A PROBABILISTIC APPROACH TO ACCOUNT FOR THE "WEAR-IN" SCATTER OF A HYBRID COMPOSITE MATERIAL

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Keywords: *Monte Carlo (MC) Simulation, Finite Element (FE) Method, PMC*

1 General Introduction

Recently, transmitting greater amounts of electric energy has become a global issue that has arisen due to the fact that current transmission line designs (based upon steel cores) are thermally limited, which restricts the amount of power that can be delivered. Utilities are faced with the difficult task of increasing power delivery under severe constraints [1]. To solve this problem the use of a hybrid polymer matrix composite (PMC) material has been proposed as the load bearing member of a new higher temperature, more efficient design.

The Aluminum Conducting Composite Core Trapezoidal Wire (ACCC/TW™) design is based upon unidirectional high strength carbon fibers reinforcing a high temperature epoxy (CFC). Electrically isolating the carbon fiber composite from the conductive aluminum strands is a sheath of unidirectional Electric Corrosion Resistant (ECR) glass fibers embedded in the same high temperature epoxy (GFC) (Fig. 1).

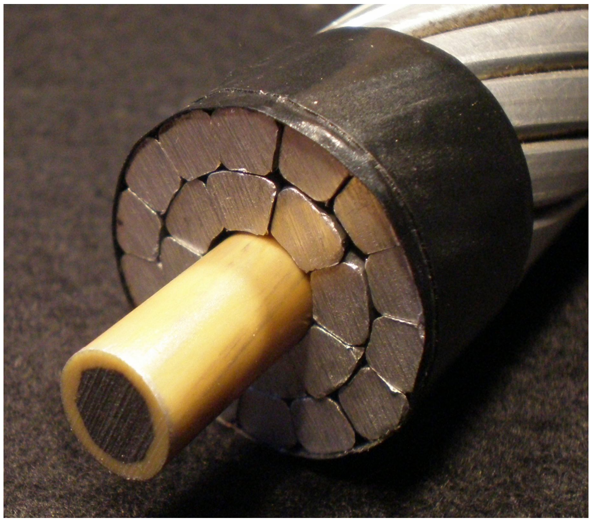

Fig. 1. ACCC/TW™ design.

The advantages of the ACCC design have been well documented in the literature [2-5]. However, for the successful long-term implementation of the ACCC technology, the fatigue properties of the hybrid composite material must be well understood. Initial work towards understanding the fatigue properties of the hybrid composite material was performed through the use of a modified rotating beam fatigue test, in which five stress amplitudes (σ_A) were considered [6]. By monitoring the deflection of the system, Burks et al. found that upon initial cycling the materials' stiffness increased as a function of a cyclic applied stress (Fig. 2). This process was termed "wear-in," and was reported to exhibit prominent scatter [6].

In that work a finite element (FE) model was developed with the simplifying assumption that the carbon fiber composite was perfectly circular and concentric with the glass fiber composite. This model will henceforth be referred to as the nominal geometry model. When viewing multiple cross-sections of the ACCC core, it is clear, that this assumption is a bit simplistic, and that the cross-sectional geometry is not perfectly circular, and is quite variable when viewing different cross-sections of the hybrid composite core (Fig. 3).

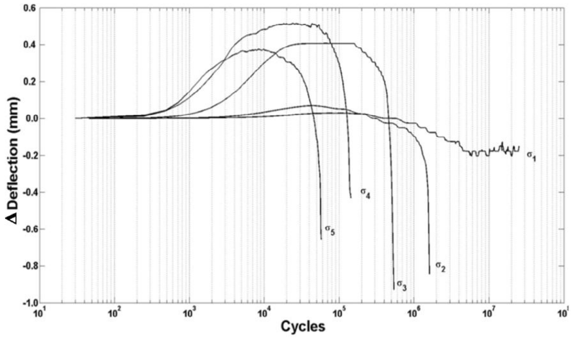

Fig. 2. Representative Δ deflection from the first cycle vs. the number of cycles, showing where the specimen stiffened upon application of the initial cyclic stress.

