PREDICTING DAMAGE ACCUMULATION IN GLASS FIBER REINFORCED PLASTICS THROUGH CUMULATIVE DAMAGE MODELS

R. Fragoudakis[1]* and A. Saigal[1]
[1]Department of Mechanical Engineering, Tufts University, Medford, MA, U.S.A.
*Corresponding author (roselita.fragoudakis@tufts.edu)

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Abstract

Three cumulative damage models are examined for the case of cyclic loading of S2 and E glass fiber/epoxy composites. The Palmgren-Miner, Broutman-Sahu and Hashin-Rotem models are compared to determine which of the three gives a more accurate estimation of the fatigue life of the two composite materials tested. In addition, comparison of the fatigue life of the materials shows the superiority of S2 over E glass fiber/epoxy.

1. Introduction

Light and durable structures are becoming the goal of many industries, as is the case of the automotive one. Composites have replaced metals in many applications, because they weigh less and have higher strength and stiffness than metals [1]. To select a material for applications involving cyclic loading, knowledge of the material's fatigue life is crucial. A statistical approach in determining the fatigue life of materials is necessary, when trying to predict when a component may fail. The Weibull distribution is used to predict the fatigue life and failure of materials using failure data from specimens subjected to certain loading conditions. It is important to be able to predict the fatigue life especially when the materials involved are brittle, as in the case of composites [1,2].
Cumulative Damage Theory is the ensemble of attempts to calculate the damage caused by cycling, as well as its accumulation when cycling includes more than one stress amplitudes [3]. There are two ways to discuss the concept of cumulative damage: residual strength, being the instantaneous static strength that the material can still maintain after being loaded to stress levels causing damage, and the estimation of cumulative damage through damage models. This latter approach is followed in this study [4].
Composites fail because of accumulated damage [1,5]. The strength of the material starts decreasing slowly early in the fatigue life, and towards the end of it, close to failure, the rate of decrease in strength becomes very rapid [6]. Even if minimum information on the fatigue life of the material is known, cumulative damage models can predict the damage generated in the material due to loading. Contrary to the case of metals, when designing composite structures it is higher stresses, defining low cycle fatigue (LCF), that are critical [7].

2. Damage Models and Materials

The following three damage models are used to predict and compare the damage caused in two composites, namely the unidirectional Glass Fiber Reinforced Plastics (GFRP) with +/-5° fiber orientation, S2 glass fiber/epoxy ($\sigma_{flexure}$ of 1.28 GPa) and E glass fiber/epoxy ($\sigma_{flexure}$ of 1.08GPa) [8], under cyclic loading conditions:

Palmgren-Miner [9-11]:

$$\left(\sum_{i=1}^{n}\frac{n_i}{N_i}\right)K=1 \tag{1}$$

Broutman-Sahu [10,12]:

$$\left(\sum_{i=1}^{n}\frac{(\sigma_{Ultimate}-\sigma_i)}{(\sigma_{Ultimate}-\sigma_{i+1})}\frac{n_i}{N_i}\right)K=1 \tag{2}$$

and Hashin-Rotem [3,10]:

$$\left(\sum_{k=1}^{i-1}\left(\frac{n_{(k-1)}}{N_{(k-1)}}\right)^{\frac{(1-S_k)}{(1-S_{(k-1)})}}+\frac{n_i}{N_i}\right)K=1 \tag{3}$$

$$S_k=\frac{\sigma_k}{\sigma_{Ultimate}} \tag{3a}$$

$$S_{(k-1)}=\frac{\sigma_{k-1}}{\sigma_{Ultimate}} \tag{3b}$$

where n_i is the number of cycles under the applied stress, N_i the cycles to failure under this same stress, σ_i and σ_k are the stresses applied, $\sigma_{Ultimate}$ is the ultimate strength of the material, S_k is the ratio of the applied stress to the ultimate strength, and K is the number of repetitions of the loading cycle. Damage is accumulated until the left hand side of the above equations equals 1, at failure [1,3,10-11].

