ELECTROMECHANICAL RESPONSE OF PIEZOELECTRIC FOAMS

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1 Introduction

Piezoelectric material (eg. Lead zirconate titanate (PZT)), by the virtue of their electromechanical coupling, plays a prominent role in modern electroceramic industry. Applications of piezoelectric materials range from sensors and actuators to hydrophones. Piezoelectric composites obtained by adding two or more constituents (eg. 1-3 type, 2-2 type, 3-3 type piezoelectric composite) exhibit improved mechanical flexibility and piezoelectric activity, and are suitable for ultrasonic imaging, while controlled porous piezoelectric materials demonstrate improved signal-to-noise ratio, impedance matching, and sensitivity, and are suitable for hydrophone applications [1]. In general the porous piezoelectric materials can be broadly classified as (i) 3-0 type, where the porosity is enclosed in all three dimensions by a matrix phase; (ii) 3-1 type where the porosity exhibits connectivity in the 1-direction, which is similar to the case of long fibers embedded in the continuous matrix phase (which is connected to itself in all three directions); and (iii) 3-3 type, where the porosity exists in an open inter-connecting network where both the matrix phase and the porosity exhibit connectivity in all three directions (foam structures) [2]. Several analytical [3], numerical [4, 5] and experimental [6, 7] studies have been conducted to understand the effect of porosity on the electromechanical response of porous piezoelectric materials with different connectivity. For example, Dunn and Taya [3] developed analytical model to predict the electromechanical response of piezoelectric material with zero-dimensional (3-0) and one-dimensional (3-1) connectivity. Kar-Gupta and Venkatesh [4] showed that the shape and orientation of the pores can significantly influence the performance of 3-1 type porous piezoelectric materials. Ramesh et al. [5] developed a finite element based numerical model to study acoustic characteristics of dense and porous piezoceramic disc hydrophones and suggested that the 3-3 type piezoelectric materials can be used for wide-band hydrophone applications. Bast and Wersing [6] synthesized porous piezoelectric materials with 3-1 type connectivity and demonstrated that the acoustic impedance decreases with increased porosity. Experimental studies by Kara et al. [7] indicate that hydrophones made of porous piezoelectric structures have better sensitivity than those of PZT-polymer. However, not much research has been done on piezoelectric foam structures (3-3 porous piezoelectric materials).

Foam structures such as open-cell foams are considered as a complex network of struts or ligaments, each connecting two vertex points. Gibson and Ashby [8] presented an excellent review on foam structures and developed a cubic cell based model for three-dimensional open-cell foams. It is shown that for low density foam structures, the Young's modulus (E*) of foam structures is related to their relative density (ρ) though the relation:

$$\frac{E^*}{E_s} = C\left(\frac{\rho^*}{\rho_s}\right)^n \tag{1}$$

where ρ^* is the density of the foam, E_s, and ρ_s are the Young's modulus and density of the solid strut, respectively. The constants C and n depends on the microstructure of the solid material and the value of n generally lies in the range $1 \le n \le 4$. For an open-cell foam, experimental results suggest that n = 2 and $C \approx 1$.

Dependency of properties of a periodic foam structure on relative density/volume fraction depends on the mechanism of deformation. If the foam structures have "straight-through" struts then the deformation is assumed to occur along the axis of strut and the properties are linearly related to the foam density [9, 10]. If the struts are finite, struts deform in bending and the structural properties are quadratically related to relative density [8, 11]. Li et al. [12] formulated effective properties of three-dimension open-cell foam using matrix method for spatial frames, assuming that the members undergo simultaneous axial, transverse shearing, flexural and torsional deformation. In addition to the property dependency on relative density and strut deformation, some efforts have been made to study the effect of cell shape [13], cell irregularity [14], and strut cross-section [12, 13] on the effective properties of foam structures. Additionally, numerical models based on idealized unit cell have also been developed to predict the creep behavior [15] of open-cell foams. However, most of the existing analytical, numerical models and experimental results predict the effective structural properties of foam structures assuming that the struts are made of isotropic material and are homogeneous. Thus a comprehensive study to characterize piezoelectric foam structures is very important to understand the effect of relative density/volume fraction, mode of deformation and foam structure on the electromechanical response of piezoelectric foam structures. Furthermore, the piezoelectric figure of merits should be studied to assess foam structures for applications such as hydrophones. Hence, the objectives of the present study are: (i) to develop a unit cell based finite element model to fully characterize foam structures; (ii) systematically study the effect of relative density/volume fraction and mode of deformation on the electromechanical response of foam structures; (iii) to quantify the effect of external strut length, and shape of foam structures on the electromechanical response and piezoelectric figure of merits.

