# NONLINEAR BUCKLING OPTIMIZATION OF LAMINATED COMPOSITES INCLUDING "WORST" SHAPE IMPERFECTIONS

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## 1 Introduction

The design problem of maximizing the load capacity of compressively loaded laminated composite structures is challenging due to the complex structural performance of general purpose engineering structures. The laminated composites are typically thin-walled shell-like structures that are sensitive to geometric imperfections when loaded in compression. In this work focus is put on this design problem for general multi-material laminated composite structures using a gradient based optimization approach, and the formulation includes the determination of the "worst" shape imperfection.

Most structural imperfections are not known in advance. To include the imperfections in a structural analysis, they have to be assumed. A convenient way to include all relevant imperfections (i.e., geometrical, structural, material, or load related imperfections) is to represent them by equivalent geometrical imperfections. In this way the geometrical imperfections are augmented by the influence of other relevant imperfections to produce the same effect on the load carrying behavior of a structure.

The idea to find the "worst" possible geometric imperfection for a given structure is as old as the discovery of the important role of imperfections itself. In practice, it is common and often recommended in technical standards to consider the "worst" imperfection as that imperfection shape which is affine to the lowest bifurcation mode. Though, recent research, see e.g. [1], shows that a combination of a number of bifurcation modes or even a simple dimple imperfection in some cases proves to be a better prediction of the "worst" imperfection. In reality large uncertainties are related in the direct determination of the real imperfection shape and amplitude since it relies on data of measured imperfections.

In engineering, the concept of the ‘‘worst” imperfections is important, since it is defined as the imperfections that yield the lowest performance of the structure and thereby a lower bound for the performance measure. In recent years, the concept of the definitely ‘‘worst” imperfection has been introduced. Within the concept, the shape of the imperfections that would lead to the lowest critical load of the structure is searched. The shape of the imperfections is additionally bounded by the given imperfection amplitude, see e.g. the works [1-5].

In this work a gradient based optimization approach is outlined for determining the "worst" shape imperfection, and it is demonstrated how this is taken into account when designing multi-material laminated composite structures for maximum load capacity, see also [5] for a detailed description of the approach for fiber angle optimization of laminated composites.

## 2 Nonlinear Buckling Analysis and Design Sensitivity Analysis

The analysis and optimization procedure for nonlinear buckling load optimization described in [6] is applied, i.e. optimization w.r.t. stability is accomplished by including the nonlinear response by a path tracing analysis, after the arc-length method, using the Total Lagrangian formulation.

Structural stability/buckling is estimated in terms of geometrically nonlinear analyses and restricted to limit point instability, despite that the presented formulas also work well for bifurcation points. In addition, bifurcation instability is in many cases transformed into limit point instability with the introduction of small disturbances/imperfections to the system.

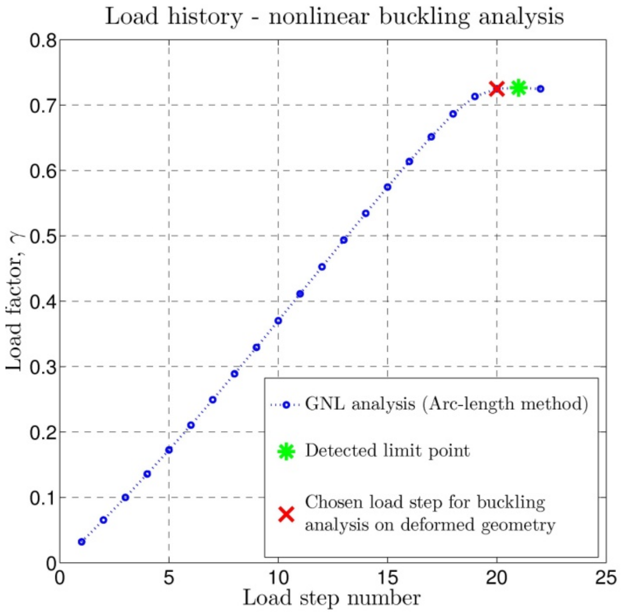


Fig. 1. Detection of limit load and chosen equilibrium point for the nonlinear buckling problem.

The proposed procedure for nonlinear buckling analysis, considering limit points, is illustrated by Fig. 1. The nonlinear path tracing analysis is stopped when a limit point is encountered and the critical load is approximated at a precritical load step, described by the load factor $\gamma^n$, by performing an eigenbuckling analysis on the deformed configuration by extrapolating the nonlinear tangent stiffness to the critical point.