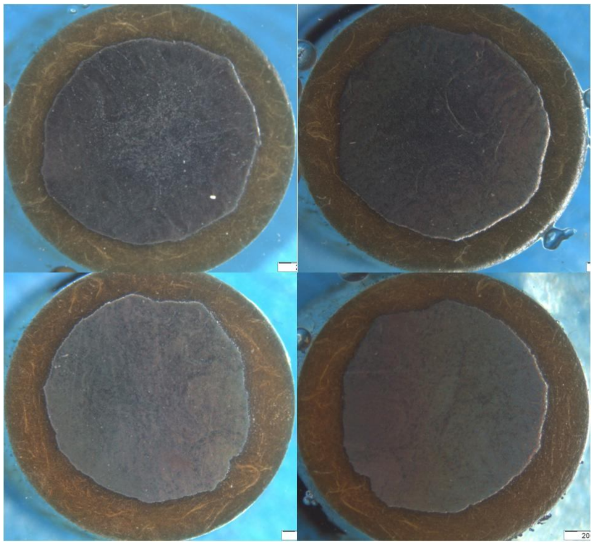

Fig. 3. Different cross sections of the ACCC core, highlighting the geometric variability of the CFC.

This cross-sectional variability along the length of the material is a result of the unique dual fiber pultrusion process that is used to manufacture the hybrid composite material. The nominal model in [6] was developed with both composites being modeled as having a volume fraction of fibers of 60%. As the material stiffened upon initial cycling, it was equivalent to the volume fraction of fibers in the composite increasing [6]. This process was reported as being highly variable in nature.

The aim of this study was two-fold. First, we developed a framework to capture the experimental variability that was observed in [6], by accounting for the two previously mentioned sources of uncertainty. Also, because of the variability in the cross-sectional geometry and the v_f that was introduced into the numeric models, we expected to observe variability of the local stress state within the GFC. Since the GFC was the composite material that failed during fatigue loading, we wished to compare the variability observed in this study, with the deterministic values calculated in [6], in order to gain a deeper understanding of the experimental results.

2 Methods

2.1 Geometric Variability

The material that was studied is a hybrid composite material that has an outer diameter of 9.50 mm. For the study conducted in [6], a nominal diameter of the CFC was taken to be 7.10 mm. Subsequent calculations and models in that work were developed with this deterministic geometry. However this was a simplification, and it has been found that the radii of the CFC was best modeled by a normal distribution (μ=3.55 mm, σ=0.14 mm) [5]. To incorporate this geometric variability into the FE model, a platform was developed between Matlab® and the FE preprocessor HyperMesh®.

Six 2D template meshes were spaced equidistantly along the axial direction of the hybrid composite material (Fig. 4a). Using the HyperMorph tool within HyperMesh, sixteen handles were created along the CFC perimeter of each template mesh. The four elements in both the CFC and GFC closest to the interface of the two composites were associated with the handles that were to be morphed. This was done to prevent severe element distortion upon morphing, and seamlessly transition from the morphed mesh to the underlying template mesh.

Matlab was called to randomly sample the CFC distribution for each of the handles, and morph the handle of the original template mesh the appropriate distance in the radial direction (Fig. 4b). Once all of the template meshes had been morphed, a solid mapping technique was used to generate the 3D hexahedral elements that would be used in the analyses, through a linear interpolation scheme that connected adjacent template meshes. Element checks were performed on all hexahedral elements in the models before analyses were run; elements had to have a minimum Jacobian of 0.7, and the maximum allowable element warpage was 2.0. The element checks insured that a high quality mesh had been generated, and that spurious stress results would not be computed. A comparison of the nominal geometry model and a morphed geometry model can be seen in Fig. 5.

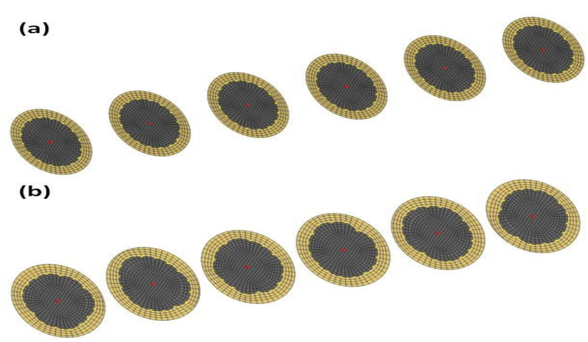

Fig. 4. Unmorphed (a) and morphed (b) template meshes.

