

# THROUGH-THICKNESS SHEAR MODULUS IDENTIFICATION FROM FULL-FIELD SURFACE MEASUREMENTS

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## 1 General Introduction

The measurement of the through-thickness (TT) moduli of composite plates is an important issue, all the more since thick composites are more and more used as primary structures in the aerospace industry for instance. In particular, the transverse or interlaminar or through-thickness shear moduli are very important to predict the stiffness of moderately thick or curved panels. This is enhanced by potentially high anisotropy ratios in composites (ratio between Young's modulus and the corresponding interlaminar shear modulus).

A first well-known method is the three-point bending test on short span beams [2]. This test is usually employed to give information on the interlaminar shear strength. There have been some attempts at using it for the interlaminar shear modulus [3] but there is the issue of indentation at the loading points. This test is usually considered not suitable for modulus measurement [2]. Another rather underexplored possibility is the torsion test on rectangular bars [4]. Ideally, it requires the use of strain gauges bonded on both the in-plane and through-thickness sides to enable the identification of both the in-plane and interlaminar shear moduli. However, this still requires rather thick specimens and the data processing relies on an analytical solution of the mechanical problem, hence a strong restriction in shape and boundary conditions. Certainly one of the most popular methods for interlaminar shear modulus identification is the Iosipescu test (or double V-notch test, ASTM standard D5379). Several studies have exhibited convincing results on the measurement of the interlaminar shear modulus of a wide range of composites, [5,6] for

instance. However, the main disadvantage is that rather thick panels have to be manufactured (about 20~mm for the standard fixture), even though this requirement may be slightly relieved if tabs are used such as in [7]. Other less popular test methods have been used, such as the cube test [8] or the inclined double notch specimen [9], for instance. Some other techniques have also been reported in [10], this list being non exhaustive. It must also be noted that inverse procedures have been attempted to relax the requirements in test geometry and loading [10].

An alternative strategy consists in performing vibration tests on plates. Initially developed for the identification of the in-plane stiffness components, these strategies, based on model updating (often finite element model updating), have been extended to the interlaminar shear moduli using higher order shear plate theories [11]. The results are satisfactory provided that the plates are thick enough for their eigenfrequencies or mode shapes to be sensitive to interlaminar shear. Nevertheless, the results are usually difficult to compare to mechanical quasi-static tests because of the effect of strain rate and strain levels, vibration tests giving rise to only small strains in the plates.

Basically, all these methods (except the short beam test and to some extent, the torsion test) require performing some kinematic measurements on the interlaminar plane or at least, need sufficiently thick specimens. For thin plates (like 1 or 2 mm thick), this may be very unpractical or altogether impossible for specimens like tubes. Moreover, all these usual test methods (except that in [10]) rely on strict specimen shape and loading arrangement for either an analytical or an

approximate solution of the associated mechanical problem.

The objective of this paper is to propose a novel method to measure the through-thickness shear moduli of composite plates from bending tests and surface full-field measurements of the three displacement components. The main advantage is that there is no need to measure strains on the through-thickness plane, which potentially enables to determine the TT shear modulus of rather thin laminates. Combined with the Virtual Fields Method [12], it also has the potential to provide an in-plane map of through-thickness shear moduli, though this will be very experimentally challenging.

## 2 Theoretical developments

A generic form of the displacement field in a moderately thick laminated plate can be written as:

$$\begin{cases} u(x, y, z) = -z \frac{\partial w(x, y)}{\partial x} + f(z) \gamma_x^0(x, y) \\ v(x, y, z) = -z \frac{\partial w(x, y)}{\partial y} + f(z) \gamma_y^0(x, y) \\ w(x, y, z) = w(x, y) \end{cases} \quad (1)$$

where  $u$ ,  $v$  and  $w$  are the components of the displacement field in the  $x$ ,  $y$  and  $z$  directions respectively ( $z$  is the thickness direction),  $f(z)$  is a continuous and differentiable function driving the through-thickness shear distribution and  $\gamma_x^0$  and  $\gamma_y^0$  the through-thickness shear strains on the mid-plane. With this formulation, it can be shown [1] that the average through-thickness shear strains can be expressed as functions of the top surface displacements by:

$$\begin{cases} \frac{1}{h} \int_{-h/2}^{h/2} 2\varepsilon_{xz}(x, y, z) dz = \frac{2}{h} u_s(x, y) + \frac{\partial w(x, y)}{\partial x} \\ \frac{1}{h} \int_{-h/2}^{h/2} 2\varepsilon_{yz}(x, y, z) dz = \frac{2}{h} v_s(x, y) + \frac{\partial w(x, y)}{\partial y} \end{cases} \quad (2)$$

This expression shows that the average TT shear strain can be interpreted as the distance between the surface displacement caused by the rotation and the actual surface displacement. For a Love-Kirchhoff theory, the right hand-side of the above equation is equal to zero.