2 Classification of Piezoelectric Foam Structures

In the present study, the effective electromechanical response of three types of piezoelectric foam structures (i.e., 3-3 type and designated as F1, F2 and F3) with and without interconnecting struts (of two types of interconnect geometry and of varying interconnect lengths) are examined and benchmarked with respect to that of piezoelectric materials with long pores (i.e., 3-1 type, designated as F4) (Fig. 1). In all the simulations, the poling axis is aligned with the 2-direction.

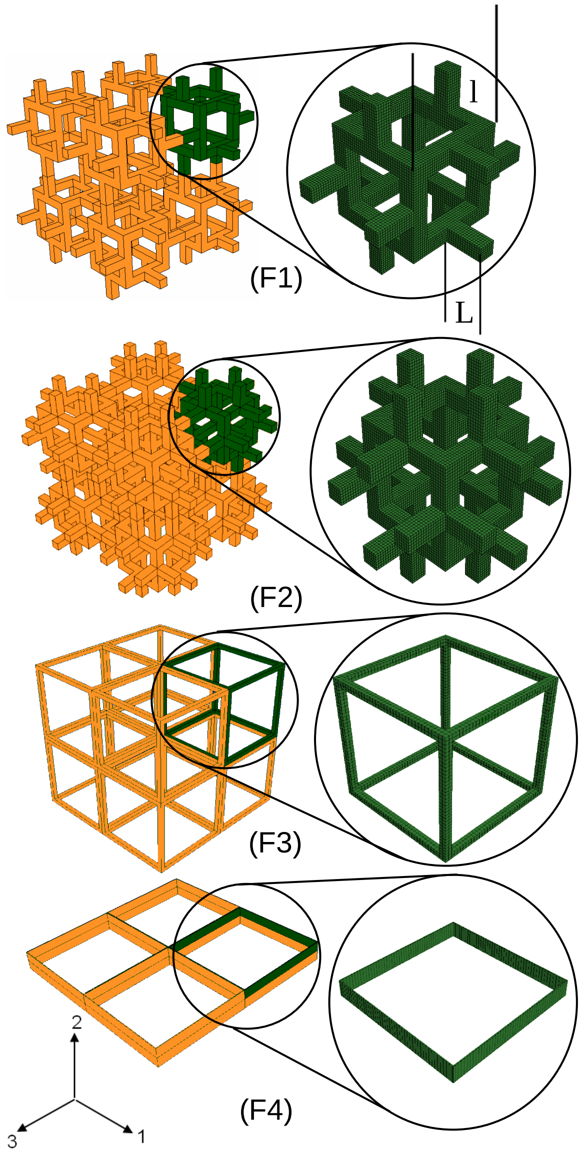

Fig.1. Piezoelectric structures with representative unit cell.

3 Constitutive Behaviour of Piezoelectric Materials

The electromechanical coupled constitutive relationships for a piezoelectric material are represented as:

$$\begin{aligned} \sigma_{ij} &= C^{E}_{ijkl}\varepsilon_{kl} - e_{ijk}E_k \\ D_i &= e_{ikl}\varepsilon_{kl} + \kappa^{\varepsilon}_{ij}E_j \end{aligned} \tag{2}$$

where σ and ε are the second-order stress and strain tensors respectively, E is the electric field vector, D is the electric displacement vector, C^E is the fourth-order elasticity tensor with the superscript "E" indicating that the elasticity tensor corresponds to measurement of C at constant or zero electric field, e is the third-order coupling tensor, and κ^ε is the second-order permittivity tensor measured at constant or zero strain.