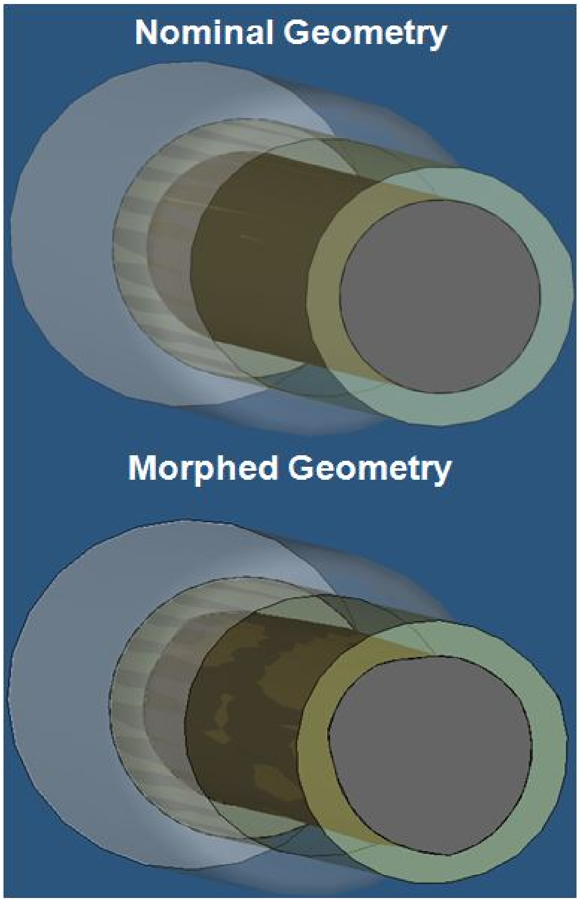

Fig. 5. Comparison of the nominal geometry model, and a morphed geometry model.

2.2 Material Property Variability

Burks et al. proposed that the materials stiffness increased due to an alignment of the reinforcing fibers under the cyclic tensile load [6]. The hybrid composite core is produced through a unique dual fiber pultrusion process, which results in some amount of fiber misalignment with the intended axial direction. Without the appropriate equipment to characterize the initial fiber misalignment, one is left to postulate whether it is the glass fibers, carbon fibers, or a combination of the two that is aligning with the axial direction. With this being the case we considered the two separate cases of either composite's fibers aligning.

To incorporate the material property variability into the platform, distributions of volume fractions of fibers were determined based upon experimental data coupled with results from the nominal geometry FE model [6]. In the nominal geometry finite element model, both composite materials were modeled as having a volume fraction of fibers of 60% [6]. Using the nominal geometry model, a minimization process was used to minimize the error between the experimentally measured deflection, and the numerically simulated deflection for all five stress amplitudes considered. The minimization process varied the volume fraction of fibers (v_f) of either the CFC or the GFC (holding the other composites' v_f constant at 60%), until the error had been minimized. This process was done for the mean value of deflection, as well as ± 1 standard deviation of deflection for each σ_A. The values for the minimized v_f were considered to be the parameters of the normal distribution of v_f for the stiffened composite material. The normal distribution parameters for each σ_A are presented in Table 1.

Table 1. Summary of the respective v_f normal distribution parameters, reported mean [standard deviation].

Stress Amplitude	Modified CFC v_f	Modified GFC v_f
σ_1	0.6324 [.0166]	0.6451 [.0229]
σ_2	0.6423 [.0263]	0.6556 [.0365]
σ_3	0.6681 [.0312]	0.6935 [.0456]
σ_4	0.6832 [.0280]	0.7154 [.0402]
σ_5	0.6509 [.0342]	0.6696 [.0477]

Monte Carlo sampling of these distributions was then used for random material property assignment via the Mori-Tanaka formulation of the Eshelby method [7, 8]. One hundred analyses for each σ_A were performed in parallel using Abaqus® v.6.9.2 on an 8 processor Linux workstation.

3 Results and Discussion

The deflection of the system was extracted for each of the analyses, and the mean and standard deviation calculated. It was found that the mean value and variability in system deflection was well captured through the use of the platform and was in good agreement with the variability observed in the experimental phenomenon. Results for the variability in the modification of both composite materials are presented in Fig. 6. The excellent agreement confirmed that it was possible to numerically capture the experimental scatter.

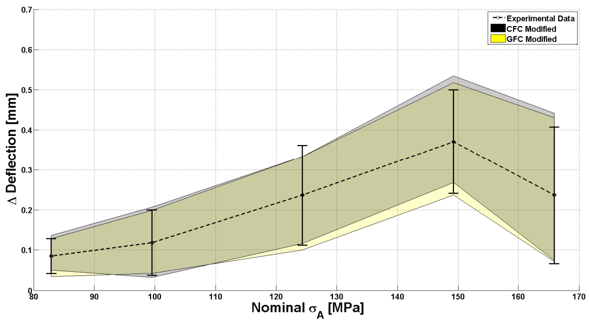

Fig. 6. Comparison of the probabilistic FE results with experimental data (±1 standard deviation scatter bands shown).

Fig. 7. Cross-section showing the accumulated damage within the GFC from the fatigue loading [6].

