ANALYTICAL MODEL FOR THE PREDICTION OF THE FRACTURE TOUGHNESS OF MULTIDIRECTIONAL LAMINATES

P.P. Camanho[1,*], G. Catalanotti[2]
[1] DEMec, Faculdade de Engenharia, Universidade do Porto, Portugal
* Corresponding author(pcamanho@fe.up.pt)

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1 Introduction

Notwithstanding the recent developments in damage models developed at the ply level for the prediction of failure initiation and structural collapse of laminated polymer composites reinforced by unidirectional fibres [1]-[3], the use of strength prediction models developed at the laminate level is widespread in the aerospace industry.

The strength prediction of notched composite laminates is typically performed using either the point-stress or the average-stress models proposed by Whitney and Nuismer [4]. The point-stress model considers that ultimate laminate failure occurs when the stress at a given distance from the hole boundary (the characteristic distance, r_{ot}) reaches the unnotched strength of the laminate, X_T.

The fact that the point-stress model is based on a linear-elastic stress analysis using only the ply elastic properties, the laminate unnotched strength, and the characteristic distance indicates that the strength prediction can be performed in a short amount of time, requiring a small effort to characterize the material. However, the characteristic distance is not a material property: it needs to be calculated every time the lay-up or the geometry are modified [5].

These facts prompted the development of cohesive zone models for the strength prediction of notched composites. Backlund [6] developed cohesive zone models for the strength prediction of composite laminates loaded in tension containing notches and cracks. The several damage mechanisms that occur at the vicinity of a crack or hole (e.g. fibre-matrix splitting, fibre fracture) are lumped into a damage zone, where a linear relation between the cohesive traction and the crack opening is assumed. The strength prediction method is therefore based on two material properties: the unnotched tensile strength of the laminate, X_T, which defines the onset of damage, and the laminate fracture toughness, $G_{Ic}(L)$.

The methods based on cohesive zone models have a great potential to replace semi-empirical methods in the design of notched composite laminates because there is no need to measure the model parameters every time the geometry changes. However, a difficulty remains: the measurement of the fracture toughness, usually performed using a center cracked specimen, is required for every lay-up used. This fact is a clear drawback in notched composite structures subjected to multi-axial loading where the simulation of cohesive cracks may be required at different angular positions along the hole boundary.

Therefore, the objective of this paper is to define a methodology to calculate the mode I fracture toughness of a multidirectional laminate, $G_{Ic}(L)$, from the fracture toughness of the 0 plies, $G_{Ic}(0)$.

2 Analytical Model

Consider a symmetric, balanced laminate containing a crack and subjected to a remote tensile stress in the x direction as shown in Figure 1.

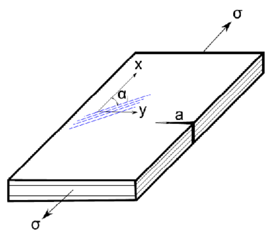

Fig.1. Cracked laminate under tensile loading.

Following the concept introduced by Vaidya and Sun [7] take $\Omega_0^{(i)}$ as the ratio between the mean remote failure stress of a group of plies that represent the balanced sub-laminate (i) and the remote failure stress of the 0 sub-laminate (i.e. a sublaminate with all plies with the fibres aligned with the loading direction), that is:

$$\Omega_0^{(i)} = \frac{\bar{\sigma}^{(i)}}{\bar{\sigma}^0} \quad (1)$$

$\Omega_0^{(i)}$ is easily calculated from lamination theory: it only depends on the elastic properties of the plies and on the lay-up of the laminate.

The mean failure remote stress of the balanced sublaminate (i), $\overline{\sigma}^{(i)}$ and that of the 0 plies, $\overline{\sigma}^{(0)}$, can also be calculated from Linear-Elastic Fracture Mechanics as functions of the corresponding fracture toughness and of the orthotropy correction factors introduced by Bao [8], χ, as:

$$\bar{\sigma}^{(i)} = \frac{\mathcal{K}_{Ic}^{(i)}}{\chi^{(i)} Y \sqrt{\pi a}} \quad (2)$$

$$\bar{\sigma}^0 = \frac{\mathcal{K}_{Ic}^0}{\chi^0 Y \sqrt{\pi a}} \quad (3)$$

Using equations (2)-(3) in (1):

$$\mathcal{K}_{Ic}^{(i)} = \frac{\chi^{(i)} \Omega_0^{(i)}}{\chi^0} \mathcal{K}_{Ic}^0 \quad (4)$$

