending stiffness; Lamination parameters for ξ_1 - ξ_4 and ξ_9 - ξ_{12} are zero and are therefore not shown. The stacking sequence of Figure 1 is representative of the repeating group of $[\theta/(\theta + \pi/2)_2/\theta]$ first introduced by Winckler [3]. Comparison of the Lamination parameters ξ_5 - ξ_8 reveals that the coupling magnitude is maximum (optimum) for standard ply orientations and approximately half this value for non-standard orientations and represents a common pattern. However, the example stacking sequences listed above reveal that there may be greater scope for tapering non-standard ply orientation designs given that solutions exist for all consecutive ply number groupings above 10 plies, which is in contract to standard ply orientation designs which occur only for multiples of 4 plies. However, comparison of the form of the odd and even stacking sequences reveals that a change in the ply number grouping from n to $n + 1$, results in a change in angle plies of $n + 2$, which therefore precludes single ply drops, i.e. from an even to an odd stacking sequence with $n + 1$ plies.

Finally, the number of solutions may be compared with laminate designs for standard ply orientations [2], which reveals 6, 524, and 35,610 with 12, 16 and 20 plies from the parent class of Extension-Bending and Shearing-Twisting coupled laminates and 410, 40,808 and 4,515,473 with 12, 16 and 20 plies, respectively, for the parent class of Extension-Bending, Shearing-Twisting and Bending-Twisting coupled laminates.

5 Conclusions.

A preliminary study of Hygro-thermally Curvature-Stable laminate designs has revealed that a much broader design space exists for non-standard ply angle orientations, i.e. +60, −60, 0 and 90°, than for their standard ply orientation counterparts. These designs offer the scope for exploiting exotic mechanically coupled laminates, free from the constraints imposed by the effects of high temperature curing, which continue to be a barrier to many coupled designs that represent an important enabling technology for new and immerging applications of composite structures.

Note that the work reported here is on-going and involves numerical simulation and experimental validation of both the Hygro-Thermally Curvature-Stable properties and the mechanical coupling strength. Additional information can be obtained from http://eprints.gla.ac.uk/53239.

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