4 Figures of Merit

In assessing the utility of piezoelectric materials for practical applications, several combinations of the fundamental material constants (i.e., figures of merit) are typically invoked. The figures of merit that are of direct interest to piezoelectric foams and their potential applications (e.g., hydrophones) are (i) the coupling constant (k_t); (ii) the acoustic impedance (Z); (iii) the piezoelectric charge coefficient (d_h); (iv) and the hydrostatic figure of merit ($d_h g_h$) [4].

5 Finite Element Modeling of Piezoelectric Foam Structures

A unit-cell based three-dimensional finite element model is developed to characterize complete electromechanical response of three types of 3-3 type piezoelectric foam structures (F1, F2, and F3) over a range of volume fractions and interconnect strut geometries, using a commercially available software ABAQUS. Eight-node, linear piezoelectric brick elements (C3D8E) are utilized for the piezoelectric foam structures where each node is allowed four degrees of freedom (three translational and one electric potential).

In general, the modeling approach for predicting the properties of piezoelectric foams involves the following five steps: (i) a unit cell that is appropriate for a foam structure with a specified relative density/volume fraction and deformation is identified; (ii) the unit cell is subjected to controlled mechanical and electrical loading conditions under carefully designed boundary conditions; (iii) the stress and electric displacements field components that developed in the unit cell as a result of applied strain and electric fields on the unit cell are measured; (iv) appropriate average procedures are invoked to capture the homogeneous coupled response of the unit cell; (v) using the matrix representation of the coupled response of piezoelectric materials, where the measured stress and electric displacements are related to the imposed strain and electric fields through the constitutive material property matrix, all the piezoelectric material constants are computed.

In invoking the unit cell approach for characterizing the electromechanical behaviour of piezoelectric foam structures, it is important to ensure that the deformation characteristics of the microscopic unit-cells are representative of the deformation of the macroscopic foam structures. Hence, particular care is taken to ensure that the deformation across the boundaries of the representative unit cell is compatible with the deformation of the adjacent unit cells. By comparing the deformation behaviour of a microscopic unit cell with the macroscopic structure (that comprised of 8 unit cells) under several loading conditions (such as face loading, corner loading, and line loading), a set of loading and boundary conditions that provide the best estimates for all the electromechanical properties of the piezoelectric foam structures are identified. (For face loading condition all the nodes on each face of the strut are loaded. In the corner loading condition all the corner nodes on each face of the strut are loaded and in line loading condition all the nodes on the middle line of each face of the struts are loaded.)

6 Results and Discussions

The numerical model developed in the present study is first applied to foams with asymmetric interconnecting strut structures (F1) and the open cell foam structure (F3) where the constituent elements of the foam structures are made of isotropic (non-piezoelectric) materials and the results are compared to those predicted by several analytical models developed earlier. Upon verifying that the results from the numerical model are in reasonable agreement with the analytical models developed earlier for isotropic (non-piezoelectric) materials, the finite element model is extended to piezoelectric foams (F1, F2 and F3) to predict their fundamental electromechanical properties and their corresponding figures of merits. The material properties are given in Table 1. The properties of 3-3 piezoelectric foams are benchmarked with those of 3-1 type porous piezoelectric materials as well.

