1 Introduction

In recent years, flywheel technology has received much attention for industrial energy storage applications. Due to advances in power electronics, loss reduction techniques such as magnetic bearings and vacuum enclosures, and the utilisation of enhanced high-strength/low-density materials, economical flywheel energy storage (FES) has become a fact. There are numerous application opportunities envisioned for those kind of flywheels, including transport systems and uninterruptible power supplies. They present many advantages compared to conventional energy storage systems based on battery technology, as fast charge and discharge operations, higher energy density (energy storage per unit weight), longer durability and minor environmental concerns. However, less experience with this technology has led to great research efforts in design optimisation aiming at the enhancement of the efficiency of FES systems [1].

The design of a composite flywheel brings along many challenges. Most of them are related to manufacturing and assembling issues and considerations on what might occur during operation. The present paper describes the parameters that have to be taken into account when designing a composite flywheel. The parameters include material properties, manufacturing aspects and assembly considerations that also involve operational issues. An optimization method is presented which takes into account these parameters and allows finding the best solution in terms of available energy.

2 Flywheel design

A schematic of the basic components for a typical flywheel system is depicted in Fig. 1. Being connected to the motor, the flywheel can be accelerated to a specific angular velocity, consuming electrical power by the motor, which then operates as generator.

As the kinetic energy is directly proportional to $\omega^2$, it is common to have very high angular velocities, which correspond to large centrifugal loads and thus lead to high circumferential and radial stresses. Composite materials have successfully been established in flywheel rotor design for their beneficial material properties, particularly their high specific stiffness and strength [2].

3 High efficiency composite flywheels

In order to improve the efficiency of flywheel rotors, stress reduction methods have been applied. The dominating stresses are generally the circumferential and radial stress. Since the fiber reinforcement is typically aligned in the circumferential direction, radial tensile stress is typically the most critical due to the weaker strength in this direction.
Instead of using only one rim material, it is also common to assemble rims of different materials as shown in Fig. 2, resulting in a hybrid composite flywheel rotor, and/or to apply compressive force between two adjacent rims via press fitting.

Fig. 2 Example of a multi-rim concept.

In this case, the radial stresses turn compressive in the region near the material interface due to the lower stiffness of the inner material which would experience greater expansion in the single material case [2].

Fig. 3 Reduction of radial stresses due to a multirim flywheel.

Press fitting one ring within another, results in residual compressive stresses in the rings which counter the tensile radial stresses that result during rotation.

For this purpose, according to literature [3], the material to be chosen should be with increasing stiffness per density value $E/\rho$ for increasing radius $r$.

In order to verify this statement, an optimization program has been created which is able to calculate the allowable maximum energy for a multi-rim rotor flywheel with interfaces [4].

The interfaces are a crucial design issue and very important considerations have to be made on the stress level that is allowed during both operation and assembly.

3 Theoretical background

A press-fitted composite flywheel rotor consists of several concentric rings made of different materials. The concentric rings are press fitted together with a certain amount of interference.

Fig. 4 Schematic description of two adjacent rings with interference.

The method of analytical stress analysis of the flywheel derived from the method proposed by Ha [5] and Krack [6].

The composite flywheel can be assumed to be cylindrically orthotropic and axis symmetric. The stress distribution in each ring is governed by the radial equilibrium equation, which is written in cylindrical coordinates [6]:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} (\sigma_{rr} - \sigma_{0r}) + \rho \omega^2 r = 0$$

where $\sigma_{rr}$ and $\sigma_{0r}$ are the radial and hoop stress, respectively, $\omega$ is the angular velocity and $\rho$ is the density of a ring.