Thus, the linearized eigenvalue problem on the deformed configuration is given as

$$\left(\mathbf{K}_0 + \mathbf{K}_\mathrm{L} + \lambda_\mathrm{j}\mathbf{K}_\sigma\right)\boldsymbol{\phi}_\mathrm{j} = \mathbf{0}, \quad \mathrm{j} = 1,2, \ldots \tag{1}$$

Here $\mathbf{K}_0$ is the global linear stiffness matrix, $\mathbf{K}_\mathrm{L}$ the global displacement vector, $\mathbf{K}_\sigma$ the global stress stiffness matrix, $\lambda_1$ the lowest eigenvalue, and $\phi_1$ the corresponding eigenvector. The critical load factor $\gamma_1^c$ is then given as the smallest value of

$$\gamma_\mathrm{j}^\mathrm{c} = \lambda_\mathrm{j}\gamma^n, \;\; \mathrm{j} = 1,2, \ldots \tag{2}$$

Design sensitivities of the critical load factor are obtained semi-analytically by the direct differentiation approach on the approximate eigenvalue problem described by discretized finite element equations. Details can be found in [6]. Adjoint DSA is currently being tested.

## 3 Parameterization of the Multi-Material Laminate Design Problem

The use of multi-material laminated composite structures for advanced load carrying applications is of growing interest due to the possibility of designing structures where the combination of high and low cost materials results in high performance structures, obtained at a reasonable cost. However, design of such structures is challenging due to the large design space. The candidate materials available for the multi-material design problem considered may be Glass or Carbon Fiber Reinforced Polymers (GFRP/CFRP) combined with lightweight materials such as foam materials or balsa wood that are typically used in sandwich structures. The materials are assumed to be distributed within a given number of layers of fixed thickness in the laminated plate/shell type of structure, and the aim is to determine the best candidate material in all layers everywhere in the design domain. The starting point of the design process is the definition of the fixed geometry of the laminated composite shell structure, i.e., shape design is not included in the approach developed as the outer shape very often is given by other considerations such as aerodynamic performance in the case of wind turbine blades. The fixed geometry, with a given number of layers, is given by a finite element discretized shell model, and the optimization approach developed is based on the use of gradient based methods where the optimization problems formulated are solved using mathematical programming.

The so-called Discrete Material Optimization (DMO) approach has been introduced by the authors [7-9] for multi-material topology design, and the basic idea is to use a parametrization that allows for efficient gradient based optimization of real-life problems while reducing the risk of obtaining a local optimum solution when solving the discrete multi-material distribution problem. The approach is related to the mixed materials strategy suggested by Sigmund and co-workers [10,11] for multi-phase topology optimization, where the total material stiffness is computed as a weighted sum of candidate materials. By introducing continuous weighting functions for the material interpolation, the discrete topology optimization problem is converted to a continuous problem that can be solved using standard gradient based optimization techniques. In the present context this means that the stiffness (or any other material property) of each layer of the laminated composite will be computed from a weighted sum of a finite number of candidate constitutive matrices. Consequently, the design variables are no longer the fiber angles or layer thicknesses but the scaling factors (or weight functions) on each constitutive matrix in the weighted sum. The purpose of the weighting functions is to obtain a distinct choice of material, i.e. at the end of the optimization the weight functions should be either 0 or 1 in order to obtain a distinct material design. Thus, continuous weight functions with penalization are used in order to penalize intermediate values, see [7-9].

As in topology optimization the parametrization of the DMO formulation is invoked at the finite element level but larger patches of elements may be associated with the same parametrization, allowing for practical design problems since laminates are typically made using fiber mats covering larger areas. In order to describe whether the optimization has converged to a satisfactory result, i.e. whether a single candidate material has been chosen in all layers of all design domains and all other materials have been discarded, a DMO convergence measure is defined, based on the convergence of the weight factors measured using the Euclidian norm.

## 3 Parameterization of Imperfections

The determination of the "worst" imperfection for a given structure is formulated as an optimization problem whereby imperfections are directly introduced in the analysis model, see also [5] where the approach is applied in connection with fiber angle optimization of composite structures. By including geometric imperfections, unstable bifurcation points, if present, are in general avoided and converted into limit points. The "worst" imperfection is in this study defined as the "worst" case imperfection shape for a structure, i.e. an imperfection shape which yields the lowest limit load. The imperfections are represented by a linear combination of base shapes $\boldsymbol{\Psi}_l, 1, .., N$, where the base shapes in this study are constructed from a number of buckling modes. The base shapes could also originate from measured imperfections, etc.

