HYGRO-THERMALLY CURVATURE-STABLE LAMINATES WITH NON-STANDARD PLY ORIENTATIONS.

C. B. York
Aerospace Sciences, School of Engineering, University of Glasgow, Scotland, UK.
(Christopher.York@Glasgow.ac.uk)

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1 Abstract
Stacking sequence configurations for hygro-thermally curvature-stable (HTCS) laminates have recently been identified in 9 classes of coupled laminate with standard ply angle orientations +45, −45, 0 and 90°. All arise from the judicious realignment of the principal material axis of laminate classes with Bending-Twisting and/or Bending-Extension and Twisting-Shearing coupling; Off-axis material alignment of these parent classes gives rise to more complex combinations of mechanical coupling behavior. However, for standard ply angle orientations +45, −45, 0 and 90°, HTCS solutions were found in only 8-, 12-, 16- and 20-ply laminates. This article considers non-standard ply angle orientations +60, −60, 0 and 90°, which lead to solutions in all ply number groupings for 10 plies and above, thus offering a possibility for tapered warp-free laminate designs.

2 Introduction.
Tailored composite laminates possessing complex mechanical couplings are beginning to find application beyond the aerospace sector, with which they have been traditionally associated, and towards new and emerging applications for which certification is less stringent and design rules have not become entrenched and risk averse. Recent research [1,2] has demonstrated that there is a vast and unexplored laminate design space containing exotic forms of mechanical coupling not previously identified, which includes all interactions between Extension, Shearing, Bending and Twisting, and that a surprisingly broad range of these coupling responses can be achieved without the undesirable warping distortions that result from the high temperature curing process. Such laminate designs may be described as hygro-thermally curvature-stable (HTCS) or warp-free.

The design of aero-elastic compliant rotor blades with tailored Extension-Twisting coupling is a well-known example laminate design concept that requires either specially curved tooling or HTCS properties in order to remain flat after high temperature curing. Winckler [3] is credited with being the first to discover a solution: an eight-ply HTCS configuration, developed by using the concept of bonding two (or more) symmetric cross-ply \([O/O/O/O]\)T sub-laminates, where each sub-laminate is counter-rotated by π/8, giving rise to the laminate: \([22.5/-67.5/22.5/-22.5/67.5/22.5]T\), which possesses Extension-Twisting and Shearing-Bending coupling. Winckler [3] recognized that the symmetric cross-ply sub-laminate represents a hygro-thermally curvature-stable configuration, which remains so after rotation and/or combining with additional sub-laminates through stacking or interlacing.

Chen [4] used an optimisation procedure to maximise the Extension-Twisting coupling of the laminate and investigated several different sub-sequence forms to achieve this. All coupled laminate results were based on 16-ply configurations, optimised for maximum mechanical coupling compliance (b16). The first configuration, based on the most general form: \([θ_1/θ_2/…/θ_{16}]T\) gave the following optimum sequence: \([14.62/16.21/-69.56/21.63/-66.34/-59.38/-55.98/-49.52/49.13/56.01/61.46/64.36/-21.3/69.04/-17.01/-14.88]T\)

