ENFORSING A SYSTEM APPROACH TO COMPOSITE FAILURE CRITERIA FOR RELIABILITY ANALYSIS

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1 Summary

Composite failure criteria have found widespread use in research and industry. In the vast majority of applications the material properties and the stresses, which serve as inputs to the criteria, are defined deterministically. However, when the reliability of composite structures is sought the input to the failure criterion will be random quantities. The reliability is efficiently identified using approximate methods such as First Order Reliability Methods (FORM) [1,2]. FORM involves an iterative optimization procedure to obtain a reliability estimate, which imposes a number of additional challenges with the use of failure criteria, since composite materials are a discontinuous medium, which invoke multiple failure modes.

Under deterministic conditions the material properties and the stress vector are constant and will result in a single dominating failure mode. When any of these input parameters are random, multiple failure modes may be identified which will jeopardize the FORM analysis and a system approach should be applied to assure a correct analysis. Although crude Monte Carlo simulation automatically may account for such effects, time constraints limit its useability in problems involving advanced FEM models. When applying more computationally efficient methods based on FORM/SORM it is important to carefully account for the multiple failure modes described by the failure criterion.

The present paper discusses how to handle this problem and presents examples where reliability assessment of ultimate failure of fiber-reinforced composites is carried out using three different failure criteria.

2 Introduction

Laminated composite structures may exhibit a number of underlying failure modes, while the failure mode which actually occurs is determined by the variation of material properties, layer orientations and loading state. These failure modes are reflected in the composite failure criteria typically used to assess strength of composite structures.

Under deterministic conditions all design input properties have a constant value, which results in a single dominating failure mode. When the input parameters are random, however, different failure modes might become dominating. In a reliability analysis these failure modes may be interpreted as separate limit states, each of them contributing to the total probability of failure of the structure.

Having multiple limit states poses a problem for reliability analyses of the type FORM/SORM (see [1,2]) because the objective of such an analysis is to obtain a failure probability estimate by finding the unique most likely failure point of a limit state function, while considering the limit state to be a straight line (FORM) or a parabola (SORM). When having multiple failure modes, the failure surface cannot be approximated by a first- or a second-degree polynomial, meaning that the estimated probability of failure might not be correct (see Figure 1, where the grey-hatched area shows the failure probability mass which a FORM analysis would not consider as part of the failure domain). To remedy this problem a system approach must be applied, where each of the failure modes are considered a component of a system.

By grouping the composite failure modes according to the geometry level on which they occur, three different levels of system behaviour can be identified:

- Multiple failure modes on lamina level: fiber failure, shear failure, matrix failure.
- Multiple failure modes on laminate level: for a multidirectional laminate a first-ply failure can occur in any of the layers (or between the layers in case of

interlaminar failure) depending on material strength, layer orientation and stress distribution.

- Multiple failure modes on structure level: failure can occur at different locations in the structure.

System behaviour on all three levels is determined by the same input parameters - material properties, loading conditions and geometry, and can therefore be approached in a similar way. The present paper is focused on the system behaviour on lamina level, which was chosen due to simplicity and the smaller amount of computational efforts required.

3 Analysis description

As an initial test case, a single ply of carbon-epoxy composite with 45° orientation is subjected to compression. In the material direction this results in both shear and normal stress, which makes the load case suitable for testing multiple failure modes.

Three failure criteria are chosen to represent different levels of complexity and physical basis: Tsai-Wu [3], Maximum Strain, and Hashin [4]. The Max-strain criterion includes five non-correlated in-plane failure modes: matrix tension, matrix compression, fiber tension, fiber compression and shear failure. The Hashin failure criterion has four of the above failure modes, and there is no independent shear failure mode, however matrix compression and fiber compression modes can be shear dominated. The Tsai-Wu criterion is a fully-interactive failure criterion, where all strength parameters are combined in a single polynomial, resulting in a single failure mode.

Failure criteria values are obtained by applying a given compressive strain to the structure and determining the in-plane stresses using Classical Lamination Theory. Then the ultimate strength of the structure is found by determining the strain level at which the current failure criterion will equal 1, indicating failure. For the Max-strain criterion this is done in a single step, because the relation between the strain and the failure criterion value is linear. For the Tsai-Wu and Hashin criteria, where the relation is quadratic, the ultimate strain to failure is found iteratively.

