ORIENTATION DEPENDENCY ON THE ELASTIC BEHAVIOR OF WIRE-WOVEN BULK KAGOME

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1 Introduction

Periodic cellular metals (PCMs) have regular internal stUUructures, which give higher strength and stiffness per unit weight than metal foams with stochastic structures. Particularly, truss type PCMs have additional benefit coming from their open cell architectures for multi-function potential. Truss PCMs are classified as Pyramid[1], Octet[2], Kagome[3], as shown, and so on in Fig. 1. Wire-woven Bulk Kagome(WBK) is a recent addition to PCMs [4-5]. WBK consists of wires and brazed joints among them. Helically formed wires are assembled in in-plane and out-of-plane directions, evenly distributed in the three-dimensional space to compose a Kagome truss-like structure with multiple layers. The slenderness ratio of the struts of WBK is a dominant factor determining relative density, compressive strength and Young's modulus. From the review of literature, we can see that very few studies are available for the geometrical WBK models and their equivalent continua properties. Several researches have been conducted on structural behaviors of WBK.

In this work, to get a better understanding of WBK structures and their mechanical characteristics, the geometrical concepts repeating unit cell/sub-unit cell are introduced. And also several geometrical parameters based on a WBK unit cell are suggested to get the three-dimensional volume. The orientation dependency on the elastic behavior of WBK assemblies is investigated hierarchically and numerically.

2 Brazed Region

2.1 Geometrical structure for WBK

The WBK unit cell consists of 6-strands helically along the three-dimensional directions and brazing joints are conjugated with 3-helical wires. There are two different types of WBK. The concave type and convex type WBK can be obtained depending on wire-assembling sequence. Fig. 2 shows geometrical shape of concave WBK structure. Three helical wires in in-plane directions were defined as 1, 2 and 3 wires, respectively. The other three helical wires in out-of-plane directions were defined as 4, 5 and 6 wires, respectively. Interval angle is 60 degree among the wires in plane and 54.74 degree between in-plan and out-of-plane. One unit cell consists of seven sub-unit cells as shown in Fig. 3.

2.2 Volume of brazed filler-metal

Volume of brazed filler-metal was difficult to calculate at each wire cross point, exactly. Geometrical shape of brazed region is complicated. Fig. 4 (a) shows tetrahedral of ideal brazed region and (b) shows cross section of brazed region. To calculate of volume of brazed filler-metal considers big tetrahedron in cross section bottom and small tetrahedron in cross section top, including partly wires, as shown Fig. 5. And then, the volume of each region was calculated by using sub-unit cells analytically as shown in Eq. 1. V_w is partly wires volume including brazed filler-metal.

$$V_{br} = V_t - V_w \tag{1}$$

Eq. 2 shows the variable a' in big tetrahedron. The a' defines distance of apex to center of mass of a triangle. The R_h is helical radius and θ is rotation angle of wire.

$$a' = \sqrt{(a - R_h \sin\theta)^2 + (R_h \cos\theta)^2} \tag{2}$$

The brazed filler-metal has a shape similar to a tetrahedron with curved ridges. So, cross section of brazed filler-metal rotates along the helical wire shape as shown Fig. 4 (b). Variable B is height of brazed filler-metal. B was measured from WBK

specimen. Using Eq. 1 and 2, V_{br} of brazed filler-metal volume was given by Eq. 3.

$$V_{br} = \int_0^B \frac{\sqrt{3}}{4}(a')^2 dz \tag{3}$$

Table 1 shows several geometrical parameters in sub-unit cell of 3D-model. V_{br} is theoretical solution using Eq. 3 and $V_{catia®}$ is volume of brazed filler-metal estimated using Catia® program.

3 Equivalent Stiffness of WBK unit cell

3.1 Compliance & transformation matrix

The unit cell of WBK consists of seven sub-unit cells with a brazed region at each middle. So, equivalent stiffness of unit cell induces from one wire. Eq. 4 shows a compliance matrix of wire's material property.

$$[S] = \begin{bmatrix} 1/E_{11} & -(\upsilon_{21}/E_{12}) & -(\upsilon_{31}/E_{13}) & 0 & 0 & 0 \\ -(\upsilon_{12}/E_{21}) & 1/E_{22} & -(\upsilon_{32}/E_{23}) & 0 & 0 & 0 \\ -(\upsilon_{13}/E_{31}) & -(\upsilon_{23}/E_{32}) & 1/E_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{31} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{12} \end{bmatrix} \tag{4}$$

And then, the result is expanded to one wire, sub-unit cell and unit cell using transformation matrix successively. Eq. 5 shows total steps of the hierarchical modeling method to determine equivalent stiffness. An infinitesimal wire volume and a global coordinate are presented in Fig. 6. Eq. 6 shows the x, y, z coordinates along the center line of one wire. The variables, θ_r and θ_c , represent the rotation- and the crimp-angles.