6.1 Verification of the Numerical Model with Existing Analytical Models

Fig. 2 compares the Young's modulus and Shear modulus computed from the finite element model developed for the foam structure F1 with external strut length equal to half the internal cube dimension (L=0.5*l) (Fig. 1) and for the foam structure F3 with external strut length equal to zero (L=0), with the predictions from the analytical models available in the literature for isotropic (non-piezoelectric) open-cell foams. It is evident that there is reasonable agreement and some differences in the predictions of the analytical models and the finite element models which can be rationalized as follows. The analytical models by Gibson and Ashby [8], Warren and Kraynik [10] assumed that the struts undergo only bending deformation whereas the model by Christensen [9] assumed that the struts undergo axial deformation (compression) while the model by Li *et al.* [16] considered three deformation mechanisms (stretching, shearing, and bending). However, the finite element model developed in the present study can accommodate simultaneous bending and axial deformation. Hence, the shear modulus predicted by the finite element model is generally lower than that predicted by the Gibson and Ashby and Christensen models. In general, it is expected that the finite element model would provide a more accurate prediction of the properties of the open cell foam structures.

Table 1. Material properties of PZT-7A

	PZT-7A (ρ=7700 kg/m^3)
$C^E_{11} = C^E_{33}$ (GPa)	148
$C^E_{12} = C^E_{23}$ (GPa)	74.2
C^E_{13}(GPa)	76.2
C^E_{22}(GPa)	131
$C^E_{44} = C^E_{66}$ (GPa)	25.3
C^E_{55}(GPa)	35.9
$e_{21} = e_{23}$ (C/m^2)	-2.324
e_{22}(C/m^2)	10.9
$e_{34} = e_{16}$ (C/m^2)	9.31
$\kappa^{\varepsilon}_{11} = \kappa^{\varepsilon}_{33}$ (nC/Vm)	3.98
$\kappa^{\varepsilon}_{22}$(nC/Vm)	2.081

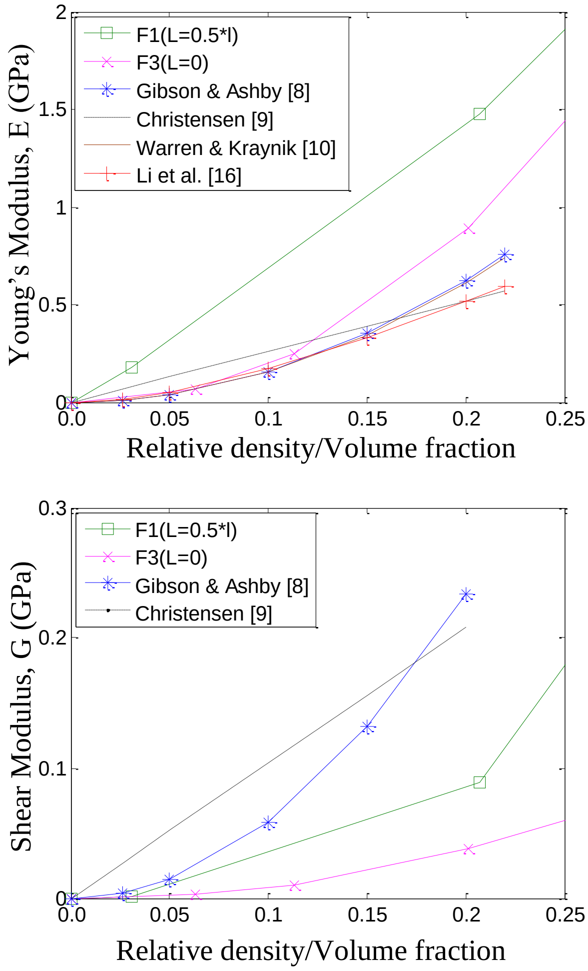

Fig. 2. Comparison of Young's modulus and Shear modulus for F1 (L=0.5*l) and F3 (L=0) obtained from the finite element model with the results predicted by the analytical models.