In the pilot fatigue study on the hybrid composite material, it was observed that all macroscopic fatigue damage was accumulated in the GFC, and that the CFC remained undamaged (Fig. 7). For this reason, the maximum axial stress within the GFC was extracted for each analysis. For the case in which the CFC was considered to stiffen, the maximum axial stress in the GFC was found to negatively correlate with the change in deflection of the system (Fig. 8a). Conversely, for the case in which the GFC was considered to stiffen, the maximum axial stress within the GFC was found to positively correlate with the change in system deflection (Fig. 8b). In both cases, the amount of correlation was found to increase with increasing stress amplitude; the correlation coefficients are summarized in Table 2.

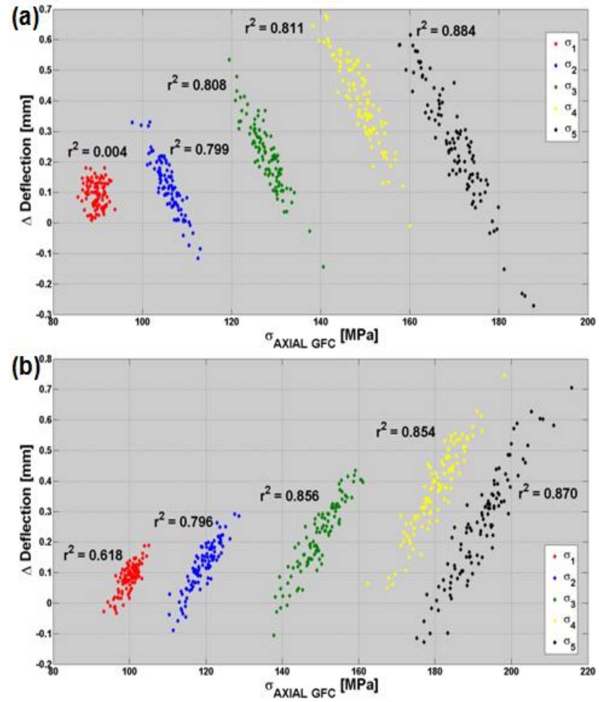

Fig. 8. Correlation between the change in system deflection and the max σ_{GFC} for (a) modified CFC v_f and (b) modified GFC v_f.

Table 2. Coefficient of determination between the maximum stress in the GFC and the change in deflection regression analysis.

Stress Amplitude	Modified CFC v_f	Modified GFC v_f
σ_1	0.004	0.616
σ_2	0.799	0.796
σ_3	0.808	0.856
σ_4	0.811	0.854
σ_5	0.884	0.870

An analytic solution for the axial stress state in flexural loading of the hybrid composite material was developed in [4]. In this derivation, it was shown that the axial state of stress within the hybrid composite is a function of the ratio of the axial stiffness's of the respective composites. In view of this, one may understand the results presented in Fig. 8. For the study where the CFC v_f was modified (i.e. – increased from its' nominal value), the ratio of the axial stiffness' was shifted towards the carbon fiber composite. The implication becomes that the CFC bears more of the bending load, reducing the state of axial stress within the GFC, hence resulting in the negative correlation that was observed with the increase in system deflection.

Table 3. Best fit distribution parameters for each stress amplitude, and the corresponding percentile of the deterministic stress value.

	CFC Modified v_f				GFC Modified v_f			
σ_A	Distribution	Scale	Shape	Percentile	Distribution	Scale	Shape	Percentile
σ_1	**Normal**	μ=89.765	σ=1.670	0.0%	**Weibull**	w = 100.866	k = 59.573	0.000831%
σ_2	**Normal**	μ=105.920	σ=2.687	0.8%	**Normal**	μ=119.385	σ=3.712	0.000004%
σ_3	**Normal**	μ=127.829	σ=3.488	15.6%	**Normal**	μ=149.629	σ=5.477	0.000187%
σ_4	**Weibull**	w=150.861	k = 46.616	44.9%	**Normal**	μ=180.425	σ=6.090	0.000015%
σ_5	**Log Normal**	λ=5.137	ζ=0.035	22.9%	**Log Normal**	λ=5.257	ζ=0.040	0.012880%

On the contrary, for the study in which the GFC v_f was modified, the ratio was slighted towards the GFC, increasing the amount of the bending load that the GFC was bearing. This in turn increased the axial state of stress within the GFC, and resulted in the observed positive correlation between the increase in deflection and the maximum axial stress within the GFC.