The coordinate transformation is performed using the direction cosine matrix as shown in Eq. 7. Global coordinate defines the center of helical radius. On the other hand, local coordinate defines the helical center line of the infinitesimal wire.

Direction cosine matrix for z and y axis shows Eq. 8. And [A] matrix can be expanded to [T] matrix

$$\begin{aligned} [C]_{local} &= [S]^{-1} \\ [C]_{global} &= [T]^T [C]_{local} [T] \Delta v \\ [C]_{wire} &= \frac{1}{V_{wire}} \left\{ \sum_{n=1}^{N} [C_n]_{global} \right\} \\ [C]_{eq.sub} &= \frac{1}{V_{sub}} \left\{ \sum_{i=1}^{3} [T]_i^T [C_{eq}]_i [T]_i V_i \right\} \\ [C]_{eq.unit} &= \frac{1}{V_{unit}} \left\{ \sum_{i=1}^{7} [T]_i^T [C_{eq.sub}]_i [T]_i V_i \right\} V_{fraction} \end{aligned} \tag{5}$$

$$\begin{aligned} & x_i = R_h \cos\phi_i \ , \ y_i = R_h \sin\phi_i \ , \ z_i = c\left(\frac{\phi_i}{\pi} - \frac{1}{2}\right), 0 \langle \phi \langle \pi \\ & \theta_r = a\cos\frac{x_i}{\sqrt{x_i^2 + y_i^2}} , \ \theta_c = a\sin\frac{z_i}{\sqrt{x_i^2 + y_i^2 + z_i^2}} \end{aligned} \tag{6}$$

with 6 by 6 components as shown in Eq. 9.[6] So, equivalent stiffness $[C]_{wire}$ of one wire was converted into local material property $[C]_{local}$ using transformation matrix [T]. In other words, Eq. 8 and Eq. 9 show a transformation matrix [A] and [T] for a helically twisted wire.

$$\begin{aligned} [T(\alpha)] &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix} \\ [T(\beta)] &= \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix} \\ [T(\gamma)] &= \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ -\sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{7}$$

The sub-unit cell consist three wires crossing at brazing point. And each wire inclines and rotates from the global coordinate. The transformed position vector of helically twisted is dependant on the rotation and slope angle in the global coordinate.[7]

$$[A] = \begin{bmatrix} \cos(\theta_c)\cos(\theta_r) & -\sin(\theta_r)\cos(\theta_c) & -\sin(\theta_c) \\ \sin(\theta_r) & \cos(\theta_r) & 0 \\ \cos(\theta_r)\sin(\theta_c) & -\sin(\theta_r)\sin(\theta_c) & \cos(\theta_c) \end{bmatrix} \tag{8}$$

$$[T] = \begin{bmatrix} a^2_{11} & a^2_{12} & a^2_{13} & a_{12}a_{13} & a_{11}a_{13} & a_{11}a_{12} \\ a^2_{21} & a^2_{22} & a^2_{23} & a_{22}a_{23} & a_{21}a_{23} & a_{21}a_{22} \\ a^2_{31} & a^2_{32} & a^2_{33} & a_{32}a_{33} & a_{31}a_{33} & a_{31}a_{32} \\ 2a_{21}\,a_{31} & 2a_{32}\,a_{22} & 2a_{23}\,a_{33} & (a_{22}a_{33} + a_{23}a_{32}) & (a_{23}a_{31} + a_{21}a_{33}) & (a_{21}a_{32} + a_{22}a_{31}) \\ 2a_{11}\,a_{31} & 2a_{12}\,a_{32} & 2a_{13}\,a_{33} & (a_{32}a_{13} + a_{33}a_{12}) & (a_{11}a_{33} + a_{13}a_{31}) & (a_{31}a_{12} + a_{32}a_{11}) \\ 2a_{11}\,a_{21} & 2a_{12}\,a_{22} & 2a_{13}\,a_{33} & (a_{12}a_{23} + a_{13}a_{22}) & (a_{13}a_{21} + a_{11}a_{23}) & (a_{11}a_{22} + a_{12}a_{21}) \end{bmatrix} \tag{9}$$

3.2 Equivalent modulus matrix

Fig. 7 shows wire element. The total number of elements is 9 and brazing height *B* is 1.7d. So, element number 4, 5 and 6 were included brazed region. It was an assumption that brazed region was rigid body. Therefore, element number 4, 5 and 6 has zero compliance and infinite stiffness. Other parts material property is listed in Table 2. (SUS304) Consequently, equivalent modulus is Table 3.

