THERMALLY STABLE LAMINATES

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1 Introduction

There is a growing interest in various couplings in the behavior of laminates:

- from a practical point of view, couplings are expected to meet the needs of adaptive, multi functional structures,
- from a theoretical point of view, their study could provide a more comprehensive understanding of laminate behavior.

As mechanical and thermal couplings are closely related, the so-called "thermal stability" is an important issue, which has been studied first by Winckler [1], then Winckler and Hill [2], Chen [3], Weaver [4], Cross et al. [5], Haynes and Armanios [6], and others. Except [2], they limit to laminates made of the same plies ("isolaminar" laminates) with unidirectional reinforcement (UD), for which they derive stacking sequences which should remain flat during the post cure cooling and more generally during uniform changes of temperature. Paper [2] also presents a thermally stable sandwich (a core with laminate facings) without discussing the restriction on the core properties.

The present paper revisits the problem, deriving general conditions, and particularizes the results to reinforcement by balanced fabrics (BF).

2 General Analysis

2.1 Review of Classical Laminated Plate Theory

The well-known relationship of the lamination theory between generalized stresses **N** (in-plane forces) and **M** (bending moments) and generalized strains $\boldsymbol{\varepsilon}^0$ (in-plane strains) and $\boldsymbol{\kappa}$ (curvatures) extends to thermoelastic behavior as:

$$\begin{cases} \mathbf{N} = \mathbf{A}\,\boldsymbol{\varepsilon}^0 + \mathbf{B}\,\boldsymbol{\kappa} - \mathbf{r}\,T_\Delta \\ \mathbf{M} = \mathbf{B}\,\boldsymbol{\varepsilon}^0 + \mathbf{D}\,\boldsymbol{\kappa} - \mathbf{s}\,T_\Delta \end{cases} \quad (1)$$

in which T_Δ is the difference in temperature between the current state and a reference state. This difference is supposed to be constant through the thickness of the plate. **A**, **B** and **D** are the well-known membrane, coupling and bending stiffnesses, while **r** and **s** are thermal coefficients.

Relation (1) is clearly limited to material linearity, but can extend to some non-linear generalized strains.

2.2 Free Deformation and Thermal Stability

Two different questions arise during a thermoelastic loading, which, although related, should not be mixed:

- is it possible to have zero stresses?
- is it possible for a plate to remain flat?

The condition for zero generalized stresses writes:

$$\begin{cases} \mathbf{0} = \mathbf{A}\,\boldsymbol{\varepsilon}^0 + \mathbf{B}\,\boldsymbol{\kappa} - \mathbf{r}\,T_\Delta \\ \mathbf{0} = \mathbf{B}\,\boldsymbol{\varepsilon}^0 + \mathbf{D}\,\boldsymbol{\kappa} - \mathbf{s}\,T_\Delta \end{cases} \quad (2)$$

and it can be met by convenient generalized loads and boundary conditions, whatever are the material properties **A**, **B**, **D**, **r** and **s**. This question is discussed below in Appendix 1.

The second question yields to the two matrix conditions:

$$\begin{cases} \mathbf{A}\,\boldsymbol{\varepsilon}^0 = \mathbf{r}\,T_\Delta \\ \mathbf{B}\,\boldsymbol{\varepsilon}^0 = \mathbf{s}\,T_\Delta \end{cases} \quad (3)$$

which stand for 6 scalar equations in 3 scalar unknowns.

Meeting these conditions is generally impossible, except when mathematical compatibility of equations is satisfied, which requires special values of the coefficients of the equations, i.e. conditions on the materials properties.

While matrix **B** can be singular, matrix **A** is positive definite for physical reasons and consequently is invertible, so the mathematical compatibility of

system (3) can be expressed as the following condition (which involves the laminate material properties but not the thermomechanical loading):

$$\mathbf{s} = \mathbf{B}\,\mathbf{A}^{-1}\mathbf{r} \qquad (4)$$

This is the general matrix condition for thermal stability, valid for any kind of laminates, i.e. isolaminar as well as hybrid laminates.

2.3 Restriction to Isolaminar Laminates

The general condition (4) is highly non linear and consequently not easy to use for designing laminates with the required property. When restricting to isolaminar laminates, simpler equivalent conditions can be derived, which are:

- **A** and **B** should have the symmetry of the square,
- **r** should be isotropic, and
- **s** should be zero.

Definition and properties of the symmetry of the square for thermoelastic behavior are presented in Appendix 2. Demonstration of the above conditions, which involves lengthy computation when using Cartesian components, can be derived easily using the polar method developed by the author.