The geometry of the imperfect structures is thus described by finite element nodal point coordinates $\mathbf{X}$ as

$$\mathbf{X} = \mathbf{X}_\mathrm{p} + \bar{\mathbf{X}}, \text{ where } \bar{\mathbf{X}} = \sum_{\mathrm{l}=1}^{\mathrm{N}} \alpha_\mathrm{l} \boldsymbol{\Psi}_\mathrm{l} \tag{3}$$

$\mathbf{X}_\mathrm{p}$ contains coordinates for the initial perfect geometry, $\alpha_\mathrm{l}$ are the unknown shape parameters, $\boldsymbol{\Psi}_\mathrm{l}$ are the base shapes, and $\bar{\mathbf{X}}$ the total imperfection vector. The unknown shape parameters $\alpha_\mathrm{l}$ are obtained as solution to an optimization problem as described in the following.

The "worst" imperfection shape is sought within the method, defined by the shape bases $\boldsymbol{\Psi}_\mathrm{l}$ and the shape parameters $\boldsymbol{\alpha}$, at which the objective function in terms of the limit load will attain a minimum. During the optimization the imperfection amplitude is constrained and formulated as a simple set of linear constraints. The set of linear constraints for the maximum amplitude of the total imperfection vector can be stated as

$$|\bar{\mathbf{X}}^m| \leq e_0^m, \quad m \in [n_1, n_2, \ldots, n_{cp}] \tag{4}$$

where $n_i$ is the index of the ith constrained component of the total imperfection vector $\bar{\mathbf{X}}$, $n_{cp}$ is the total number of constrained components, and $e_0^m$ is the amplitude value of the mth constrained value. Different constraint values can be applied for different parts of the structure and e.g. set according to manufacturing tolerances for the structure.

The optimization starts either with the first base shape $\boldsymbol{\Psi}_1$, normalized by the allowable amplitude $e_0$, as the initial guess for the total imperfection vector or as an even averaging of all base shapes, $\boldsymbol{\Psi}$, such that at least one amplitude constraint is active.

## 4 Optimization Formulations

Two different optimization problems are solved in the proposed procedure for multi-material design of laminated composite structures for maximum load capacity, taking the "worst" shape imperfection into account. These two optimization problems are solved separately.

### 4.1 Laminate Optimization Problem

The mathematical programming problem for maximizing the lowest critical load is a max–min problem. The direct formulation of the optimization problem can give problems related to differentiability and fluctuations during the optimization process since the eigenvalues can change position, i.e. the second lowest eigenvalue can become the lowest. An elegant solution to this problem is to make use of the so-called bound formulation, see [12-14]. A new artificial variable $\beta$ is introduced and a new artificial objective function $\beta$ is chosen. An equivalent problem is formulated, where the previous non-differentiable objective function is transformed into a set of constraints.

The optimization formulation in the case of laminate optimization, for a max–min problem with the use of the bound approach, is formulated as follows

$$
\begin{aligned}
&\text{Objective: } \underset{\boldsymbol{x},\beta}{Maximize}\ \beta \\
&\text{Subject to: } \gamma_j^c \geq \beta\,, \quad j = 1,2,\dots, N_\lambda \\
&\qquad (\mathbf{K}_0 + \mathbf{K}_L + \lambda_j \mathbf{K}_\sigma)\boldsymbol{\phi}_j = \mathbf{0} \\
&\qquad \gamma_j^c = \lambda_j \gamma^n, \qquad j = 1,2,\dots \\
&\qquad \mathrm{M} \leq \overline{\mathrm{M}} \\
&\qquad (\mathrm{C} \leq \overline{\mathrm{C}}) \\
&\qquad 0 < \underline{x_i} \leq x_i \leq \overline{x_i} < 1, \quad i = 1,..,I
\end{aligned}
\tag{5}
$$

The laminate design variables are stored in the vector $\boldsymbol{x}$, and the mass M must be below a prescribed value $\overline{\mathrm{M}}$. In several situations the end compliance C is also limited by an upper value $\overline{\mathrm{C}}$ in order to obtain a structure with sufficient stiffness. The mathematical programming problem is solved by the Method of Moving Asymptotes (MMA) [15]. The closed loop of analysis, design sensitivity analysis and optimization is repeated until convergence in the design variables or until the maximum number of allowable iterations have been reached.

The laminate optimization problem is solved for a fixed, imperfect geometry described by (3), i.e. for fixed shape parameters $\alpha_l$.