6.2 Identifying Optimum Unit-Cell Boundary Conditions and Loading Conditions

Fig. 3 compares the stress developed in the foam structure F1 upon application of mechanical strain along the 2-direction on face 1 for the unit cell under different boundary conditions obtained from simulations of a microscopic unit cell with that of a macroscopic structure (with eight unit cells) for the foam structure F1 with external strut length equal to half the internal cube dimension (L=0.5*l). After careful analysis, face loading conditions are used to characterize the fundamental properties C_{11}, C_{12}, C_{13}, C_{22}, C_{33}, C_{23}, e_{21}, e_{22}, and e_{23}, line loading conditions are used to characterize C_{44}, C_{55}, C_{66}, and e_{16}, and corner boundary conditions are used to characterize dielectric constants κ_{11}, κ_{22}, and κ_{33} as these loading conditions provide the best match between the properties obtained from the microscopic unit cell and the macroscopic foam structure.

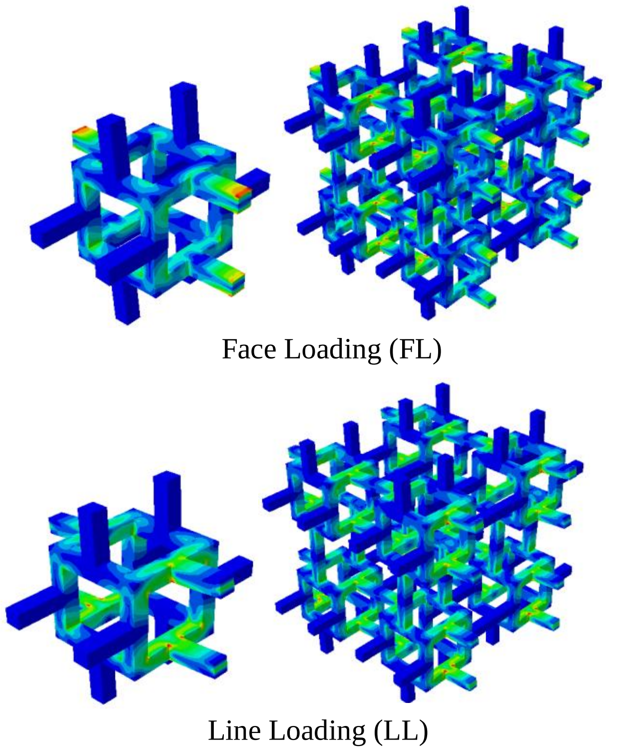

Fig. 3. Spatial evolution of stress in the foam structure F1 upon application of mechanical strain along 2-direction on face 1 for microscopic unit cell with that of a macroscopic structure (with eight unit cells) with external strut length equal to half the internal cube dimension (L=0.5*l).

6.3 Figure of merits

As discussed in Section 4, piezoelectric coupling constant, acoustic impedance, piezoelectric charge coefficient, and hygrostatic figure of merit are analyzed in assessing the utility of cellular solids and porous structures for hydrophone applications. From the study (Figs. 4, and 5) the following observations are made:

(i) Piezoelectric foam structure (F3) exhibits better piezoelectric coupling constant compared with F1, and F2 structures with varying interconnected strut lengths and F4 foam structures. In general the piezoelectric coupling constant increases as the interconnected strut length increases for F1 and F2 foam structures.

(ii) Acoustic impedance of F4 (3-1 type) piezoelectric foam structure increase linearly with volume fraction where as for F1, F2 and F3 foam the acoustic impedance increase non-linearly. The acoustic impedance for F4 structure is higher compared to F1, F2 and F3 foam structures.

