TRUSS WAVINESS EFFECTS ON MECHANICAL BEHAVIORS OF WIRE-WOVEN BULK KAGOME

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1 Introduction

Many studies have been conducted for various cellular mUUaterials, such as truss Periodic Cellular Metal (PCM). Because they provide not only high strength per density, but also make one to use interior space for additional function like heat transfer, catalyst support and storage. The pyramid truss [1], octet truss [2], and woven textile topologies [3] have been studied about mechanical performance, optimal designs for specific applications, and fabrication techniques.

Kagome truss is a recent addition to lattice truss structures. Since the truss elements of the Kagome truss PCM have half the length of those of the Octet, it has excellent resistance to buckling which is a main failure mode of the truss structure, and also has high internal space utilization. [4, 5]

Lee et al. [6] introduced a new technique for fabricating multi-layered Kagome truss-like structures using wires. Helically formed wires were systematically assembled in 6 directions evenly distributed in the 3D space, and then the cross points among the wires were fixed by brazing to be a robust Kagome truss-like PCM which was named WBK after Wire-woven Bulk Kagome in Fig.1.

Since the mechanical strength and stiffness of WBK have been theoretically estimated on basis of assumption that WBK is composed of straight struts, [7] the analytic solutions sometimes give substantial errors compared with experimental results. In fact, WBK is assembled with helically-formed wires. Consequently, the struts are curved, which resulted in errors in estimation based on the previous theoretical solution. Recently, Queheillalt et al. [8] derived the equation considering waviness effect of strut to predict the mechanical performance about the metal textile lattice core. The wires were modeled to have a sinusoidal shape. They reported that the strength and stiffness of textile core were 20% lower than those of collinear core due to the waviness effect.

In this study, to improve the theoretical solutions, the truss waviness and brazed portion are taken into account to estimate the strength and stiffness of WBK. And the results are compared with those measured by experiments and estimated by finite element analysis.

2 Basic Analytical Solutions

Lee at el. [9] derived the analytic solution of compressive strength, assuming that WBK has an ideal Kagome truss structure. Fig.2 shows unit cells of the ideal Kagome truss and the WBK.

$$\sigma_y^c\Big|_{\substack{elastic\\bucking}} = \frac{\sqrt{2}\pi^3}{128}k^2E\left(\frac{d}{c}\right)^4 \quad (1)$$

$$\sigma_y^c\Big|_{\substack{inelastic\\bucking}} = \frac{\sqrt{2}\pi^3}{128}k^2E_t\left(\frac{d}{c}\right)^4 = \frac{\sqrt{2}}{8}\pi\sigma_t\left(\frac{d}{c}\right)^2 \quad (2)$$

$$\sigma_y^c\Big|_{\substack{plastic\\bucking}} = \frac{\sqrt{2}}{8}\pi\sigma_o\left(\frac{d}{c}\right)^2 \quad (3)$$

In this equation, σ_o and E are yield strength and Young's modulus of the wire material, d is the diameter of a wire and c is the length of a strut. E_t defined the slope ($\partial\sigma/\partial\varepsilon$) on stress-strain curve and k is the constants depending on the boundary conditions at the ends. They also derived the equivalent Young's modulus of the ideal Kagome truss.

$$E_e = \frac{3\sqrt{2}}{40}\pi \times E\left(\frac{d}{c}\right)^2 \quad (4)$$

3 Modified Analytical Solutions

3.1 Maximum bending moment

Park et al. [10] calculated the maximum bending moment in a helically formed strut of WBK core taking account of the constraints at the both ends due to brazed filler metal. They applied Prager's equation [11] for failure strength, σ_w of a strut subjected to bending moment and axial force. The

equivalent compressive strength is calculated as replacing σ_o with σ_w to consider the waviness effect of the strut Eq.(3).

We derive the moments considering not only the axial force and bending moment but also the shear forces acting one of out-of-plane struts in a tetrahedron consisting WBK truss core in Fig.3(a). The axial and shear forces in the strut are related to each other by elementary beam theory as follows [12]; See Fig.3(b).

$$F_s = \frac{3}{4}\left(\frac{d}{c-2B}\right)^2 \frac{\cos\omega}{\sin\omega} F_a \tag{5}$$

A helical wire is projected as trigonometric functions on 2-D planes in Fig.3(c). For each of the axes, $M_y(z)$, $M_x(z)$, T, The following equations are derived:

$$M_y(z) = -F_a\left(C_1\cos\lambda z + C_2\sin\lambda z + a(J-1)\cos\frac{\pi(z+B)}{c}\right) \tag{6}$$

$$M_x(z) = -F_a\left(C_3\cos\lambda z + C_4\sin\lambda z + a(1-J)\sin\frac{\pi(z+B)}{c} - \frac{3\sqrt{2}}{16}\{2z-(c-2B)\}\left(\frac{d}{c-2B}\right)^2\right) \tag{7}$$

$$T = R'\cdot a\left[\sin\frac{\pi(z+B)}{c} - \sin\frac{\pi B}{c}\right] \tag{8}$$