### 4.2 "Worst" Imperfection Formulation

The unknown shape parameters $\alpha_l$ are obtained as solution to an optimization problem where the lowest critical load factor is minimized in order to determine the "worst" imperfection shape. When solving a min-min problem there is no need for a bound formulation or similar techniques. The following optimization problem is solved:

$$
\begin{aligned}
&\text{Objective: } \underset{\boldsymbol{\alpha}}{Minimize}\ \min \gamma_j^c, \quad j = 1,2,\dots \\
&\text{Subject to: } (\mathbf{K}_0 + \mathbf{K}_L + \lambda_j \mathbf{K}_\sigma)\boldsymbol{\phi}_j = \mathbf{0} \\
&\qquad \gamma_j^c = \lambda_j \gamma^n, \qquad j = 1,2,\dots \\
&\qquad |\overline{\mathbf{X}}^m| \leq e_0^m \\
&\qquad \overline{\mathbf{X}} = \textstyle\sum_{l=1}^{N} \alpha_l \boldsymbol{\Psi}_l \\
&\qquad \underline{\alpha_l} \leq \alpha_l \leq \overline{\alpha_l}, \qquad l = 1,..,N
\end{aligned}
\tag{6}
$$

This problem is solved to full convergence for fixed laminate design variables $\boldsymbol{x}$. Thus, the design optimization approach iterates back and forth between the two different optimization problems, keeping the design variables of the other problem fixed while solving the current problem.

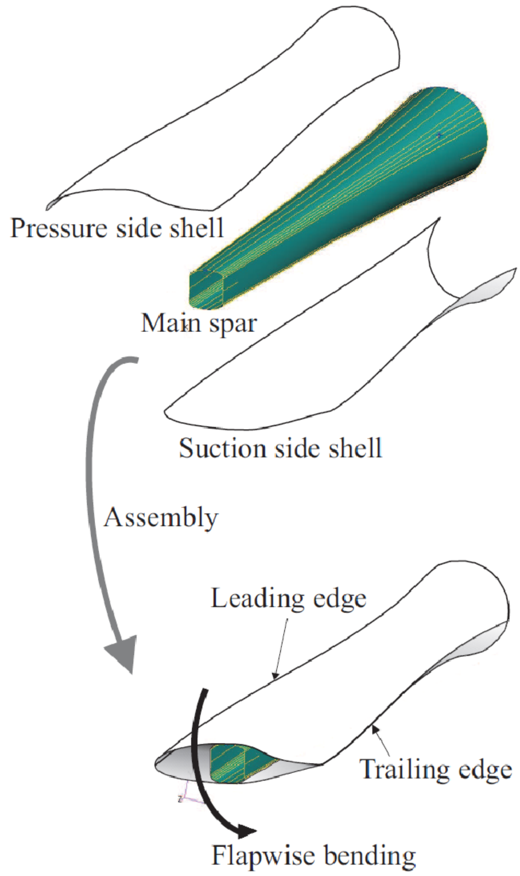


Fig. 2. Main spar from wind turbine blade (courtesy of Lennart Kühlmeier [16]).

## 5 Main Spar Example

The optimization approach has been demonstrated on several engineering examples including nonlinear buckling load optimization of parts of wind turbine blades. One example considered is a generic main spar as illustrated on Fig. 2. When subjected to the maximum flapwise bending load case it may fail due to local buckling as illustrated on Fig. 3.

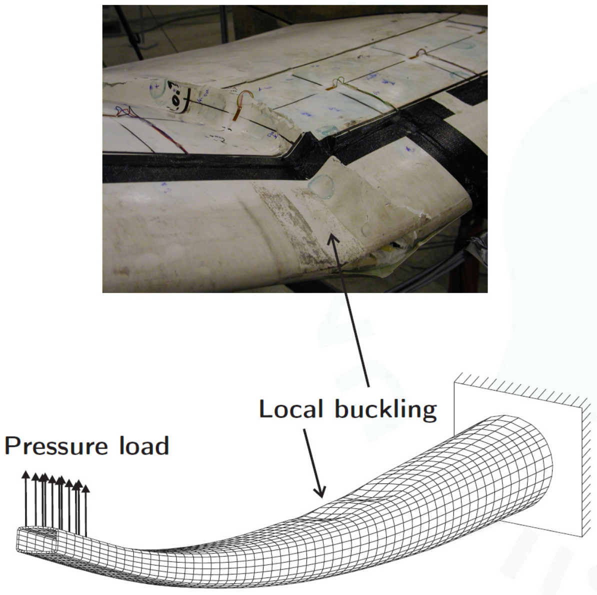


Fig. 3. Local buckling of main spar.

The midsection of the main spar is divided into 16 patches as illustrated on Fig. 4 together with the cross sections defining the generic main spar. 14 m of the main spar is included in the model where 9 node isoparametric shell finite elements with large displacement and rotation capabilities are used. Each patch consists of 10 layers of equal thickness. The candidate materials are GFRP UD oriented at $0^0$, $45^0$, $-45^0$, and $90^0$, GFRP $\pm 45^0$ and $0^0/90^0$ combi mats and foam material. The laminate design problem is thus a multi-material design problem.