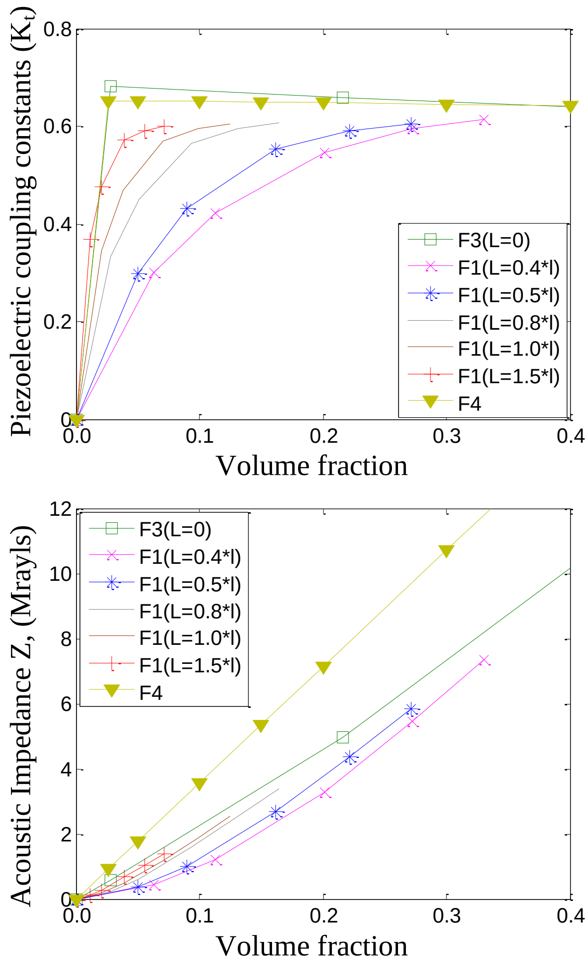

Fig. 4. Variation of piezoelectric coupling constant and acoustic impedance for F1 (with varying external strut lengths), F3, and F4 foam structures.

7 Conclusions

Foam structures such as open-cell foams have been widely recognized for there potential application in light weight structures, crash protection, thermal insulators etc. However, a comprehensive study on the electromechanical response of foam structures made of piezoelectric materials is not yet available. Furthermore, the application of piezoelectric foam structures for hydrophone has not been examined. Hence, a finite element model has been developed to systematically study the effect of relative density/volume fraction, shape and mode of deformation on the electromechanical properties and figures of merit of piezoelectric foam structures.

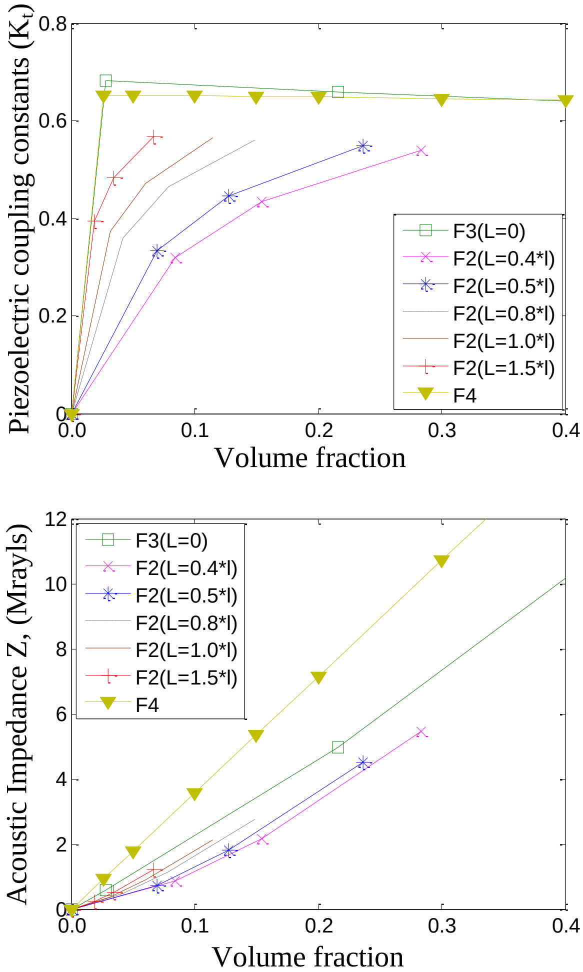

Fig. 5. Variation of piezoelectric coupling constant and acoustic impedance for F2 (with varying external strut lengths), F3, and F4 foam structures.

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