A specimen may undergo cycling while being subjected to one or more stress levels. At two stress levels, where σ_1 and σ_2 are imposed on the specimen for an amount of n_1 and n_2 cycles, respectively, n_2 is the number of cycles that will lead the specimen to failure. This parameter is called the residual lifetime. The residual lifetime of a component can be predicted using all three of the above models at failure, i.e. when their mathematical expression equals 1. The couples σ_i and n_i, being the stress and respective number of cycles, are used to create a damage curve. A damage curve shows what is the ultimate damage caused to the specimen at a residual life of zero. Such a curve is called an S-N curve. The life fraction of a component, stressed at σ_i, is represented by the ratio n_i/N_i [3].

The Palmgren-Miner model defines damage in the material, in the form of life fractions. Each such ratio represents a percentage of life consumed [3,9,11-12]. The sum of these ratios defines failure of the material when it equals to 1. At this point no more residual life remains to be expended. Palmgren-Miner does not account for the order in which the stresses are applied to the specimen [11]. The other two models, which also define damage in the form of life fractions, account for the loading sequence.

Broutman and Sahu gave a modified Miner's sum. They used the linear strength reduction curves, and assuming that the residual strength is a linear function of the fractional life spent when the component is loaded at a given stress level, predicted more accurately the fatigue behavior in GFRP, especially at higher stress levels [12].

Hashin and Rotem developed a cumulative damage model to predict damage in two-stress level loading, which can be expanded for use under multi-stress level loadings, using the concept of damage curve families to represent residual lifetimes for two-stress level loading, and the fact that equivalent residual lives are expended by specimens that undergo different loading schemes[1] [3].

Palmgren-Miner and Hashin-Rotem rules have been initially designed and tested on metals, although later used in GFRP damage predictions. The Broutman-Sahu model was developed and tested on GFRP.

The damage models can be classified as linear or non-linear and according to the parameters required for their calculation [1]. Consequently, Palmgren-Miner is a linear stress independent model, Broutman-Sahu a linear stress dependent model, and Hashin-Rotem a non-linear stress dependent model.

The two composites investigated, S2 glass fibre/epoxy and E glass fibre/epoxy, are very common composite alternatives to steel in the

[1]Equivalent loading postulate: "cyclic loadings which are equivalent for one stress level are equivalent for all stress levels." [3]

manufacturing of leaf springs for heavy-duty vehicles [8]. The fiber orientation plays a crucial role in determining the fatigue life of the material, especially in loading conditions that involve bending [5, 13]. Composites with different fiber orientations may have different fatigue lives, and failure may occur at a different part of the composite.

3. Results and Discussion

Experiments were conducted on a single composite leaf spring mounted on a test rig. The load was applied to the center of the leaf spring. The specimens were cycled under each load until failure occurred.
Damage is calculated for a range of mean stresses between 256 MPa and 560 MPa, and for a loading ratio (R) of approximately 0.2. The stress ratios correspond to both low cycle (LCF) and high cycle fatigue (HCF) in both composites.
A standard two-parameter Weibull analysis (Eqn. 4) [14] was performed, in order to decide upon the model that gives more realistic results, for damage and fatigue life, when compared to experimental data [8].

$$F(t) = 1 - exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right] \quad for \quad t \geq 0 \tag{4}$$

The scale (α) and shape (β) parameters were calculated for each composite (Table 1). The mean value of damage caused in the material after one loading cycle is given by the scale parameter, and damage per cycle is larger for E glass fibre/epoxy, by at least one order of magnitude, for all damage models.
The cumulative distribution of damage predicted by the two linear models is similar at almost all mean stresses for S2 glass fibre/epoxy (Fig. 1(a)).
Hashin-Rotem gives a lower probability of failure than Broutman-Sahu and Palmgren-Miner, for mean stresses between 256 MPa and 350 MPa. For stresses from 350 MPa to 485 MPa the probability of failure, from the Hashin-Rotem model, is higher, but at 485 MPa the failure probability is very close to that of the other two models.