Cross et al. [5] augmented the theoretical proofs of Chen [4] for the necessary conditions for hygro-thermally curvature-stable coupled laminates, focussing also on maximising the mechanical coupling response, but now with the smallest
possible ply number groupings. A 5-ply anti-
symmetric configuration was derived: [76.3/33.6/0/33.6/-76.3]T. The article also included
numerical and experimental validation to assess the
robustness of the designs due to ply orientation
errors. However, conclusions were drawn entirely
on the basis of the anti-symmetric 6-ply solution:
[15/-75/45/45]T.
A number of subsequent articles have substantially
extended this work; the focus, however, remaining
almost entirely on maximising the mechanical
compliance (b11) using free form rather than
standard ply orientations. Only the most recent
work [6] has considered combined mechanical
coupling, i.e. Extension-Twisting and Bending-
Twisting coupling behaviour at the laminate level.
Weaver [7] developed a small number of laminate
configurations containing repeating groups of four-
ply symmetric sub-sequences with orthogonal
orientations which serve to validate a number of the
laminate forms proposed by Winckler [3]. The
resulting configurations are repeated here for
completeness: [0/90/90/0/45/-45/45/45/45]T, [90/0/90/0/90/60/-30/30/60]T, [0/45/90/45/90/0/45/0/45]T,
[0/90/45/45/90/0/45/-45/0/45/0/45/45/90/45/45/45]T, where the repeating 0/90/90/0
sub-laminate is rotated by 45° in the first solution,
and by 90° and 60°, respectively, in the second. The
concept of sub-laminate ‘splicing’, proposed by Tsai
[8], was also shown to be applicable to coupled
hygro-thermally curvature-stable laminates with repeating sub-laminate groupings, whereby an
underscore identifies the plies of one sub-laminate
which have been ‘spliced’ or, more appropriately,
‘interlaced’ with another.
Cross et al. [5] provide an important clue to
discovering an entire range of non-standard ply
orientations that are the basis for this study. Listed
are HTCS configurations developed in the absence
of repeating cross-ply sub-laminates assumed by
others [3,7]. Instead, solutions contain combinations
of π/3 extensionally isotropic and cross-ply sub-
sequences; the result being a transformation from
uncoupled isotropic properties to coupled, but warp
free solutions. This transformation can be
understood from the well-known fact that the
addition of cross-plies to an otherwise uncoupled
laminate renders the laminate coupled in Extension-
Bending and Shearing-Twisting; which is a parent
class for the majority of the forgoing solutions.
The concept can be seen clearly from one of two
new 13-ply laminate solutions described later in the
paper: [+/-/O/-/O/+/-/O/O/0/O][T, where +, -, O and ● become +60, -60, 0 and 90° (or +30, -30, 90 and 0°), i.e. [60/-60/0/60/0/60/-60/0/60/90/90/0]T. Here, the first nine plies of the
stacking sequence represent a quasi-isotropic
laminate: [60/-60/0/60/0/60/0/60/-60]T, with Bending-Twisting coupling, but the addition of the
cross-ply sub-laminate [0/90/90/0]T to the outer
surface, results in a hygro-thermally curvature stable
laminate, possessing Extension-Bending, Shearing-
Twisting and Bending-Twisting coupling when axis
aligned.
By contrast, the single 8-ply laminate solution:
[+/-/O/-/O/+/-/O/O][T, i.e. [60/0/60/60/60/0/60/0]T,
which possesses Extension-Bending, Shearing-
Twisting and Bending-Twisting coupling, illustrates
an example where the cross-ply and π/3 extensionally isotropic sub-sequences are interlaced
rather than added. Several 8-ply solutions were
presented by Cross et al. [5], but in fact correspond
to the above sequence with off-axis rotation, reversal
of the stacking sequence and sign switching.

3 Development of hygro-thermally curvature-
stable (HTCS) laminates designs.

The necessary conditions for hygro-thermally curvature-stable behaviour can be found in
numerous articles [2,4,5,7,9], but are summarized
here in terms of the well-known lamination
parameters and the equivalent form of the
extensional and coupling stiffness matrices, which
vary with material axis alignment, β, as follows:
β = mπ/2 and π/8 + mπ/2 (m = 0, 1, 2, 3)

ξ = \begin{bmatrix}
A_{11} & A_{12} & 0 \\
A_{12} & A_{11} & 0 \\
0 & 0 & A_{66}
\end{bmatrix}
(1)

β = mπ/2 (m = 0, 1, 2, 3)
ξ = \begin{bmatrix}
B_{11} & -B_{11} & 0 \\
-B_{11} & B_{11} & 0 \\
0 & 0 & -B_{11}
\end{bmatrix}
(2)

β = π/8 + mπ/2 (m = 0, 1, 2, 3)
The calculation of non-dimensional coupling stiffnesses $B_{ij}$ of coupling stiffness matrix, $B_{ij}$, the coupling stiffness contribution from the angle and subscripts $i, j = 1, 2, 6$.

Terms of the uniform ply thickness $B_{ij} = \{Q_k\}_{ji}$ independent of both material properties and fibre orientations.

Definitive listings of coupled laminates have recently been derived for all classes of coupled laminate described above. These definitive listings are presented in symbolic form, together with non-dimensional parameters by the following expressions:

$$B_{ij} = \{(Q_{ki})_{j}((4t^2 - (3t)^2) + Q_{kij}(2t^2 - (2t)^2) - (2t)^2 + Q_{kij}(2t)(t) + Q_{kij}((t)^2 - (0)^2) - (2t)^2) + Q_{kij}((t)^2 - (0)^2) - (2t)^2 + Q_{kij}((t)^2 - (3t)^2)\}/2$$

and subscripts $i, j = 1, 2, 6$.

