STRENGTH OF MULTI-AXIAL LAMINATES WITH MULTIPLE RANDOMLY DISTRIBUTED HOLES

S. Kazemahvazi*, D. Zenkert
Department of Aeronautical and Vehicle Engineering, KTH Royal Institute of Technology, Stockholm, Sweden
* Corresponding author (sohrabk@kth.se)

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1 Introduction

Due to the increasing use of fibre reinforced plastics in the construction of naval ship hulls the ballistic and blast performance of these materials has gained particular interest during the past decade.

A typical scenario of a composite ship hull being exposed to hostile fire can be described as follows; shortly after detonation a scatter of fragments will travel at high speed creating patterns of penetration and perforation damages on the ship hull. Subsequent to these fragment damages a high intensity pressure wave will cause the ship hull panels to deform at an elevated strain rate. Hence, the high intensity pressure wave hits an already damaged structure motivating the study of notched laminates at high rate loading. Due to the very high intensity of the pressure loading, the panel will most probably exhibit large deformations implying build up of membrane stresses so that tensile stresses will exceed compressive stresses. Thus, initially tensile loads will be studied.

A previous study [1] investigated the notch and strain rate sensitivity of glass fibre reinforced vinyl-ester laminates with a single notch. Two types of notches were tested; drilled circular notches and notches from fragment simulating projectile impacts. It was found that drilled circular holes give similar reduction in tensile strength as notches from fragment simulating projectile impacts. The strength of unidirectional laminates with multiple holes was investigated in [2]. The study was based on an extensive experimental programme which was used to develop a semi-analytical model to predict the strength of unidirectional laminates with multiple holes. The model was able to predict the strength of unidirectional laminates with multiple randomly distributed holes with good accuracy.

In the present paper we use percolation theory [3] to develop a cost-effective phenomenological residual strength model. The model provides a closed form expression for the residual strength as function of the hole density (the amount of hole area / specimen area). The benefit of this model is that it provides a good first estimate of the residual strength using only one input variable (hole density). The drawback is, however, that the standard deviation of the strength estimations is larger (since the differences in hole patterns and shapes are not accounted for) and that a significant amount of experiments have to be performed to calibrate the model. To calibrate the model a number of experiments are performed on multi-axial laminates with randomly distributed holes. Further, a finite element model is developed in order to make numerical experiments and get more experimental points for the calibration of the theoretical model.

2 Experimental Protocol

Quadriaxial glass fibre non-crimp fabrics infused with vinylester resin have been used exclusively. The laminates have approximately the same amount of fibres in 0, ±45 and 90-degree. The specimen dimensions are length x width = 150 mm x 50 mm with a gauge length of 100 mm. A random hole pattern was applied to the central patch of the specimen (50x50 mm). All holes had a diameter of 5 mm. Fig. 1 show pre-test photographs and FE-model hole geometry for five different hole patterns. Each specimen was tested in a screw-driven test machine at a quasi-static loading rate. The load was measured using a 30 kN load cell and full strain field

measurement was obtained using digital image correlation.

3 Finite Element Analysis

A finite element model was developed in order to make numerical experiments for a number of different hole configurations and hole densities. The numerical experiments, together with the actual experiments, were used as input for the statistical percolation theory model.

3.1 Model description

The finite element simulations were performed in the commercial finite element code ABAQUS. The specimen was modelled as a shell with composite layup. Constitutive material properties were chosen as in table 1 and calibrated against tensile test experiments for an un-notched specimen. In order to model the onset and progression of damage the in-built damage model for fibre reinforced composite materials was used. This is based on the failure criteria developed by Hashin and Matzenmiller and is described in more detail by Lapczyk and Hurtado [4]. The energy release rates were estimated and calibrated against experimental results for a number of hole configurations – thus the energy release rates were not experimentally measured. Further a small material viscosity, η, was used to improve the convergence of the analysis. In an implicit analysis (quasi-static) the viscosity only have minor effects on the global load-displacement response but improves the convergence rate of the simulation significantly. A discussion on this is given by Lapczyk and Hurtado [4].