3 Discussion

3.1 Consequences

Due to the symmetries which result from the stability condition, the corresponding laminates have a limited anisotropy for membrane and coupling stiffness (**A** and **B**). Even the bending stiffness **D** is constrained, as discussed in Appendix 3. So, high anisotropy, as found in orthotropic materials, and high coupling cannot be obtained.

Balanced fabrics reinforcement, which leads to ply with square symmetry, can be used to build easily thermally stable laminates: contrary to unidirectional reinforcement, it gives the required property whatever are the number of plies and their orientations.

3.2 Applications

All the published results from the literature satisfy the general and isolaminar conditions presented in this paper. Which is more, all the stacking sequences using unidirectional reinforcement (UD) from the literature can be advantageously replaced by simpler sequences using balanced fabrics (BF) reinforcement.

As an example, the 8-ply 4-direction UD sequence first published by Winckler [1]:

$$[+\theta\,/(\theta\text{-}90)_2\,/+\theta\,/\text{-}\theta\,/(90\text{-}\theta)_2\,/\text{-}\theta]$$

can be replaced by a 2-direction BF sequence:

$$[+\theta\,/\text{-}\theta]$$

Both laminates have the same final properties for membrane and coupling, with only small difference for bending stiffness.

4 Further comments

Some questions are still pending :

Warp and weft might behave slightly differently, so balance might be imperfect in « balanced fabrics », possibly deviating from exact thermal stability.

Extension to other swelling phenomena (hygral, chemical, etc.) might be questioned, as uniform values in the thickness of the corresponding variables is not assured as for the temperature.

No practical results with hybrid laminates are available up to now.

All the present results are derived in the linear case. From the work of Hyer and others authors (see for instance [7]), it is known that in the case of thermally warping laminates, the linear shape is not always stable and non linear equilibrium positions can appear. For the case of thermally non-warping laminates presented here, no experimental evidence of similar behavior has been reported yet.

References

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Appendix 1: Free thermal deformation

System (2) which has 6 equations for 6 unknowns can be solved, as **A** and **D** are non-singular matrices. It follows expressions of the generalized strains $\boldsymbol{\varepsilon}^0$ and $\boldsymbol{\kappa}$, which are proportional to the temperature difference T_Δ .

It is well known from continuum mechanics that strains cannot receive arbitrary values but must satisfy the so-called Saint-Venant's compatibility equations. In the plane case, there is only one of such equations.

In the same way, curvatures cannot receive arbitrary values and must satisfy two compatibility equations.

Analysing these compatibility equations, it can be shown that for general anisotropy they are satisfied when the temperature field is constant over the plate (remember that temperature is also supposed to be constant in the thickness), while for the isotropic case linear temperature field is also acceptable. When this condition is not satisfied, it means that thermal deformation is not free and induces stresses. As a consequence, compared to isotropic structures, anisotropic laminates are more prone to thermal stresses.

Appendix 2: Square symmetry

A2.1 General

Square symmetry is the two-dimensional equivalent of cubic symmetry in three dimensional space. It can be seen as a special case of plane orthotropy. While in general terms of symmetry elements, it only needs to have a 4-fold rotation axis, in most of the cases it also has a center of symmetry and 4 mirrors. A first set of principal directions (say at 0° and 90°) is directed along two orthogonal mirrors, and a second set is directed along the two other mirrors, at -45° and +45°. A typical example of materials with such a square symmetry is a plain weave fabric (with identical warp and weft).

Due to the symmetries, the properties have the same values in two principal (orthogonal) directions (of any set). More specific results can be derived taking into account the tensorial nature of the considered properties, which controls the variation with the direction.

A2.2 Thermomechanical properties

For second-order properties, such as thermal expansions, square symmetry reduces to isotropy, i.e. the values are the same in any direction.

For fourth-order properties, such as elastic stiffness and compliance, the values are the same in any two orthogonal directions:

If X and Y are a set of principal orthogonal coordinates, the matrix of the components of some fourth-order tensor **Q** (with the usual matrix notation for stiffness) writes:

$$\begin{pmatrix} Q_{XX} & Q_{XY} & 0 \\ Q_{XY} & Q_{XX} & 0 \\ 0 & 0 & Q_{SS} \end{pmatrix} \tag{5}$$

and in arbitrary orthogonal axes 1 and 2, it takes the form:

$$\begin{pmatrix} Q_{11} & Q_{12} & +Q_{16} \\ Q_{12} & Q_{11} & -Q_{16} \\ +Q_{16} & -Q_{16} & Q_{66} \end{pmatrix} \tag{6}$$

which reduces again to a special form in the other principal axes :

$$\begin{pmatrix} \frac{Q_{XX}+Q_{XY}+2Q_{SS}}{2} & \frac{Q_{XX}+Q_{XY}-2Q_{SS}}{2} & 0 \\ \frac{Q_{XX}+Q_{XY}-2Q_{SS}}{2} & \frac{Q_{XX}+Q_{XY}+2Q_{SS}}{2} & 0 \\ 0 & 0 & \frac{Q_{XX}-Q_{XY}}{2} \end{pmatrix}$$