The coupling stiffness contribution from the angle plies is therefore:

$$B_{ij} = -2t^2/2 \times Q_{ij}$$

and from the cross-plies:

$$B_{ij} = t^2/2 \times Q_{ij}$$

These coupling stiffness terms may also be written in alternative form:

$$B_{ij} = \chi_{ij} t^2/4 \times Q_{ij}$$

Similar non-dimensional parameters can be developed for the Extensional and Bending Stiffnesses. These non-dimensional parameters, together with the transformed reduced stiffness, $Q_{ij}$, for each ply orientation and constant ply thickness, $t$, facilitate simple calculation of the elements of the extensional, coupling and bending stiffness matrices from:

$$B_{ij} = \{(n_{ij}Q_{ij} + n_{ij}Q_{ij} + n_{ij}Q_{ij} + n_{ij}Q_{ij})\}/t$$

3.2 Lamination parameters for coupled laminates

Lamination parameters, originally conceived by Tsai and Hahn [10], offer an alternative set of non-dimensional expressions when ply angles are a design constraint. For optimum design of angle-ply and cross-ply laminates, lamination parameters offer a convenient tool, since they allow the stiffness terms to be expressed as linear design variables. The optimized lamination parameters may then be matched against a corresponding set of stacking sequences with given laminate thickness $H (= n \times t)$.

The lamination parameters are related to the non-dimensional parameters by the following expressions:

$$\xi_1 = \xi_{1n} = \{n_{ij}\cos(2\theta_i) + n_{ij}\cos(2\theta_i) + n_{ij}\cos(2\theta_i) + n_{ij}\cos(2\theta_i)\}/n$$

$$\xi_2 = \xi_{2n} = \{n_{ij}\cos(4\theta_i) + n_{ij}\cos(4\theta_i) + n_{ij}\cos(4\theta_i) + n_{ij}\cos(4\theta_i)\}/n$$

$$\xi_3 = \xi_{3n} = \{n_{ij}\sin(2\theta_i) + n_{ij}\sin(2\theta_i) + n_{ij}\sin(2\theta_i) + n_{ij}\sin(2\theta_i)\}/n$$

$$\xi_4 = \xi_{4n} = \{n_{ij}\sin(4\theta_i) + n_{ij}\sin(4\theta_i) + n_{ij}\sin(4\theta_i) + n_{ij}\sin(4\theta_i)\}/n$$
\[ \xi_5 = \xi_1^B = \left( \chi_x \cos(2\theta_0) + \chi_y \cos(2\theta_0) + \chi_o \cos(2\theta_0) + \chi_o \cos(2\theta_0) \right) / n^2 \]
\[ \xi_6 = \xi_2^B = \left( \chi_x \cos(4\theta_0) + \chi_y \cos(4\theta_0) + \chi_o \cos(4\theta_0) + \chi_o \cos(4\theta_0) \right) / n^2 \]
\[ \xi_7 = \xi_3^B = \left( \chi_x \sin(2\theta_0) + \chi_y \sin(2\theta_0) + \chi_o \sin(2\theta_0) + \chi_o \sin(2\theta_0) \right) / n^2 \]
\[ \xi_8 = \xi_4^B = \left( \chi_x \sin(4\theta_0) + \chi_y \sin(4\theta_0) + \chi_o \sin(4\theta_0) + \chi_o \sin(4\theta_0) \right) / n^2 \]

(18)

\[ \xi_9 = \xi_1^D = \left( \chi_x \cos(2\theta_0) + \chi_y \cos(2\theta_0) + \chi_o \cos(2\theta_0) + \chi_o \cos(2\theta_0) \right) / n^3 \]
\[ \xi_{10} = \xi_2^D = \left( \chi_x \cos(4\theta_0) + \chi_y \cos(4\theta_0) + \chi_o \cos(4\theta_0) + \chi_o \cos(4\theta_0) \right) / n^3 \]
\[ \xi_{11} = \xi_3^D = \left( \chi_x \sin(2\theta_0) + \chi_y \sin(2\theta_0) + \chi_o \sin(2\theta_0) + \chi_o \sin(2\theta_0) \right) / n^3 \]
\[ \xi_{12} = \xi_4^D = \left( \chi_x \sin(4\theta_0) + \chi_y \sin(4\theta_0) + \chi_o \sin(4\theta_0) + \chi_o \sin(4\theta_0) \right) / n^3 \]