The performance of the Tsai-Wu criterion is strongly influenced by the stress interaction factor *F12** (see [3] for definition of the stress interaction factor). Values of *F12** that differ substantially from zero result in higher failure loads in biaxial loading. However, in order for the Tsai-Wu criterion to produce results similar to the other two criteria, the value of *F12** is chosen equal to zero.

Reliability analysis is carried out using First-Order Reliability Method (FORM), which is supplemented by importance sampling and crude Monte Carlo simulations (see [5] and [6] for description of methods). Importance sampling simulations are done by using the design point obtained from FORM analysis as a sample center point, and with a sample size of 5000. Monte Carlo simulations are done with a sample size of $5 \cdot 10^6$ samples.

Table 1 lists the input parameters with their stochastic variation. The mean values for the stiffness and strength properties represent some typical values for carbon fiber composites, while the coefficients of variation are chosen by engineering judgement. All parameter values are chosen in a way that they well illustrate the problem which the authors are trying to present. Using different input values would certainly change the results of the analysis, however the principles described here will still be valid.

The material properties cannot be treated as independent values as they are often highly correlated. The correlation between input parameters is given by the correlation matrix, shown in Table 2. The values in the table are taken from [7].

Reliability analyses using different failure criteria result in different reliability estimates as seen from the data shown in the next section of this paper. This is due to the differences in the failure criteria formulations, and not due to the reliability analysis methods used. Therefore a comparison of the reliability estimates is meaningful only when done between reliability indices calculated with the same failure criterion, and not between calculations with different failure criteria.

4 Results from component analysis

4.1 Multiple failure-mode criteria

In order to be able to illustrate the influence of multiple failure modes on the reliability analysis, first a set of analyses where all failure modes are activated is carried out. Then component reliability analyses where only one failure mode is activated at a time are carried out. This set of analyses is applied to the two failure criteria with multiple failure modes – Max-strain and Hashin. The values which are compared are the reliability indices obtained from the reliability analyses. Each calculation is repeated with the three different reliability methods mentioned above - FORM, importance sampling and crude Monte Carlo. The results when using the Max-strain criterion are the following (β_{FORM}, β_{IS} and β_{MC} are the reliability indices obtained from FORM, importance sampling and Monte Carlo analyses respectively):

Analysis	All modes	Fiber	Shear	Matrix
β_{FORM}	3.17	3.17	3.38	4.69
β_{IS}	3.08	3.17	3.37	4.69
β_{MC}	3.06	3.18	3.40	-

Due to the very small probability of failure it was not possible to obtain a reliable result from Monte Carlo simulation for the matrix failure mode (a sample size of about $100 \cdot 10^8$ is required for obtaining a converged Monte Carlo solution at that reliability level).

The same set of results for the Hashin criterion is given below (for this criterion there is no pure matrix or shear failure mode as described earlier):

Analysis	All modes	Fiber	Shear/Matrix
β_{FORM}	3.03	3.03	3.47
β_{IS}	2.99	3.02	3.47
β_{MC}	2.97	3.03	3.47

From these results it is visible that for the component reliability analyses of single failure modes all three analysis methods agree very well, while when all failure modes are included the results do not match that closely. The reason for this mismatch is the fact that despite having multiple failure modes, FORM analysis would converge to a design point laying on the closest limit state surface, which represents a single failure mode, and the estimated probability of failure will reflect only this failure mode. This explains the exact match between the results of FORM analysis with all failure modes included and the results with the fiber failure mode - it means that the FORM analysis with all failure modes actually still represents only the fiber failure mode. On the other hand, importance sampling and Monte Carlo are both simulation methods where the sample is randomly chosen, therefore not limited to representing a single failure mode. This leads to multiple failure modes occurring within the samples chosen for the presented analysis. However, the results given by the importance sampling cannot be considered accurate, because the sampling density for this analysis is centered at a design point obtained from FORM, thus it is expected that the sample is strongly biased towards one of the failure modes. Therefore from the three analysis methods only the Monte Carlo analysis is expected to show a realistic estimate of the failure probability and the reliability index. Figure 2 illustrates the presence of two distinct failure modes occurring in a Monte Carlo simulation.