Table 1: Shape and Scale Parameters for all Damage Models

	Palmgren-Miner	Broutman-Sahu	Hashin-Rotem
β E glass/fibre	0.22	0.28	0.38
β S2 glass/fibre	0.28	0.37	0.42
α E glass/fibre	1.29×10^{-3}	2.68×10^{-4}	0.41
α S2 glass/fibre	5.06×10^{-5}	2.08×10^{-5}	8.49×10^{-3}

The Hashin-Rotem failure probability at 560 MPa, for S2 glass fibre/epoxy composite, is approximately 95%, 2% lower than that estimated by the two linear models at the same mean stress.
For E glass fibre/epoxy composite the cumulative distribution of damage predicted by the Palmgren-Miner model is almost identical to that predicted by the Broutman-Sahu model (Fir. 1(b)). Both linear models show a constant probability of failure at low mean stresses up to 280 MPa. In both cases of the linear models, failure probability at these mean stress levels is approximately 19%. At mean stresses between 350 MPa and 400 MPa, the Broutman-Sahu model gives a bit lower probability of failure, and higher values at 460 MPa to 485 MPa. When these values of failure probability are compared to the results of the Palmgren-Miner model, they differ by at most 1%. 560 MPa is the mean stress level where all three models give a prediction of total failure, a cumulative damage of 1. The reason all models coincide at this point, is due to the fact that this mean stress level, for a loading ratio of 0.2, corresponds to a maximum stress of 1.1 GPa, which is higher than the ultimate tensile strength of the material. The non-linear model starts with lower probability of failure, than the other models, but the cumulative damage increases between 280 MPa and 360 MPa, giving a probability 5% higher than the Palmgren-Miner model. Above 460 MPa it agrees with the two linear models within less than 1%.

Table 2: Average Cumulative Distribution of Damage for S2 glass fibre/epoxy composite

Mean Stress [MPa]	Palmgren-Miner	Broutman-Sahu	Hashin-Rotem
256-350	0.281	0.293	0.261
360-485	0.537	0.544	0.571
560	0.974	0.976	0.954

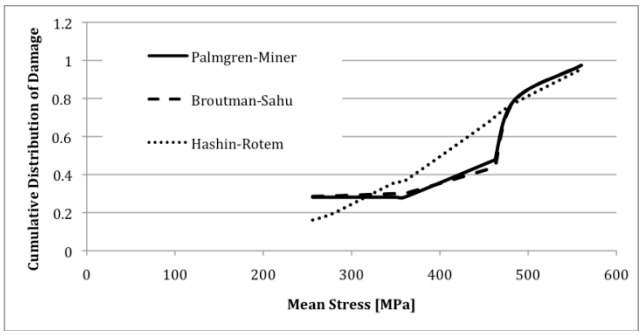
Table 3: Average Cumulative Distribution of Damage for E glass fibre/epoxy composite

Mean Stress [MPa]	Palmgren-Miner	Broutman-Sahu	Hashin-Rotem
256-358	0.246	0.241	0.262
360-485	0.570	0.568	0.598
560	1	1	1