For the 2D plane stress analysis of a cylindrically orthotropic ring, the equation the original material
laws for an anisotropic material in a cylindrical coordinate system can be simplified as:

\[ \begin{bmatrix} \sigma_{\theta \theta} \\ \sigma_{rr} \end{bmatrix} = \frac{1}{1 - v_{12}^2} \begin{bmatrix} E_1 & v_{12}E_2 \\ v_{12}E_2 & E_2 \end{bmatrix} \begin{bmatrix} \varepsilon_{\theta \theta} \\ \varepsilon_{rr} \end{bmatrix} \]

The kinematic laws (strain-displacement relations) are given by:

\[ \varepsilon_{\theta \theta} = \frac{u_r}{r}, \quad \varepsilon_{rr} = \frac{\partial u_r}{\partial r} \]

Substituting equations (2) and (3) into equation (1) results in the following equation for \( u_r \):

\[ \frac{d^2 u_r}{dr^2} + \frac{1}{r} \frac{du_r}{dr} - \frac{Q_{11}}{Q_{33}} u_r = -\frac{\rho \omega^2}{Q_{33}} r \]

After solving the differential equation (4), the following results will be obtained [6]:

\[ u_r = -\rho \omega^2 \gamma_0 r^3 + C_1 r^k + C_2 r^{-k} \]

\[ \sigma_{rr} = -\rho \omega^2 \gamma_r r^2 + C_1 \gamma_2 r^{k-1} + C_2 \gamma_3 r^{-k-1} \]

\[ \sigma_{\theta \theta} = -\rho \omega^2 \gamma_\theta r^2 + C_1 \gamma_3 r^{k-1} + C_2 \gamma_6 r^{-k-1} \]

where \( C_1 \) and \( C_2 \) are unknown constants and will be determined from boundary conditions and:

\[ \kappa = \sqrt{\frac{Q_{11}}{Q_{33}}} \]

\[ \gamma_0 = \left( \frac{9 - k^2}{9} \right) Q_{33} \]

\[ \gamma_1 = (Q_{13} + 3Q_{33}) \gamma_0, \quad \gamma_2 = Q_{13} + k Q_{33} \]

\[ \gamma_3 = Q_{13} + k Q_{33}, \quad \gamma_4 = (Q_{11} + 3Q_{33}) \gamma_0 \]

\[ \gamma_5 = Q_{11} + k Q_{33}, \quad \gamma_6 = Q_{11} - k Q_{33} \]

This set of equations allows the calculations of the stresses and radial displacements of one single rim and can be expanded for a series of rims with interfaces.

### 4 Optimisation

For a hybrid flywheel rotor composed of \( N_{rim} \) composite rims, the total available energy \( E_{total} \) of the flywheel rotor can be calculated as follows:

\[ E_{total} = \frac{\pi}{4} h \omega^2 \sum_{i=1}^{N_{rim}} \rho_i \left( r_{i,\text{outer}}^4 - r_{i,\text{inner}}^4 \right) \]

In Eq. (9) \( h \) is the height, \( \omega \) is the angular velocity of the rotor, and \( \rho_i \) is the density of the i-th rim. Strength ratio \( R \) is used for failure analysis, where a value of \( R \) greater than 1 indicates material failure. In the present program, the maximum stress failure criterion is used, the Strength ratio \( R \) for this case can then be computed by:

\[ R_{\text{inner}} = \frac{\sigma_{h}}{X^T}, \quad \text{for } \sigma_{h} \geq 0 \]

\[ R_{\text{outer}} = \frac{\sigma_{h}}{X^C}, \quad \text{for } \sigma_{h} \leq 0 \]

\[ R_{\text{inner}} = \frac{\sigma_{r}}{Y^T}, \quad \text{for } \sigma_{r} \geq 0 \]

\[ R_{\text{outer}} = \frac{\sigma_{r}}{Y^C}, \quad \text{for } \sigma_{r} \leq 0 \]

Herein \( X^T, Y^T, X^C, Y^C \), denote the material strengths in the longitudinal and transverse direction for tensile and compressive values respectively.