4.2 Tsai-Wu criterion

Tsai-Wu criterion differs from the first two criteria considered, as it does not specify multiple failure modes. The absence of multiple failure modes means that there will not be any discrepancy between results obtained from FORM analysis and Monte Carlo simulation (the small differences seen in the table below can be attributed to the curvature in the limit state surface which the FORM method cannot capture). However, the fact that the material failure is described by a single polynomial including all material strength properties leads to another issue: all strength variables will have an influence on the failure criterion/failure envelope, regardless of the stress state. As an example, changing the fiber compressive strength will also lead to change in the failure surface in biaxial tension (see Figure 3).

For the problem discussed in this paper this behaviour of the Tsai-Wu criterion leads to the tensile strength in matrix and fiber direction also influencing the reliability of the structure, despite the fact that the structure is loaded in compression. Removing the fiber tensile and matrix tensile strength from the set of stochastic variables resulted in a change of the reliability estimate:

Analysis	All variables	Tensile strength excl.
β_{FORM}	2.13	1.96
β_{IS}	2.09	1.93
β_{MC}	2.10	1.93

It is therefore advised that when using the Tsai-Wu criterion within the framework of reliability analysis careful consideration should be taken on how the specifics of this failure criterion influence the results of the analysis.

5 System Analysis

The results from the reliability analyses described previously in this paper show that using the First-Order Reliability Method to carry out reliability analysis will not give accurate results when failure is determined by failure criteria with multiple failure modes. It is however still very desirable from computational efficiency point of view to use iterative reliability methods such as FORM/SORM, as these methods require much less number of function calls compared to a crude Monte Carlo simulation. A way to achieve an accurate solution using FORM/SORM methods is to apply system analysis, where each failure mode is considered a component in a series system.

The probability of failure of a series-connected system (large intersection) is given by the union of the failure probabilities of the two components (see [1]):

$$1-p = P\left[\min_{k=1}^{m}\{M_k\} > 0\right] = P\left[\bigcup_{k=1}^{m}(M_k) > 0\right] \qquad (1)$$

This union probability can be approximated by using the results from a component-wise FORM analysis, where each of the limit state surfaces is evaluated independently. The probability of the union of the components is approximated by:

$$p \approx p^{FORM} = 1-\Phi_{\kappa}(\beta, \mathbf{R}) \qquad (2)$$

Where

$\beta = [\beta_1, \beta_2, \ldots, \beta_N]^T$ is the vector with the component reliability indexes

R is the component correlation matrix

Φ_{κ} is the multivariate normal distribution in standard normalised space.

The correlation coefficients are derived from the component **α**-vectors (line slopes indicating sensitivity) derived from the FORM component reliability analyses:

$$r_{ij} = \boldsymbol{\alpha}_\mathbf{i}^{\mathbf{T}}\boldsymbol{\alpha}_j \qquad (3)$$

Using the formulas above it is determined that the correlation coefficient for the problem using the Max-strain criterion equals 0.568, and for Hashin 0.604. To complete the calculation of the system reliability, the multidimensional normal integral must be evaluated. Due to the low number of degrees of freedom the computational requirements for determining multidimensional normal integral are not high, and it is therefore chosen that the multi-normal integral is evaluated by direct numerical integration.

The system reliability estimates are compared to the reliability estimates from the Monte Carlo analysis:

	Fiber fail.	Shear/matrix fail.	System analysis	Monte Carlo
Max strain	3.17	3.38	3.07	3.06
Hashin	3.03	3.47	2.98	2.97

Reliability estimates from the system analysis are in good agreement with the Monte Carlo simulation results. It is therefore possible to determine the reliability of a composite structure with multiple failure modes in a computationally efficient way by carrying out component analysis for each failure mode using FORM. These component analyses are then combined into a system analysis to obtain the total system reliability.

6 Conclusions

The anisotropic material behaviour and the presence of multiple failure modes impose challenges to applying reliability analysis to composite materials. When failure is assessed using a multi-modal failure criterion it is not possible to obtain a correct probability estimate using a single FORM analysis. A correct solution is obtained by splitting the failure modes into separate component analyses for each failure mode, followed by a system reliability analysis. Using a single-mode interactive failure criterion such as Tsai-Wu does not lead to the same situation, however due to the interactive nature of the criterion the reliability estimate is influenced by all strength parameters in all directions, regardless of the stress state.