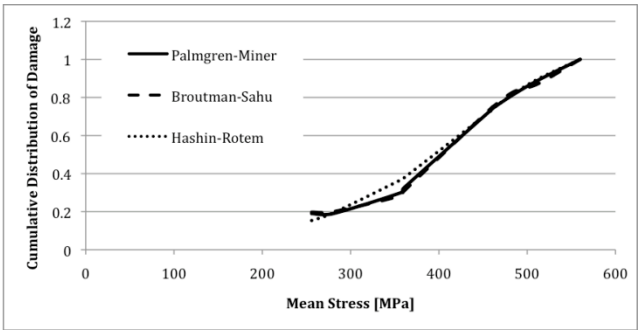
For both composites, but especially for the E glass fibre/epoxy, the curve of the Hashin-Rotem predictions is a smooth curve resembling a best-fit line for the two curves of the linear damage models. Hashin-Rotem deviates the most from the other two models in the case of the S2 glass fiber/epoxy composite. Hashin-Rotem deviates the most from the two linear models between 462 MPa to 480 MPa. At 462 MPa the non-linear model calculates a failure probability approximately 31% more than Palmgren-Miner and Broutman-Sahu models. The highest deviation of the Hashin-Rotem model from the two linear ones, in E glass fibre/epoxy, is 7% at 358 MPa.

An average of the cumulative distribution of damage predicted by all three models, at various mean stress ranges, is given for both composites in Tables 2 and 3.

Although it may not be clear at a low mean stress range, as the stress level increases E glass fibre/epoxy suffers more damage than S2 glass fibre/epoxy. The reason E glass fibre/epoxy composite shows less cumulative damage at low mean stresses is due to the shape parameter (β), of the Weibull analysis that defines the shape of the cumulative distribution curve. When the shape parameter is small the distribution starts at lower values than those for larger parameters, but has a more rapid ascend.



(a)



(b)

Fig. 1: Cumulative Distribution of Damage versus Mean Stress: (a) S2 glass fibre/epoxy composite (b) E glass fibre/epoxy composite

Cumulative damage distribution for one cycle (K=1) is shown in Fig. 1, and discussed above. To calculate the fatigue life of the materials, the value K when each of the three models equals 1, i.e. at failure, needs to be determined. Fig. 2 gives the mean stress versus cycles to failure, where the short dashed line in each graph is experimental data from the literature [8].

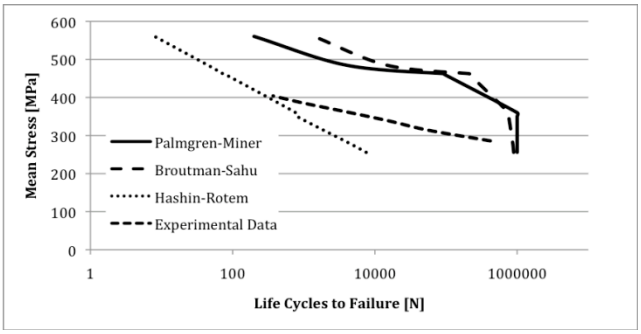
For S2 glass fibre/epoxy composite, up to 360 MPa Broutman-Sahu and Palmgren-Miner give similar results, and differ at 560 MPa by 89%. The Hashin-Rotem model underestimates the composite's life by two orders of magnitude at low mean

stresses, when compared to experimental data.

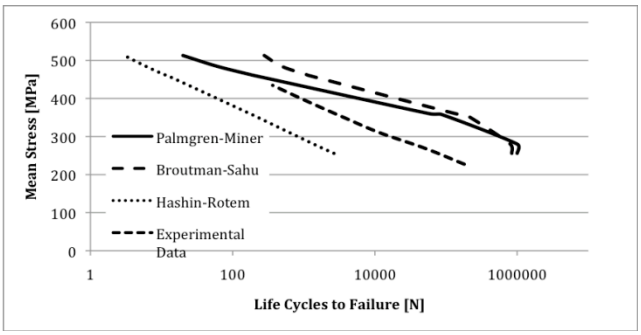
The stress dependent linear model, Broutman-Sahu, compared to Palmgren-Miner, predicts lower fatigue life at lower stresses and higher life at higher stresses. This is explained by the fact that the Palmgren-Miner model is not sensitive to changes in stress, since it is a stress independent model. These changes in stress may be small but are important in a material that fails by accumulating damage under cyclic loading. Comparison of the predictions of the damage models to experimental data, shows that the two linear model predictions are higher by at most one order of magnitude, at very low mean stresses, while this is the case of experimental data compared to the non-linear model predictions at high mean stresses, above 400 MPa.