The above expression was validated using finite element simulations. A simple cantilever beam (Fig. 1) was modeled using 3D bricks. A unidirectional carbon/epoxy material was considered

to enhance the effect of TT shear. The span length  $L$  drives the relative amount of TT  $xz$  shear strain with respect to bending strains. A gauge length of 20 mm was selected, for a thickness of 2 mm and a width of 14 mm, in order to enhance the effect of through-thickness shear. Fig. 2 shows the comparison between the through-thickness average of the through thickness  $xz$  shear strain calculated (a) from the  $z=h/2$  surface displacements with equation 2 and (b) directly from the element strains. The derivative of the deflection was obtained by simple point-to-point differentiation. The match between the two is obvious, confirming the validity of equations 2.

## 3 Experimental implementation

The next step is to validate this approach experimentally. For this purpose, speckle interferometry was selected to provide the 3 directional surface deformation measurements (Fig. 3). The cantilever configuration is shown in Fig. 1. A span of 20 mm was selected to promote TT shear.

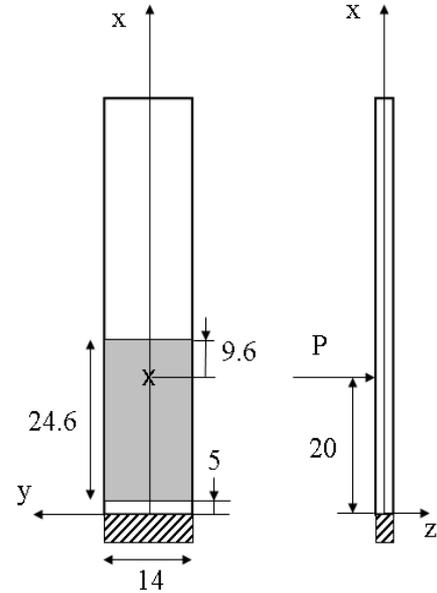


Fig.1. Short span cantilever bending.

170 images were taken to reach a global load of 237 N. This should give rise to an average  $xz$  shear strain of about 0.2 %. The 170 load steps were necessary because of the large difference between in-plane and out-of plane deformations, the later being nearly two orders of magnitude higher than

the former. The derivative of the deflection was calculated by point to point differentiation after a Gaussian smoothing over a 10 pixels kernel. However, the difficulty is that speckle Interferometry only provides relative deformation values. This is not a problem when strains are obtained by differentiation but in equations 2, the absolute u displacement has to be known. Here, a constant was added to the u field so that the average over the surface above the loading point of the xz shear strain in zero (zero shear force). This was the reason for extending the field of view to that region.

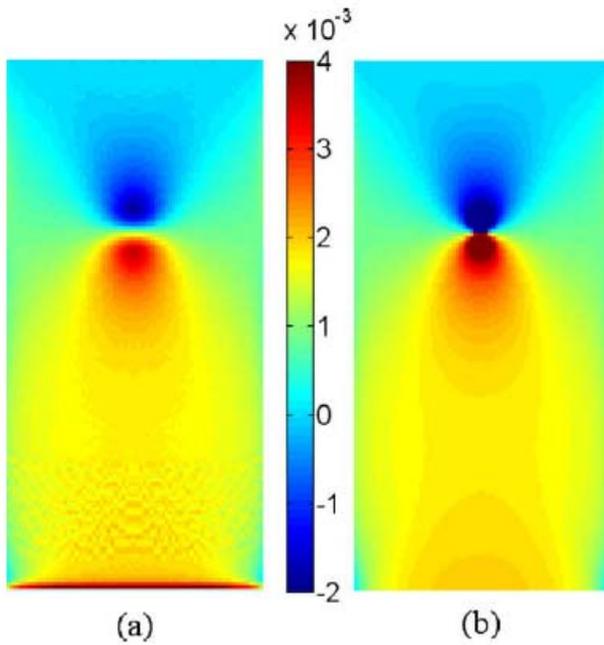


Fig.2. Comparison between the through-thickness average of the through thickness xz shear strain calculated (a) from the z=h/2 surface displacements and (b) directly from the element strains.