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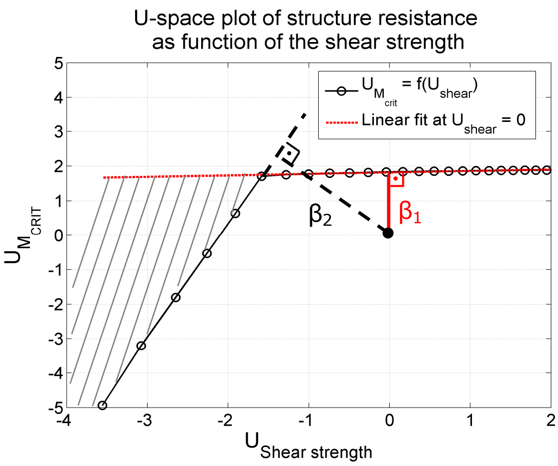

Fig.1. Shifting failure modes with Hashin criterion

Table 1. Stochastic input parameters

Parameter	Unit	Mean	CoV
Strain (load)	-	0.012	0.15
E_1	GPa	126	0.1
E_2	GPa	11	0.1
G_{12}	GPa	6.6	0.1
ν_{12}	-	0.28	0.1
$\varepsilon_{1c,\ crit}$	-	0.0145	0.15
$\varepsilon_{1t,\ crit}$	-	0.016	0.15
$\varepsilon_{2c,\ crit}$	-	0.02	0.15
$\varepsilon_{2t,\ crit}$	-	0.0045	0.15
$\gamma_{12,\ crit}$	-	0.019	0.15

Table 2. Correlation between parameters

ρ	Load	E_1	E_2	G_{12}	ν_{12}	ε_{1c}	ε_{1t}	ε_{2c}	ε_{2t}	γ_{12}
Load	1	0	0	0	0	0	0	0	0	0
E_1	0	1	0.8	0.8	0	0.6	0.6	0.2	0	0.2
E_2	0	0.8	1	0.8	0	0.6	0.6	0.2	0	0.2
G_{12}	0	0.8	0.8	1	0	0.6	0.6	0.2	0	0.2
ν_{12}	0	0	0	0	1	0	0	0	0	0
ε_{1c}	0	0.6	0.6	0.6	0	1	0.8	0.2	0	0.2
ε_{1t}	0	0.6	0.6	0.6	0	0.8	1	0.2	0	0.2
ε_{2c}	0	0.2	0.2	0.2	0	0.2	0.2	1	0.8	0.8
ε_{2t}	0	0	0	0	0	0	0	0.8	1	0.8
γ_{12}	0	0.2	0.2	0.2	0	0.2	0.2	0.8	0.8	1

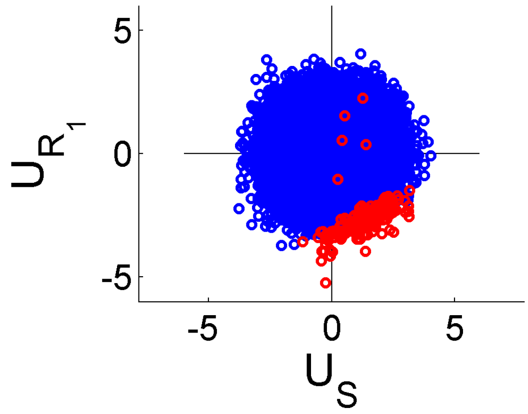

Fig.2. Multiple failure modes in a Monte Carlo simulation using max strain criterion.

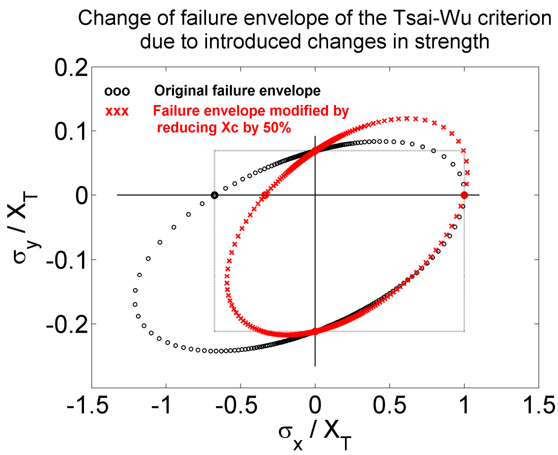

Fig.3. Influence of strength parameters on Tsai-Wu failure envelope.