Here, a is the radius of the helix, B is the height of the brazed portion, c represents the length of the element. C_1, C_2, C_3, C_4 are constants calculated by a fixed boundary conditions under the influence of brazed portions. To simplify the formula by λ, J as a substitute, R' is as follows:

$$\lambda = \sqrt{\frac{F_a}{EI}} \qquad J = \lambda^2 \Big/ \left(\lambda^2 - \left(\frac{\pi}{c}\right)^2\right) \tag{9}$$

$$R' = -F_a\left(\frac{2a\cos\frac{\pi B}{c}}{c-2B} - \frac{3\sqrt{2}}{8}\left(\frac{d}{c-2B}\right)^2\right)$$

Finally, the equivalent maximum bending moment can be calculated by the following equation:

$$\overline{M} = \frac{1}{2}\times\sqrt{M_x{}^2 + M_y{}^2} + \frac{1}{2}\times\sqrt{M_x{}^2 + M_y{}^2 + T^2} \tag{10}$$

3.1 Equivalent compressive strength

Prager's equation is used to determinate whether the strut fails or not. We also used the equation combined the axial force and the bending moment on the assumption that perfectly plastic behavior occurs in the wire.

$$\left(\frac{F_a}{F_o}\right)^2 + \left(\frac{\overline{M}}{M_o}\right) = 1 \quad F_o = \frac{\pi d^2\sigma_{ys}}{4}, \quad M_o = \frac{d^3\sigma_{ys}}{6} \tag{11}$$

Here, σ_{ys} is the yield strength of the material (200MPa in SUS304), d and A are the diameter and cross section area of the strut.

Because the force and moments are expressed as functions of the F_a in all the above equations, we calculate the critical value of the axial force, $F_{a,cr}$, to satisfy Prager criterion by substituting Eq. (10) into Eq. (11). Then, the critical vertical load, P_{cr}, applied at the top of a tetrahedron consisting WBK in Fig.3(b) can be calculated. Finally, the equivalent compressive strength follows as:

$$\sigma_c = \frac{3P_{cr}}{A_s} = \frac{3}{2\sqrt{3}c^2}\left(F_{acr}\sin\omega + \frac{3\sqrt{2}}{8}F_{acr}\left(\frac{d}{c-2B}\right)^2\cos\omega\right) \tag{12}$$

3.2 Equivalent Young's modulus

Park et al. applied Castigliano's second theorem to obtain Young's modulus of the curved strut, E_w. They also calculated the equivalent Young's modulus by replace E in Eq. (4) with E_w. In the similar way, we applied Castigliano's 2nd theorem to derive the displacement at the top of the helically formed strut, δ_a.

$$\delta_a = \left[\frac{1}{AE_s}\int_0^{c-2B} F_a\frac{\partial F_a}{\partial F_a}dz + \frac{1}{E_sI}\int_0^{c-2B} M_x\frac{\partial M_x}{\partial F_a}dz + \frac{1}{E_sI}\int_0^{c-2B} M_y\frac{\partial M_y}{\partial F_a}dz + \frac{1}{AG_s}\int_0^{c-2B} R'\frac{\partial R'}{\partial F_a}dz + \frac{1}{JG_s}\int_0^{c-2B} T\frac{\partial T}{\partial F_a}dz\right] \tag{13}$$

Here, F_a axial force acting on the wire, c is an element length of the tetrahedral configuration and d is the diameter of the wire, B is the height of the brazing part, I is the moment of inertia, G_s is the shear modulus, J is the polar moment of inertia, E_s is Young's modulus of the wire. And we calculated the equivalent Young's modulus E_e of WBK core follows as:

$$E_e = \frac{3F/A_s}{\delta/h} = \frac{3F_a}{\delta_a\cdot c}\left(\frac{\sqrt{2}}{6}\sin^2\omega + \frac{1}{8}\left(\frac{d}{c-2B}\right)^2\sin\omega\cos\omega\right) \tag{14}$$

4 Finite Element Analysis

Finite element analysis was performed using the Periodic Boundary Condition (PBC) in order to verify the equivalent compressive strength and Yong's modulus. Fig.4 shows the PBC model of the WBK unit cell composed the wires and brazed filler

metal. We made the finite element models using a commercial graphics code as PATRAN 2005 and performed the finite element analysis using ABAQUS ver. 6.9. The wire and brazing part were simulated solid elements of 15nodes (C3D15) and the model had a total of 8,280 elements and 31,284 nodes. As show in Table.1, the models with four types of slenderness ratio (*d/c*) were analyzed. In the models the length of struts differs with the diameter kept constant. The yield stress was σ_{yp} =200MPa, the Young's modulus was E=200GPa, and the Poisson's ratio was assumed to be υ=0.3. The material properties was modeled to be elastic-perfectly plastic to clarify the yield point.