A mesh convergence study was performed and an element size of approximate 0.2 mm was required in the vicinity of the hole in order to get sufficiently high resolution of the stress field (and thereby a converged value for the specimen strength). Fig. 2 shows an example of a mesh for a specimen with a large number of holes.

3.2 Benchmarking of FE-model to experimental data

FE-simulations were benchmarked against experimental observations for a variety of hole densities and hole configurations. Fig. 3 and Fig. 4 show examples of comparison between FE-simulations and experimental observations. The specimens in these examples have 5 and 10 randomly distributed holes respectively. It is observed that the FE-model is able to predict the global load-deflection response with good accuracy. Comparisons between the full strain fields also show good agreement both in terms of areas of strain concentrations and the quantitative level of strains.

A summary of all benchmarking FE-simulations are found in table 2. The majority of the simulations show good agreement with the experimental observations. However, hole configurations 5-1 and 20-1 show significantly larger discrepancy. An explanation for this is that each hole configuration was only tested once. Considering the possibility of small deviations in the drilled hole pattern geometry and the natural deviation in the strength of the material it can be concluded that there is a statistical possibility for a large discrepancy between experimentally measured strength and FE-simulations.

3.3 Residual strength predictions for laminates with large number of randomly distributed holes

The described FE-model was used in order to generate a bulk mass of residual strength data as input for the statistical strength prediction model described in section 4. The benefit of using numerical experiments is that the time consuming process of manufacturing specimens with a large number of holes is reduced. The drawback is however that there will be somewhat larger uncertainties in the input data. Fig. 5 shows examples of 4 different hole geometries.

4 Statistical phenomenological model

Percolation theory has been used in order to develop a fast and effective model to predict the residual strength of laminates with randomly distributed holes. The normalized fracture strength, F, as

function of hole density, P, can be described by [5,6],

$$F(P) = k\left(\frac{P^c - P}{P^c}\right)^{2.2}\left[1 - m\left(\frac{P^c - P}{P^c}\right) + n\left(\frac{P^c - P}{P^c}\right)^2\right] \quad (1)$$

where k, m and n are fitting coefficients. P is the hole density, i.e. the total area of holes divided by the total area of the plate. P^c is the percolating threshold. It is defined as the hole density at which the plate will have zero strength. For an infinite plate this value is 0.64 (64 % area of holes). Since the model is only based on one variable, P, no consideration is taken to changes in residual strength as function of different hole patterns. Further, if the fitting of eq. 1 is performed solely on numerical simulations – potential deviations in residual strength due to deviations in material properties will not be captured.

By assuming that P^c = 0.64 and employing a least squares fit based on the Marquardt-Levenberg algorithm the solid curve in Fig. 6 is obtained. The fitting coefficients in eq. 1 are then; k = 1.935, m = 2.14 and n = 1.593. Fig. 6 now shows the normalized strength as function of the hole density. The crosses are data points from experiments and the diamonds are strength predictions by numerical simulations. Although some outliners can be observed the overall fit accuracy of the fit is good. Obviously, the potential outliners can be identified easier the more experimental and numerical data that is used in the curve fitting model. Thus, the quality of the fit and the accuracy of the strength predictions (eq. 1) will increase with number of observations. Since the present study is only based on a limited number of experiments the quantitative strength predictions will have lower accuracy. Typically hundreds of simulations are required in order to obtain high accuracy fit.

5 Concluding remarks

An experimental program has been performed were multi-axial laminates with multiple randomly distributed holes have been tested in tensile loading. A finite element model has been developed and calibrated to fit the experimental observations. The FE-model has then been used to perform numerical experiments in order to get a larger number of statistic data points for the phenomenological model. The phenomenological model is based on percolation theory and provides a fast and efficient way to predict the residual strength of the laminate as function of hole density.