(19)

Elements of the thermal force and moment resultants are related to the laminate parameters [10], laminate invariants and thermal coefficients by:

\[ N_x^\text{Thermal} = \Delta T U_1^\text{Thermal} + \xi_x U_2^\text{Thermal} \times H/2 \]
\[ N_y^\text{Thermal} = \Delta T U_1^\text{Thermal} - \xi_y U_2^\text{Thermal} \times H/2 \]
\[ N_{xy}^\text{Thermal} = \Delta T \xi_y U_2^\text{Thermal} \times H/2 \]
\[ M_x^\text{Thermal} = \Delta T \xi_x U_2^\text{Thermal} \times H^2/8 \]
\[ M_y^\text{Thermal} = \Delta T \xi_y U_2^\text{Thermal} \times H^2/8 \]
\[ M_{xy}^\text{Thermal} = \Delta T \xi_y U_2^\text{Thermal} \times H^2/8 \]

(20)

where the laminate invariants are calculated from the reduced stiffness terms, \( Q_i \):

\[ U_1 = 3(Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}) / 8 \]
\[ U_2 = (Q_{11} - 2Q_{22}) / 3 \]
\[ U_3 = 3(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}) / 8 \]
\[ U_4 = 3(Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}) / 8 \]
\[ U_5 = 3(Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}) / 8 \]

(22)

and thermal coefficients (Chen, 2003):

\[ U_1^\text{Thermal} = \alpha_1 Q_{11} + (\alpha_{11} + \alpha_{22})Q_{12} + \alpha_{22} Q_{22} \]
\[ U_2^\text{Thermal} = \alpha_1 Q_{11} + (\alpha_{11} + \alpha_{22})Q_{12} + \alpha_{22} Q_{22} \]

(23)

Finally, the reduced stiffness terms are calculated from the material properties:

\[ Q_{11} = E_1(1 - \nu_{12}v_{21}) \]
\[ Q_{12} = v_{12}E_2(1 - \nu_{12}v_{21}) \]
\[ Q_{22} = E_2(1 - \nu_{12}v_{21}) \]
\[ Q_{66} = G_{12} \]

(24)

It is recognized that behavior due to changes in temperature and moisture content are synonymous in the context of hygro-thermally curvature-stable design, where the associated thermal and moisture expansion coefficients are interchangeable. However, discussion is restricted to the thermal loading condition in this article.

4 Results and Discussion.

In addition to the two new 13-ply laminate solutions previously mentioned, which in fact differ only by a change in the sign of the last four plies: [+/-/O/-/O+/O/+/-/O/O/O/O], the number of solutions from this Extension-Bending and Shearing-Twisting coupled parent class increases to 19, 76, 89, 177, etc., for 15-, 16-, 17- and 18-ply laminates, respectively. Examples for each ply number grouping, are given below, noting that they represent the minimum and maximum coupling stiffness, \( B_{16} \) and \( D_{26} \), when the off-axis material alignment, \( \beta = \pi/8 \):

\[ [+/-/O/O/O/O/#/O/#+/O/#+/O/#+]_T \]
\[ [+/-/O/+/-/O/O/O/#/#/#/#+]_T \]
\[ [+/-/-/-/-/O/#/#/#/#/#+]_T \]
\[ [+/-/-/-/+/-/O/#/#/#/#/#+]_T \]
\[ [+/-/-/#/-/#/#/#/#/#/#]_T \]
\[ [+/-/-/#/#/#/#/#/#/#]_T \]
\[ [+/-/-/+/-/#/#/#/#/#]_T \]
\[ [+/-/-/+/-/#/#/#/#/#]_T \]