5. Results and Comparison

By using the above analytic solutions, i.e., Eqs. (12) and (14), the equivalent compressive strength and Young's modulus of WBK were calculated as a function of *d/c*. The results were compared with them from finite element analyses and experiments in Figs. 5 and 6. The experiment results of the strength were based on the initial yield points observed in the compression tests on WBK cores made by SUS304 wire. [9]

5.1 Equivalent compressive strength

In Fig. 5, the results estimated by Eq. (12) are substantially lower than those by Eq. (3) for the ideal Kagome truss PCM but higher than those in the previous study by Park et al. [10]. The new results agree fairly well with those from the finite element analyses but the discrepancy with the experimental results tend to increase with the slenderness ratio, which seems to be attributed to material strain hardening of the SUS wires.

5.2 Equivalent Young's modulus

In Fig. 6, the results estimated by Eq. (14) are also lower than those by Eq. (4) for the ideal Kagome truss PCM and higher than those calculated in the previous study by Park et al. [10]. The new results are constantly higher than those from the finite element analyses. The solution is derived from the elastic deformation of the out-of-plane struts only, but the deformation of in-plane struts is not considered. According to Park [13], the deformation of in-plane struts induces 10% decrease in the stiffness. The discrepancy with the experimental results is irregular depending on the slenderness ratio. Also, we suspect that the experiments might have a technical problem. That is, in the experiments the displacement was not measured directly from an independent sensor installed on the specimens, but calculated by subtracting displacement due to compliance of the load track of the test system from the stroke measured at the bottom of the hydraulic ram.

6. Conclusions

In this study, the new analytic solutions of the equivalent compressive strength and Young's modulus of WBK were derived considering waviness of the struts and brazed portion. The shear force as well as the axial force and moments acting in one of the three out-of-plane struts of the tetrahedron-like structure composing WBK were taken into account to predict a critical force acting at the top of the structure and in turn the equivalent yield strength. To verify the solution, the results calculated with the slenderness ratio of the struts (*d/c*) were compared with those from the finite element analysis performed using the PBC model and the experimental data. As the results, the following conclusions were drawn;

i) The equivalent compressive strengths estimated by the solution agree fairly well with those from the finite element analyses but the discrepancy with the experimental results tend to increase with the slenderness ratio, which seems to be attributed to material strain hardening of the SUS wires.

ii) The equivalent Young's moduli estimated by the solution are constantly higher than those from the finite element analyses, which seems to be attributed to the deformation of in-plane struts which is ignored in the new solution.

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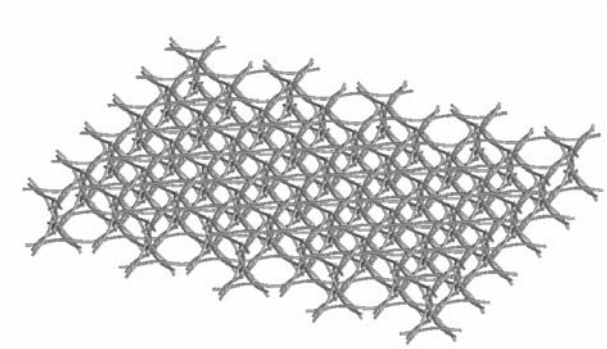

Fig.1. WBK truss core.

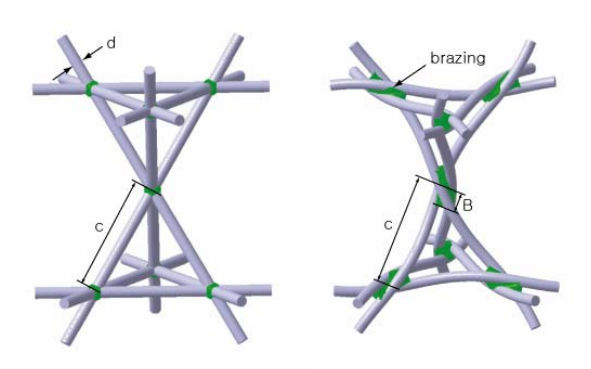

Fig.2. The unit cell of the ideal Kagome truss and the WBK truss including brazing part.