When inner and outer flywheel radii are constant parameters, the optimization problem for a hybrid flywheel rotor of \( N_{rim} \) press fitted rims is thus formulated as follows:

\[ \text{Find } x = [T_1, T_2, ..., T_{N_{rim}}, \delta_1, \delta_2, ..., \delta_{N_{rim}-1}, h] \]

\[ \text{Maximize } f(x) = E_{total}(x) \]

\[ \text{Subjected to } \]

\[ g(x) = R(x) - 1 \leq 0; \]

\[ 10 \text{mm} \leq T_{N_{rim}} \leq 450 \text{mm} \]

\[ 13000 \text{rpm} \leq n \leq 17000 \text{rpm} \quad 0 \leq \delta_i \leq t_{st} \]

and \( 0 \leq \delta_i \leq t_{comp}, i=2, ..., N_{rim}-1 \)
In Eq. (11) \( n \) is a rotating speed, \( T \) is the thickness of each rim, and \( \delta \) is the interference at each interface.

The maximum interference between steel and composite can be calculated considering a coefficient of thermal expansion between \( 11 \times 10^{-6}/°C \) and \( 13 \times 10^{-6}/°C \) and a cooling temperature difference of about 200 °C. The interference is therefore depending on the dimensions of the steel ring but could be increased when also force is applied in order to press fit.

The maximal interference between two adjacent composite rings is delimited by two factors, the internal capability for press fitting of the manufacturer who decides its own limit, and a mere question of strength of the rings while press fitting, as the forces due to the interference when no rotational forces are applied could exceed the strength of the material.

Another factor of great importance that has to be taken into account in the optimisation is that the radial forces at the interferences between two different rims have to be always negative at maximum speed.

The rims in fact are not connected to each other by any mechanical fastener or adhesives, therefore in order to stay connected they have to rely on the compressive forces that are initially applied during press fitting. The forces diminish while spinning but cannot become positive to avoid a separation of the rims. This is an important parameter in the optimisation which, when it is not applied is giving much better results that in practice are not applicable.

### 4 Results

The following results are based on a large flywheel of 1 m length, with a required internal steel ring. The material characteristics, as well as their strength and the safety factor considered in the optimisation routine are shown in Table 1. For steel, an extremely high strength has been considered. As this high grade steel is difficult to find and manufacture a sensitivity analysis will be shown later.

A safety factor of 1.5 has been applied on the strength of all composites to account for manufacturing flaws and production, including assembly. For the steel a safety factor of 2 is taken into account. This is mainly to account for fatigue issues occurring during service life. This factor of safety highly depends on the type of steel grade that is used and might be insufficient for such a high strength steel as considered in the beginning. Tests will be necessary to verify those assumptions.

Fig. 5 shows the results of a parameter study whose aim was to understand which mechanical characteristics are influencing the performances of a large flywheels. Three carbon fibres grades and one glass grade have been considered for flywheels made of one up to three composite rings, as shown in the plot.

From this parameter study it is possible to come to the following conclusions:
- There is no need of more than one ring made of GFRP
- Medium stiffness high strength CFRP (as T800) is preferable to high stiffness low strength CFRP (as M40J)
- Higher stiffness CFRP is needed in the outer layer for better performance

When four rings are used, at least one outer ring of T800 is necessary. An inner ring made of GFRP could be useful as interface between the CFRP rings and the steel ring, but it is not necessary.

As mentioned before, the initial steel strength is 1400 MPa, in which a safety factor of 2 is considered; therefore the limit strength is 700 MPa. A factor of 2 is a simple approach to calculate the fatigue endurance limit.

To be able to judge which steel strength grade is sufficient for the production of such a flywheel, the ultimate strength of the steel has been varied from 300 MPa to 700 MPa for a flywheel with three rings of T300 and three rings of T800 and different values of pretension for the three rings.

![Fig. 6 Influence of steel strength on available energy.](image)

Fig. 6 shows the effect of the steel strength on both available energy and speed. From the picture it appears that below limit strength of 400 MPa both energy and speed are decreasing. For a steel ring with limit strength of 300 MPa the available energy decreases of more than 30% the energy obtained with a steel ring of limit strength equal to 700 MPa.

### 5 Conclusions

This paper presents an optimisation method aiming at finding the maximal energy of a multi-rim flywheel rotor. The optimisation method takes into consideration also production issues as well as assembly and operational ones.

It is shown here that, at equal specific stiffness, strength plays a much more important role and therefore a material with the highest specific strenght should be preferred to high stiffness one for the outermost rims.

### References