Similarly for the parent class with Extension-Bending, Shearing-Twisting and Bending-Twisting coupling the number of solutions increases from the previously identified [5] single 8-ply laminate solution: [+/-/O/-/O/-/O/-/+O/O/O], to 8, 14, 40, 135, 494, 1,188, etc. for 10-, 11-, 12-, 13-, 14-, 15-ply laminates, respectively. Examples for each ply number grouping, using the same criteria described above, are given below:

\[ [+/-/O/O/O/#/#/#/#/+O/O/O]_T \]
\[ [+/-/-/-/-/#/#/#/#/#/#]_T \]
\[ [+/-/-/#/#/#/#/#/#/#/#]_T \]
Comparison of the magnitudes of the maximum couplings achievable by these non-standard ply orientation, where stacking sequence symbols $+$, $-$, $\circ$ and $\bullet$ become $±60$, $-60$, $0$ and $90^\circ$ (or $±30$, $-30$, $90$ and $0^\circ$), in contrast to standard $±45$, $-45$, $0$ and $90^\circ$ ply orientations, as in previous studies [2,11], reveals a general reduction.

Figure 1 - Polar plot of Lamination parameters $\xi_5$ - $\xi_8$, corresponding to off-axis material alignment, $\beta$, for the 16-ply HTCS laminate with Isotropic extensional and bending stiffness: $[+/-+/-+/-O/\circ/\circ/O/\circ/O/\circ/O/\circ/O/\circ/O/\circ/O]_T$, after Ref. [11], with standard ply orientations $±45$, $0$ and $90^\circ$ in place of symbols $+$, $-$, $\circ$ and $\bullet$, respectively.

Figure 2 - Polar plots of Lamination parameters (top) $\xi_1$ - $\xi_4$; (middle) $\xi_5$ - $\xi_8$; (bottom) $\xi_9$ - $\xi_12$ corresponding to off-axis material alignment, $\beta$, for the 16-ply hygro-thermally curvature-stable laminate stacking sequence $[+/-/-/+/+/-O/\circ/\circ/O/\circ/O]/\bullet/O/\circ/O/\circ/O]_T$, with non-standard ply orientations $±60$, $0$ and $90^\circ$ in place of symbols $+$, $-$, $\circ$ and $\bullet$, respectively.

This comparison is demonstrated by polar plots of the Lamination parameters, illustrated in Figs 1 and 2. Here, the coupling magnitude achievable from the
non-standard 16 ply solutions may be compared with that of a laminate developed elsewhere [11], with standard ply orientations ±45, 0 and 90° and with Isotropic Extensional and Bending stiffness; Lamination parameters for \(\xi_1 - \xi_4\) and \(\xi_9 - \xi_{12}\) are zero and are therefore not shown. The stacking sequence of Figure 1 is representative of the repeating group of \([\theta/(\theta + \pi/2)];/\theta]\) first introduced by Winckler [3]. Comparison of the Lamination parameters \(\xi_5 - \xi_8\) reveals that the coupling magnitude is maximum (optimum) for standard ply orientations and approximately half this value for non-standard orientations and represents a common pattern. However, the example stacking sequences listed above reveal that there may be greater scope for tapering non-standard ply orientation designs given that solutions exist for all consecutive ply number groupings above 10 plies, which is in contrast to standard ply orientation designs which occur only for multiples of 4 plies. However, comparison of the form of the odd and even stacking sequences reveals that a change in the ply number grouping from \(n\) to \(n+1\), results in a change in angle plies of \(n+2\), which therefore precludes single ply drops, i.e. from an even to an odd stacking sequence with \(n+1\) plies.

Finally, the number of solutions may be compared with laminate designs for standard ply orientations [2], which reveals 6, 524, and 35,610 with 12, 16 and 20 plies from the parent class of Extension-Bending and Shearing-Twisting coupled laminates and 410, 40,808 and 4,515,473 with 12, 16 and 20 plies, respectively, for the parent class of Extension-Bending, Shearing-Twisting and Bending-Twisting coupled laminates.

5 Conclusions.

A preliminary study of Hygro-thermally Curvature-Stable laminate designs has revealed that a much broader design space exists for non-standard ply angle orientations, i.e. +60, −60, 0 and 90°, than for their standard ply orientation counterparts. These designs offer the scope for exploiting exotic mechanically coupled laminates, free from the constraints imposed by the effects of high temperature curing, which continue to be a barrier to many coupled designs that represent an important enabling technology for new and immersing applications of composite structures.

Note that the work reported here is on-going and involves numerical simulation and experimental validation of both the Hygro-Thermally Curvature-Stable properties and the mechanical coupling strength. Additional information can be obtained from http://eprints.gla.ac.uk/53239.

References