The fact that the correlation between the change in deflection and the maximum axial stress in the GFC was not perfect (i.e. $-r^2 \neq 1$) for any of the stress amplitudes indicated that the cross-sectional geometric variability along the lengths had a significant impact on both response variables. Interestingly, as the stress amplitude increased, the level of correlation increased, implying that the numeric platform was accounting for more of the observed experimental variability.

Additionally, since the level of correlation increased with increasing stress amplitude, we conclude that the geometric variability along the lengths of the specimen became less significant at higher stress amplitudes. The deflection of the system is a function of the stiffness of the materials, as well as their moments of inertia, as is the local state of stress. Since the moment of inertia for the CFC and the GFC was changing along the lengths of the model, this was impacting the deflection of the system. However, at the point of maximum stress within the GFC the moment of inertia will take on a single value, and that will be the only geometric parameter affecting the state of stress at this point. Hence, we observed that at higher stress amplitudes the geometric variability along the length of the model became less significant.

Finally, for all ten cases considered in this research the maximum axial stress within the GFC was fit with a distribution that best described the data. Normal, log-normal, and Weibull distributions were considered. The appropriateness of the fit was checked at the 5% significance level via the Kolmogorov-Smirnoff goodness of fit procedure [9]. An example of the distribution fitting process for σ_3 of the CFC modified v_f can be seen in Fig. 9. Table 3 presents all distributions, and their respective parameters, that best described the maximum axial stress within the GFC.

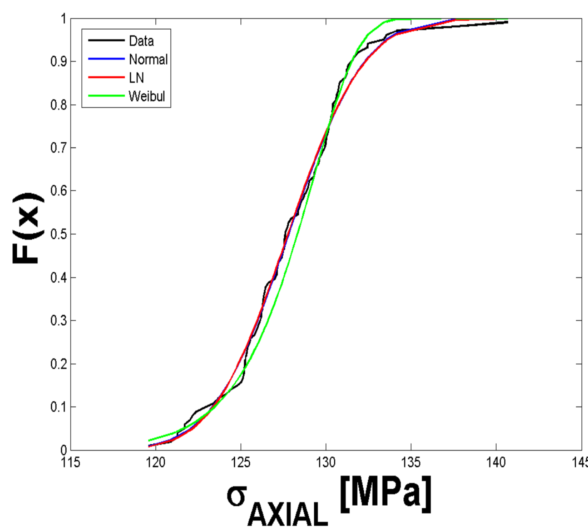

Fig. 9. Representative empirical CDF with all distribution fits. Results are presented for σ_3 in the CFC modified v_f study.

For each stress amplitude the axial state of stress analytically calculated and numerically verified in [6] was compared to the appropriate distribution. The aforementioned deterministic stress value was calculated from the nominal geometry and material parameters.

It was found for each σ_A considered, that the experimental deterministic maximum axial stress in the GFC fell below the 45^{th} percentile for all of the Monte Carlo FE simulation distributions. This finding shows that an understanding of the local variability in both geometry and material properties is essential for understanding the axial state of stress within the GFC. An understanding of the variability in the axial state of stress of the GFC can then lead to a conservative fatigue design procedure for the ACCC transmission lines to avoid costly in-service failures.

4 Conclusion

In this study, a framework to account for the experimental variability of "wear-in" data of a hybrid composite material with a numeric platform was developed. Both the variability in material properties, and geometry was accounted for. The material property variability was introduced via Monte Carlo sampling of the Mori-Tanaka formulation of the Eshelby method.

The variability in cross-sectional geometry was captured through a mesh morphing scheme that utilized an interface between Matlab and the FE preprocessor HyperMesh.

It was found that the numeric framework was able to quite nicely capture the experimental scatter in "wear-in" data. By extracting the maximum state of stress within the GFC (the portion of the hybrid composite material found to fail in fatigue loading), a great deal of insight was gained. It was found that the maximum state of stress within the GFC and the change in deflection of the system correlated quite well with one another. When the CFC was assumed to be the material stiffening, the correlation was found to be negative, while when the GFC was assumed to stiffen, the correlation was positive. This was explained by recognizing that the axial state of stress within the hybrid composite is directly related to a ratio of the respective materials' stiffness. Additionally, it was concluded that the impact of the geometric variability was significant.

Finally, it was found that the deterministic maximum axial stress within the GFC for each σ_A, calculated using the nominal geometry and volume fraction of fibers, fell below the 45^{th} percentile when compared to the stress distributions gathered from the numeric models that incorporated variability. Hence, use of the deterministic axial stress value would result in a non-conservative analysis. This finding emphasizes the need to characterize the physical variability inherent to the hybrid composite material, and understand the impact that the variability will have on the composites' fatigue performance.

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