3.3 FEA for WBK unit cell

Fig. 8 shows Periodic Boundary Condition (PBC) model. The model was made using Patran 2005 and FEA was performed by ABAQUS ver. 6.9. The wire and brazed region have same material property. The element type was C3D15. The modulus of elasticity was 200GPa. Fig. 9 shows stress-strain curve of the FEA result.

3.4 Comparison of hierarchical & numerical solution

From this study, we have predicted the equivalent stiffness for WBK unit cells, as shown in Eq. 10 and Table3. Elastic moduli of 33 direction are very similar to each other. But, the shear moduli are different from each other. We guess that the error was caused by inadequate B.C.(boundary condition) of the PBC model.

4 Conclusion

In this work, the orientation dependency of the elastic behavior of WBK assemblies is investigated by the hierarchical approach and FEA. First of all, geometry of WBK is studied in detail. Also several geometrical parameters based on a WBK unit cell are suggested to get the three-dimensional volume. Therefore, volume of brazed region compares theoretical solution with that measured by Catia® modeling.

And transformation matrix is defined using direction cosine. Also, FEA was performed on PBC model of WBK by ABAQUS program. And then, equivalent stiffness compared between hierarchical approach and numerical solution.

$$[C]=\begin{bmatrix} 448.3243 & 192.5071 & 174.8500 & 2.5524 & 6.1208 & -2.4232 \\ 192.5071 & 446.0268 & 196.8088 & 1.1034 & 0.8834 & -4.3401 \\ 174.8500 & 196.8088 & 504.9026 & -21.5992 & -4.6299 & 21.5852 \\ 2.5524 & 1.1034 & -21.5992 & 136.6132 & 4.5636 & 0.8659 \\ 6.1208 & 0.8834 & -4.6299 & 4.5636 & 133.4044 & -2.0391 \\ -2.4232 & -4.3401 & 21.5852 & 0.8659 & -2.0391 & 136.5294 \end{bmatrix} \quad (10)$$

Table1. Geometrical parameters of sub-unit cell model and volume of brazed filler-metal

(Unit:mm)

c	d	B	V_{br}	$V_{catia®}$
8.1	0.78	1.326	0.2928	0.3200

Table2. Isotropic material property

(Unit:GPa)

E_{11}	E_{22}	E_{33}	G_{12}	G_{23}	G_{31}	ν
200	200	200	86	86	86	0.3

Table3. Result of equivalent stiffness, estimated by the hierarchical approach and FEA

(Unit:MPa)

	E_{11}	E_{22}	E_{33}	G_{12}	G_{23}	G_{31}
Theory	345	329	388	135	133	135
FEA	-	-	381	-	74	-

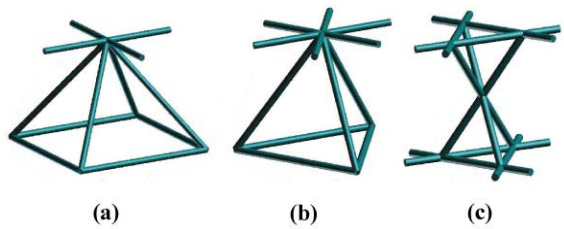

Fig.1. Configurations of unit cells of ideal trusses (a) pyramid (b) octet, and (c) Kagome