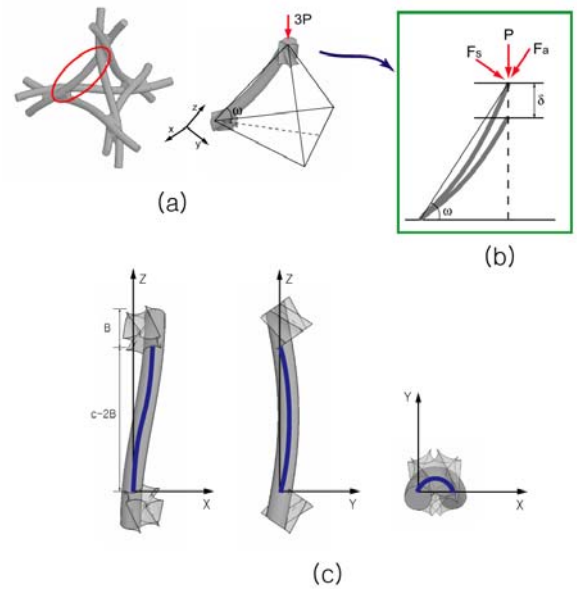

Fig.3. (a) A single strut of WBK unit cell (b) sketch of deformation of a single strut of the WBK core under compression load (c) the configurations of half pitch of single wire projected on two dimensional planes except brazed part.

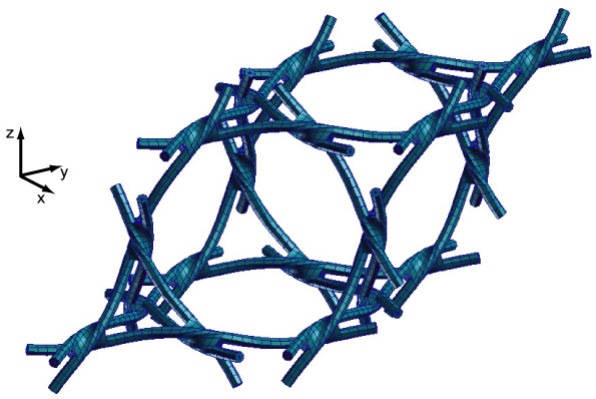

Fig.4. Finite element model of WBK unit cell with Periodic Boundary Conditions (PBCs)

Table 1 The geometric parameters of the wires composing WBK used for finite element analyses

		1	2	3	4
Geometric parameters	*d (mm)*	0.78	0.78	0.78	0.78
	c (mm)	5.35	6.45	8.1	12.6

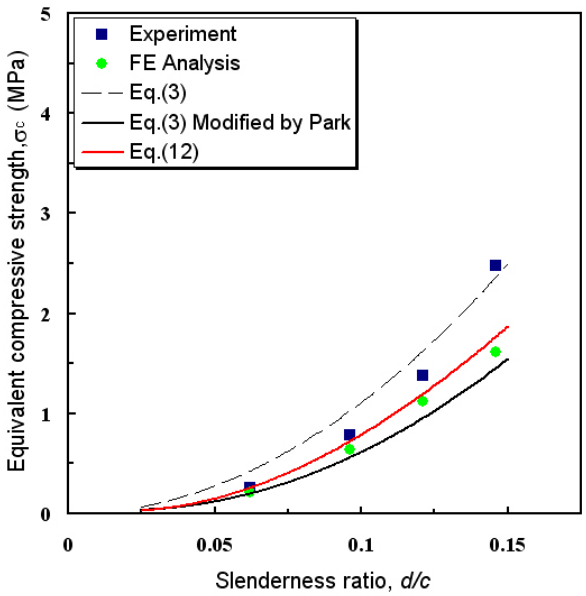

Fig.5. Analytic solutions, FE Analysis and experiment results of equivalent compressive strength of WBK truss core.

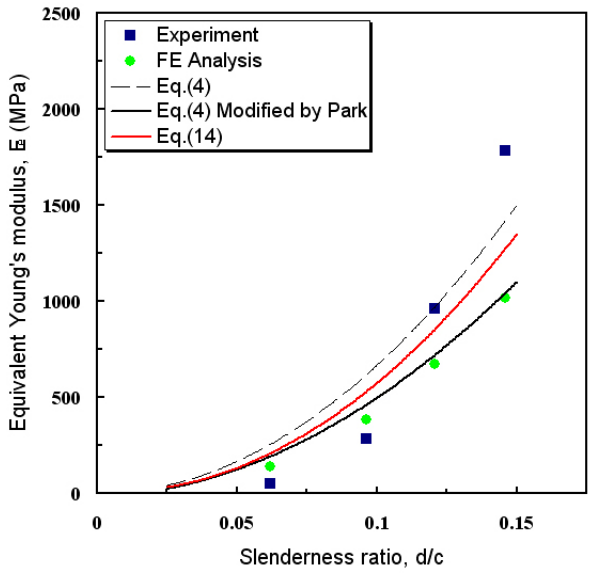

Fig.6. Analytic solutions, FE Analysis and Experiment results of equivalent Young's modulus of WBK truss core.