The objective is to maximize the limit load factor, while taking constraints on compliance and mass into account. The constraint on the mass is set such that 20% of the mid section must be filled with foam material. The compliance constraint allows for a maximum tip displacement of 4.25 m.

For the initial design, the material parameters are determined using equal weighting of all candidate materials in the DMO parameterization.

The limit load factor for the perfect initial geometry is 1.98. If the lowest buckling mode $\boldsymbol{\phi}_1$, normalized to the allowable amplitude $e_0$, is taken as the initial guess for the total imperfection vector, then the limit load factor is reduced to 1.90. Here the maximum imperfection amplitude in the analysis and optimization problem is taken as $e_0 = 0.002$ m which is 10% of the laminate thickness at the mid section.

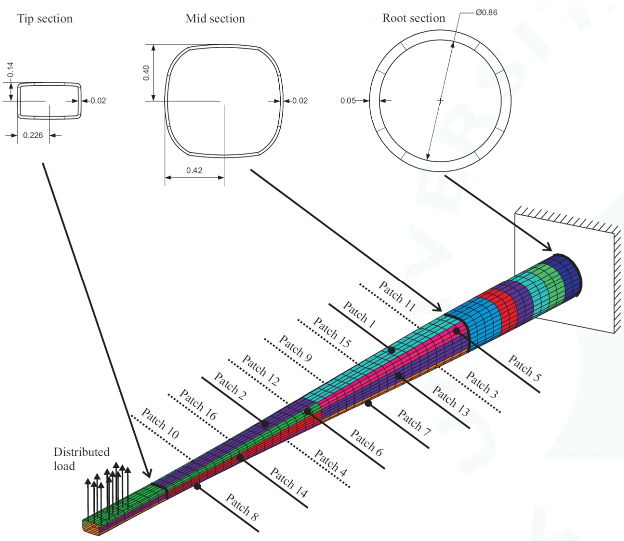


Fig. 4. Parameterization of main spar.

In the first iteration, the limit load factor is reduced from 1.98 to 1.60 by determining the "worst" shape imperfection for fixed laminate variables. The 50 lowest nonlinear buckling modes obtained when solving Eq. (1) for the initial design are taken as base shapes for the parameterization given by Eq. (3). The "worst" shape imperfection is seen on Fig. 5. It is interesting to note for this example that the scaling factor $\alpha_1$ on buckling mode 1 is nearly zero when solving for the "worst" imperfection shape. Thus, the common approach of considering the "worst" imperfection shape as a scaling of the lowest bifurcation mode would not give the most critical shape in this case. This is also seen from the estimated limit point which is 1.90 when using mode 1 as imperfection shape whereas the optimization approach finds a "worst" imperfection shape that reduces the limit load factor to 1.60.

Next the limit load factor is increased from 1.60 to 3.16 by laminate optimization for fixed imperfect geometry while fulfilling mass and compliance constraints for the problem. This optimization process of switching between the two subproblems can be continued, and in general convergence is reached within 3-4 global iterations, see other examples in [5]. In the third optimization iteration where an updated “worst” shape imperfection is determined, the limit load factor is only reduced by less than 2% for this example, and the laminate design of iteration four is similar to the DMO design obtained in iteration two.

This procedure leads to robust designs, i.e. optimal laminate designs that are least sensitive to geometric imperfections. Furthermore, the method of “worst” shape imperfections leads to more conservative estimates than limit loads determined otherwise. The example illustrates the importance of taking imperfection into account in the design process.

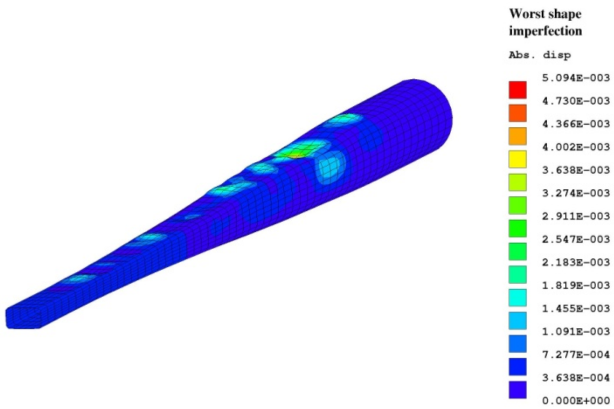


Fig. 5. “Worst” shape imperfections (scaled with a factor of 25) for the initial design of the main spar.

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