Finally, it was shown [1] that the change of sensitivity vector because if the beam bending had to be taken into account, as well as the initial position of the beam. This resulted in a constant average of the xz shear strain in each slice between load and clamp. Finally, from the experimental map in Figure 4, it is possible to identify the through-thickness shear modulus  $G_{xz}$  using the Virtual Fields Method [12]. Equation 3 is obtained using a constant through-thickness virtual shear strain field. It can also be obtained by writing the equilibrium of a cross-section between the clamp and loading point

and integration it between 0 and L.  $\overline{\epsilon_{xz}}$  stands for the spatial average of  $\epsilon_{xz}$  over the (x-y) area between loading point and clamp.

$$G_{xz} = \frac{P}{wt\overline{\epsilon_{xz}}} \quad (3)$$

A  $G_{xz}$  modulus value of 5.37 GPa was obtained with this test, which is a sensible value for this type of material. Fig. 4 shows the maps of xz shear strain from experiment and simulation, with very good match considering the very small strains involved.

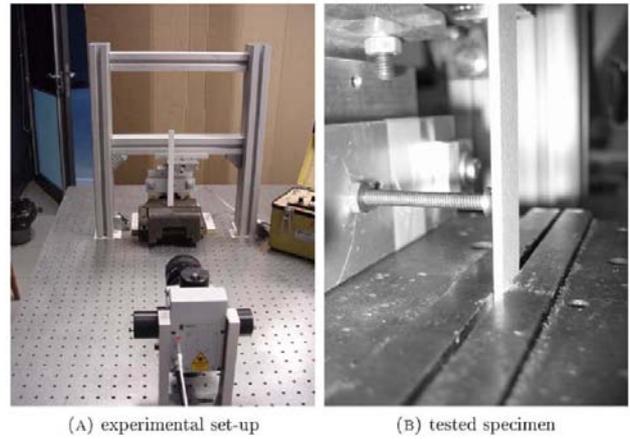


Figure 3 – Experimental set-up.

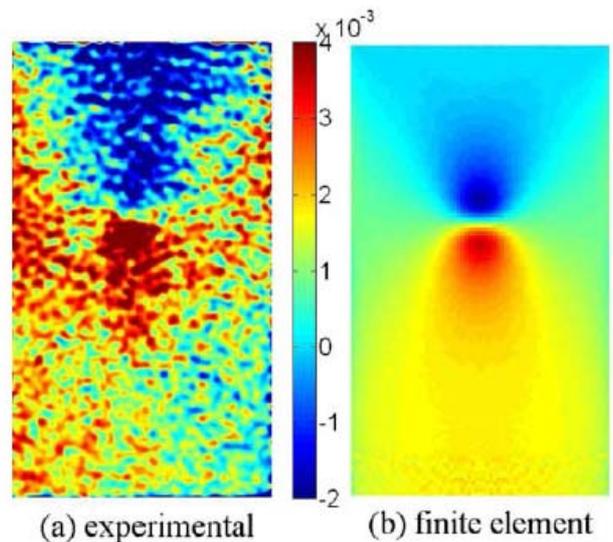


Figure 4 – Comparison of average xz shear strains components from experiment (Eq. 2) and simulation.

Finally, it is interesting to underline the sensitivity of this technique to small changes in the sensitivity

vector. It was previously stated that one had to take into account the change of sensitivity vector because of the deformation of the beam but also the initial position of the beam with respect to measurement plane. Figure 5 shows a plot of through-thickness shear strain on the central vertical line of the specimen. The line tagged 'corrected' corresponds to the data corrected by the change of sensitivity vector during the test but assuming perfect initial position. One can see that the experimental data does not fit the prediction. Because the shear force is constant in the gauge section, the left hand-side of the curve should be flat, as the FE model predicts. Introducing a small  $0.28^\circ$  initial misalignment provides a distribution that follows expectation while a  $-0.56^\circ$  does not. As shown in [1], a  $-0.28^\circ$  correction was used here to produce the data but in the future, a full validation will be sought by measuring the initial position of the specimen.

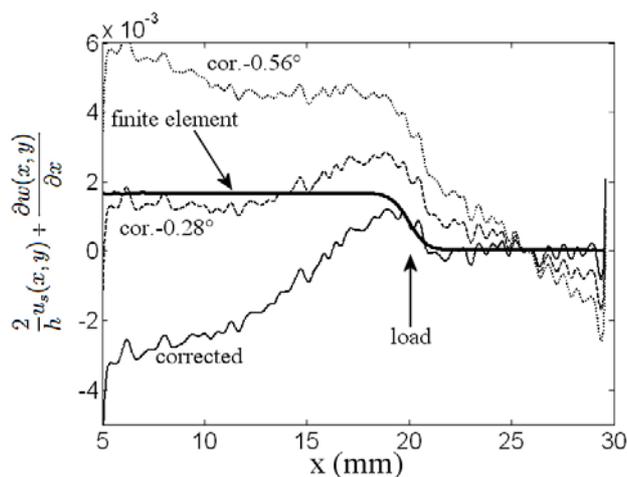


Figure 5 – Effect of the correction for initial misalignment on the distribution of TT shear (average through the width along the longitudinal axis).

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