In the case of the fatigue life predictions for E glass fibre/epoxy, not much difference can be observed from that of S2 glass fibre/epoxy, as far as comparison among the three damage models is concerned. The two linear models give a fatigue life differing from each other by less than one order of magnitude at low mean stresses up to 280 MPa (Palmgren-Miner gives a fatigue life 15% higher than Broutman-Sahu). Broutman-Sahu shows better results at higher stresses, 92% higher than Palmgren-Miner at 512 MPa. As is the case of this model in the previous composite, the fatigue life of the composite is underestimated by the Hashin-Rotem model.

The fatigue life predicted by the non-linear model is two orders of magnitude smaller than that predicted by the Palmgren-Miner model at low mean stresses, while at 560MPa these predictions are one order of magnitude smaller than the Palmgren-Miner predictions, and two orders of magnitude less than the fatigue life given by the Broutman-Sahu model. A similar pattern for S2 glass fibre/epoxy composite can be observed when comparing the predictions of the damage models to experimental data at low mean stresses. At higher mean stresses, however, it is the Palmgren-Miner model that is closer to experimental data of E glass fiber/epoxy, while the Hashin-Rotem model remains within one order of magnitude below experimental data, at all mean stresses.



(a)



(b)

Fig. 2: Mean Stress versus Life to Failure: (a) S2 glass fibre/epoxy composite, (b) E glass fibre/epoxy composite

Experimental results for both composites fall between the linear and non-linear models. It is also worth observing that the predicted fatigue life is higher at the HCF region. The effect of damage accumulation in composite materials can be seen if close attention is paid to what happens when, as is the case of this study, the mean stress rises above 460 MPa. At this stress level the fatigue life drops by 72%, in S2 glass fibre/epoxy and 99% in E glass fibre/epoxy.

An average fatigue life at different mean stress ranges for all examined models and experimental data is presented for both materials in Tables 4 and 5. The fatigue life of E glass fibre/epoxy is lower than that of S2 glass fibre/epoxy by at least one order of magnitude at most mean stress ranges. Comparison of the predicted fatigue life to experimental data is also shown in the tables.

Table 4: Average Fatigue Life for S2 glass fibre/epoxy composite

Mean Stress [MPa]	Palmgren - Miner	Broutman - Sahu	Hashin-Rotem	Experi-mental Data
256-350	10^6	$8.3x10^5$	$4.2x10^3$	$2.2x10^5$
360-485	$5.0x10^5$	$3.9x10^5$	408	$2.2x10^3$
560	200	$1.4x10^3$	8	No Data
	Compared to experimental data			
	Above	Above	Below	

Table 5: Average Fatigue Life for E glass fibre/epoxy composite

Mean Stress [MPa]	Palmgren - Miner	Broutman - Sahu	Hashin-Rotem	Experi-mental Data
256-358	$5.4x10^5$	$5.1x10^5$	$1.4x10^3$	$1.5x10^4$
360-485	$2.9x10^4$	$6.9x10^4$	93	$1.3x10^3$
560	20	278	3	No Data
	Compared to experimental data			
	Above	Above	Below	

4. Conclusions

Results show that the larger the damage accumulation, the larger the probability of failure and the smaller the fatigue life of the material. S2 glass fiber/epoxy suffers less damage accumulation than E glass fiber/epoxy (Tables 2 and 3), and as a result has better fatigue life (Tables 4 and 5).

It is hard to decide upon the optimal damage model among those investigated here. The linear models gave more accurate but overestimated predictions of the fatigue life of the composites. The non-linear model predictions were significantly lower. The stress-dependence of the model was important at lower stresses. For these reasons damage model predictions should always be compared to the experimental data of the material under consideration.

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