The previous equation relates the fracture toughness of a balanced sub-laminate (i) to that of the 0 plies using the ply elastic properties and the lay-up of the laminate. Therefore, the fracture toughness of the sub-laminate (i), represented by the energy dissipated per unit area by the sub-laminate (i), is given as:

$$\mathcal{G}_{Ic}^{(i)} = \frac{\left[\mathcal{K}_{Ic}^{(i)}\right]^2}{E_{eq}^{(i)}} \quad (5)$$

Having defined the fracture toughness of the sub-laminate (i) according to the previous equations, and knowing the fracture toughness of the 0 plies, the laminate fracture toughness is calculated assuming self-similar crack propagation along all the plies of the laminate as:

$$\mathcal{G}_{Ic}^{L} = \frac{\sum_{(i)}^{N} \mathcal{G}_{Ic}^{(i)} t^{(i)}}{t^L}, \ (i) \neq 90^\circ \quad (6)$$

where $t^{(i)}$ is the thickness of sublaminate (i), N is the number of sub-laminates considered, and t^L is the total thickness of the laminate. A remarkable characteristic of this model is that it predicts the fracture toughness of a general fibre dominated laminate using only the ply elastic properties and the fracture toughness of the 0 plies.

3. Experiments and model validation

The material selected for the experiments is the Hexcel IM7-8552 carbon epoxy unidirectional laminate supplied as a pre-impregnated tape.

The fracture toughness of the individual 0 plies was measured using compact tension (CT) tests on cross-ply laminates [9]. The tests were performed using the $[0/90]_{8s}$ lay-up proposed by Pinho [10] and the mean value of the fracture toughness measured for the 0 plies is $134.7 kJ/m^2$. This value will be used to predict the fracture toughness laminates with two different lay-ups: $[90/0/\pm 45]_{3s}$ (nominal thickness of 3mm) and $[0/\pm 60/\pm 80]_{2s}$ (nominal thickness of 2.5mm).

Center cracked specimens are typically used to measure the laminate fracture toughness required for the strength prediction of open-hole laminates loaded in both tension and compression. The center cracked specimens are used in this work to verify whether the model presented in the previous section is able to predict the laminate fracture toughness from that of the 0 ply.

The centre-cracked specimens were loaded in tension up to failure, at a loading rate of 2mm/min. The load-displacement relations were approximately linear up to final failure. Figure 2 shows the load-displacement relation measured in the $[90/0/\pm45]_{3s}$ specimen.

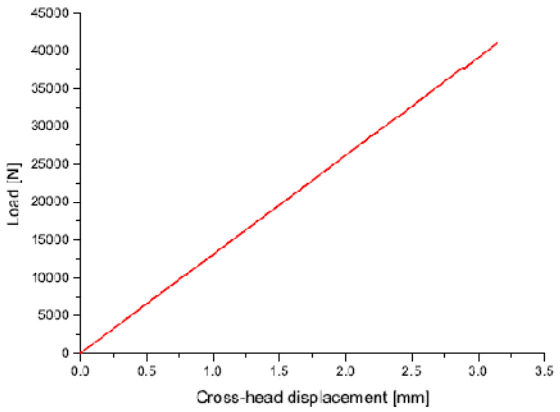

Fig.2. Load-displacement relation - $[90/0/\pm45]_{3s}$ specimen.

Figure 2 shows one of the $[90/0/\pm45]_{3s}$ failed test specimens, where a net-section tensile failure is observed.

Fig.2. $[90/0/\pm45]_{3s}$ failed test specimen.

From the peak load measured for each laminate, and using Linear-Elastic Fracture Mechanics, it is possible to measure the fracture toughness for each laminate and compare it with the values predicted using the fracture toughness of the 0 plies and equations (1)-(6).

Table 1 compares the experimentally measured fracture toughness of the two laminates with the analytical predictions.

	Experimental $K_{Ic(L)}$ $MPa\sqrt{m}$	Analytical $K_{Ic(L)}$ $MPa\sqrt{m}$	Error (%)
$[90/0/\pm45]_{3s}$	45.1	48.5	+8
$[0/\pm60/\pm80]_{2s}$	35.9	36.5	+2

Table 1: Comparison between analytical model and experiments.

The results shown in Table 1 indicate that the model proposed accurately predicts the fracture toughness of multidirectional laminates from the fracture toughness of the 0 ply.

4. Conclusions

From the comparison between the model predictions and experimental data, obtained in center-cracked specimens, it is concluded that the model predictions are accurate, especially considering its simplicity and ease of use.

Therefore, the model is useful to generate the material property required for the strength prediction of multidirectional laminates under multi-axial loading using cohesive zone models. In fact, using the model proposed here there is no need to perform tests to measure the fracture toughness of different lay-ups: the fracture toughness of the 0 ply and the ply elastic constants are the only material properties required.

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