E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	$\hat{\sigma}_{1t}$ (MPa)	$\hat{\sigma}_{1c}$ (MPa)	$\hat{\sigma}_{2t}$ (MPa)	$\hat{\sigma}_{2c}$ (MPa)	$\hat{\tau}_{12}$ (MPa)
41	9.8	2.8	1250	600	20	100	60

X_{1t} (N/mm)	X_{1c} (N/mm)	X_{2t} (N/mm)	X_{2c} (N/mm)	η
15	15	0.5	0.5	< 0.0005

Table 1: Material data properties used in the finite element model. The indexes 1 and 2 refer to the axes along and transverse the fibre direction respectively. The indexes t and c refer to tensile and compressive loading. E is the Young's modulus, G the shear modulus, σ the strength, X the energy release rate and η the viscosity coefficient.

Specimen	Failure load Exp. (kN)	Failure load FE (kN)	Discrepancy
5-1	13.5	11.0	-18.5%
5-2	10.7	10.7	0.3%
5-3	12.4	12.8	3.4%
5-4	14.7	13.7	-6.9%
5-5	13.5	13.3	-1.5%
10-1	9.1	9.5	3.4%
10-2	9.9	9.0	-8.4%
10-3	9.2	9.5	3.0%
10-4	11.0	9.9	-10.1%
10-5	10.1	10.3	1.9%
20-1	4.1	4.8	16.1%
20-2	6.9	7.5	7.9%
20-3	6.8	6.6	-2.9%

Table 2: Summary of simulations benchmarked against experimentally measured failure load.

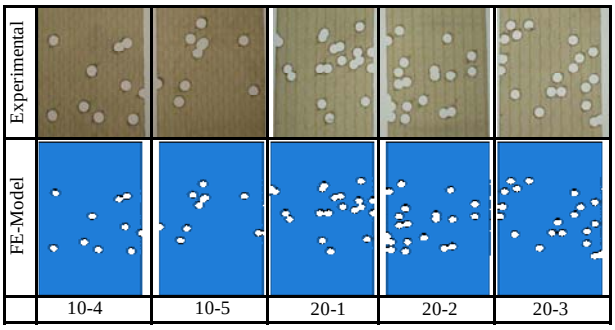

Fig.1. Comparison between experimental and FE-model geometries for five different hole patterns.

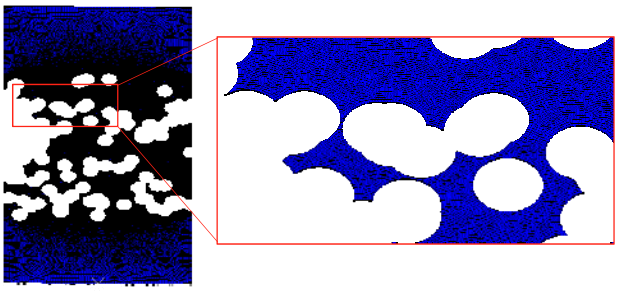

Fig.2. Mesh refinement in the vicinity of holes. Typical mesh size was 0.2 mm.

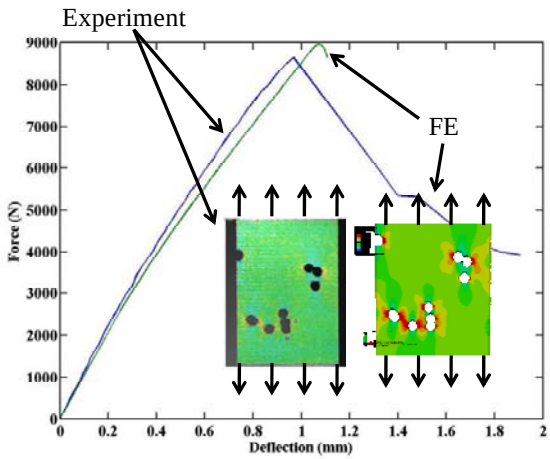

Fig.3. Comparison between experimental and FE simulation load-deflection curve for a specimen with 10 randomly distributed holes.