As seen from these expressions, the shear-tension coupling terms are opposite in directions 1 and 2 and vanish in the principal axes X and Y, as well as in the other set of principal axes.

A2.3 Invariant character

An invariant condition for square symmetry of the stiffness in arbitrary axes writes:

$$R_1 = 0 \tag{7}$$

where R_1 is the orthotropy invariant parameter, defined by the author as:

$$64\, R_1^2 = \left(Q_{11} - Q_{22}\right)^2 + 4\left(Q_{16} + Q_{26}\right)^2 \tag{8}$$

A2.1 Coupling effects

From the point of view of coupling behavior, square symmetric materials appear as intermediate between isotropic and non-isotropic materials.

As in any non-isotropic material, shear-tension coupling can exist: in fact, as seen from the above expressions (5) and (6), shear-tension coupling terms appear in off-axis tensile or shear loading, while they vanish in the principal axes X and Y, as well as in the other set of principal axes.

However, as in isotropic materials, **in any axes** the isotropic strain produces an isotropic stress, with no shear, while a shear strain produces a shear stress with no isotropic stress. So isotropic and deviatoric terms are uncoupled, as checked easily:

$$\begin{pmatrix} Q_{11} & Q_{12} & +Q_{16} \\ Q_{12} & Q_{11} & -Q_{16} \\ +Q_{16} & -Q_{16} & Q_{66} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \left(Q_{11} + Q_{12}\right) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \tag{9}$$

and

$$\begin{pmatrix} Q_{11} & Q_{12} & +Q_{16} \\ Q_{12} & Q_{11} & -Q_{16} \\ +Q_{16} & -Q_{16} & Q_{66} \end{pmatrix} \begin{pmatrix} +u \\ -u \\ v \end{pmatrix} = \begin{pmatrix} +U \\ -U \\ V \end{pmatrix} \tag{10}$$

A2.1 Application to laminates

Using the classical laminated plate theory, it can be shown that the square symmetry for all the plies of a laminate induces isotropy for all the thermal coefficients and square symmetry for all the in-plane, bending and coupling stiffness:

$$\underset{\text{all plies}}{R_1^k = 0} \quad \Rightarrow \quad \underset{\text{membrane}}{\overline{R}_1 = 0}, \quad \underset{\text{coupling}}{\widehat{R}_1 = 0}, \quad \underset{\text{bending}}{\widetilde{R}_1 = 0}$$

Specially, for isolaminar laminates, the membrane-bending coupling matrix **B** has a special form depending on two parameters only:

$$\mathbf{B} = \widehat{R}_0 \begin{pmatrix} +c & -c & +s \\ -c & +c & -s \\ +s & -s & -c \end{pmatrix} \tag{11}$$

where c=cos 4a and s= sin 4a, with a some angle resulting from the stacking sequence.

Together with properties (9) and (10) this particular form (11) explain that condition (4) is satisfied for square symmetric isolaminar laminates: square symmetric $\mathbf{A}^{-1}$ acting on isotropic **r** gives an isotropic result, then **B** acting on this intermediate result gives zero, which is the value of **s**.

Appendix 3: Limitation on anisotropy

Laminates made of BF plies are square symmetric in bending too, so the bending anisotropy is obviously limited. The situation is not so easy to analyse for laminates made of UD plies. However it can be expected that bending anisotropy is limited, because the conditions to obtain convenient values for the membrane and coupling stiffness **A** and **B** create constraints on the values of ply angles: limiting the field of values of the angle parameters limits consequently the field of values reached by the variables, including the bending stiffness.

This has been checked in the simple case of the basic Winckler's solution and its equivalent BF laminate given in paragraph 3.2. It was found that compared to the square symmetric values, the relative deviation for D_{11} or D_{22} in the Winckler case is less than 10%:

$$\frac{\Delta D}{D} < \frac{\left|\sin\left(2\theta\right)\right|}{8} < 10\ \% \tag{12}$$