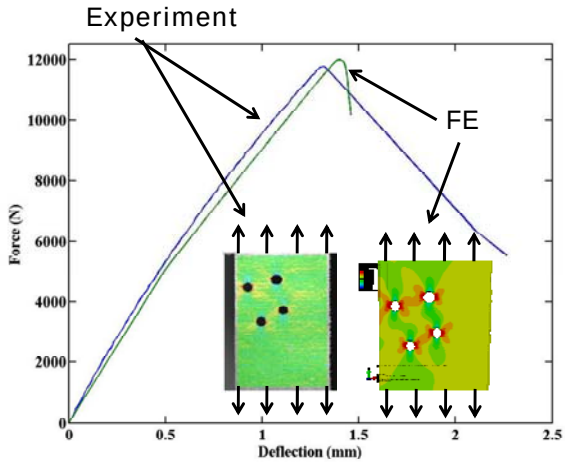

Fig.4. Comparison between experimental and FE simulation load-deflection curve for a specimen with 5 randomly distributed holes.

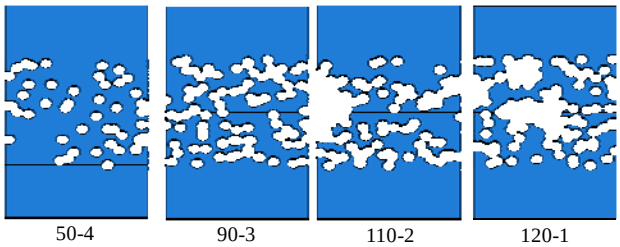

Fig.5. Example of four different hole geometries with 50, 90, 110 and 120 randomly distributed holes respectively.

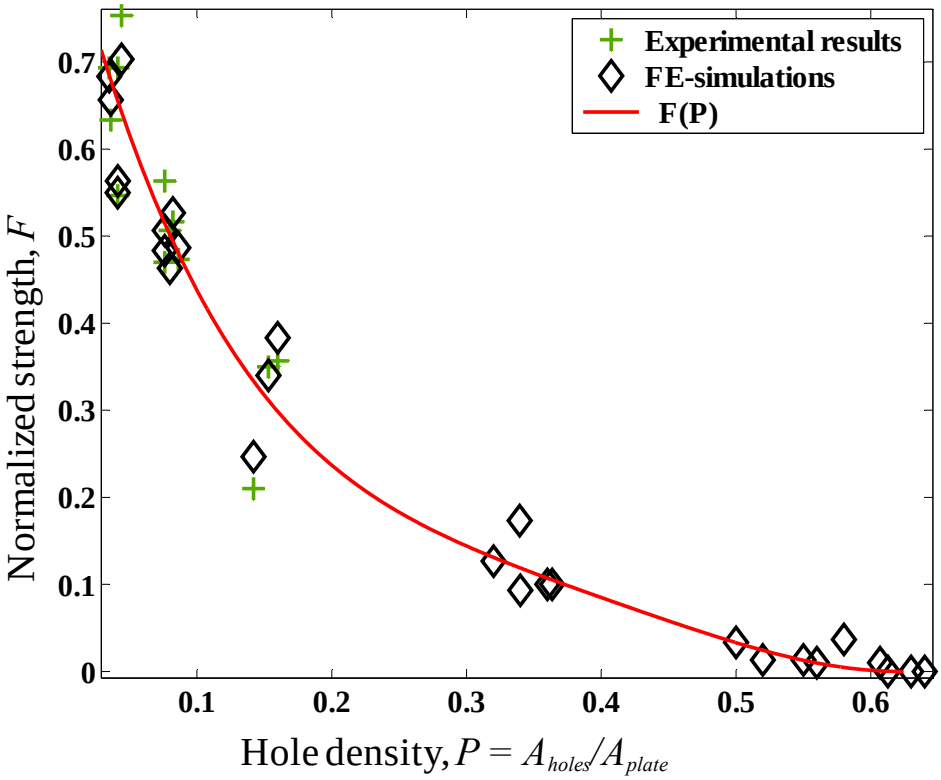

Fig.6. Example of least square fit of eq. 1 to experimental and numerical results.

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