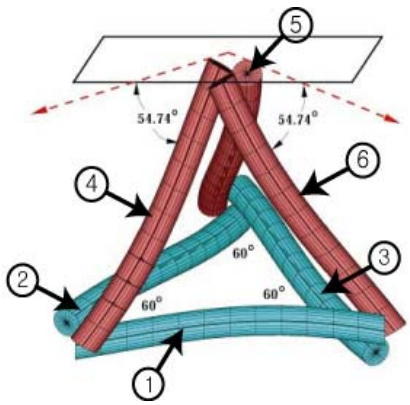

Fig.2. Geometrical shape of a tetrahedral wire cell

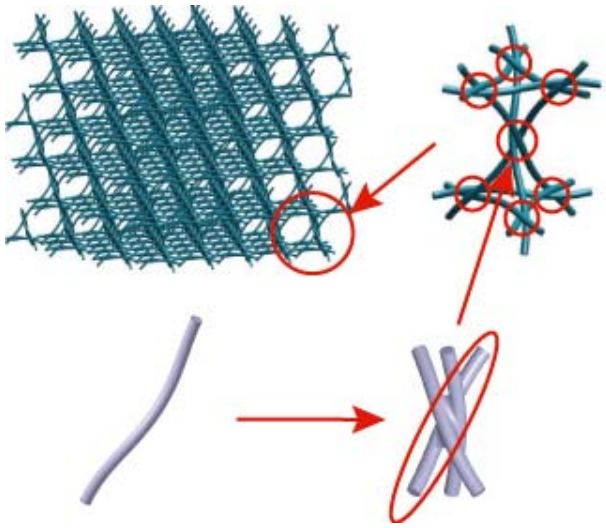

Fig.3. Hierarchical shapes of a WBK assemble

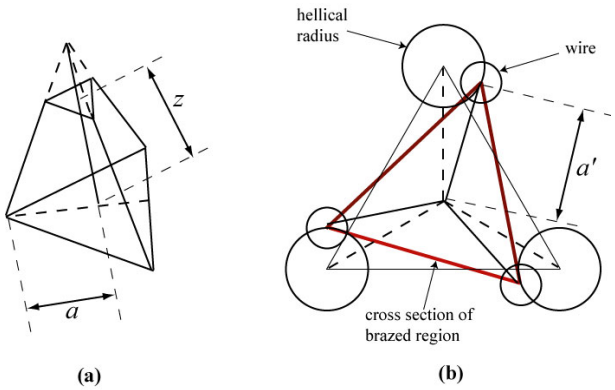

Fig.4. (a) Tetrahedral of ideal brazed filler-metal, (b) cross section of brazed filler-metal

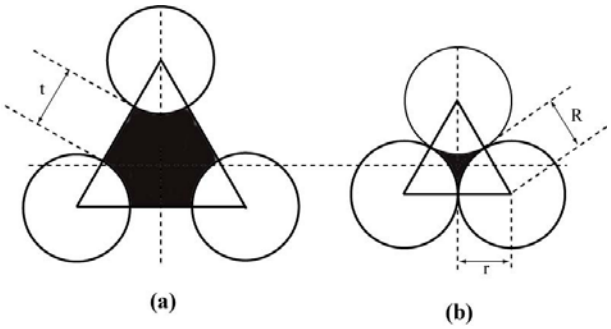

Fig.5. Brazed region of (a) bottom cross section, (b) top cross section

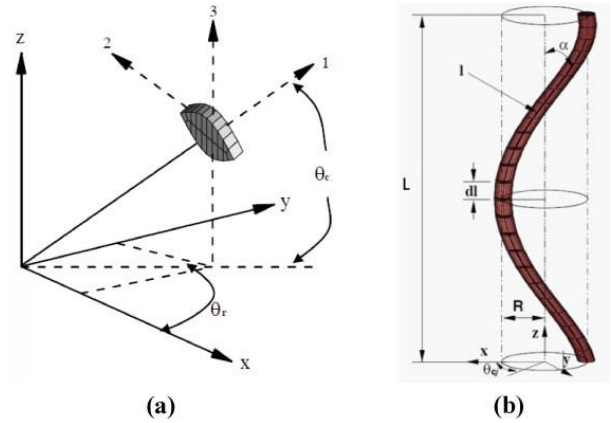

Fig.6. (a) Infinitesimal wire volume, (b) helically twisted wire

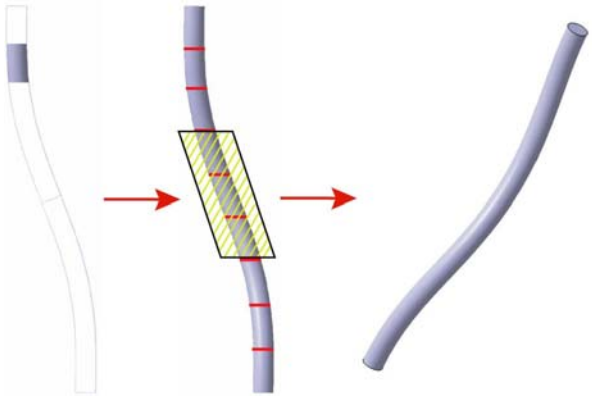

Fig.7. Brazed region in the one wire

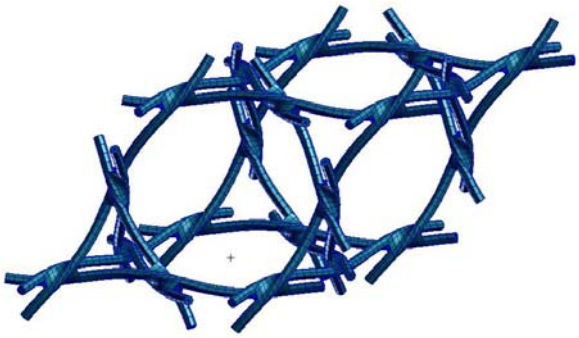

Fig.8. Finite element model of WBK unit cell

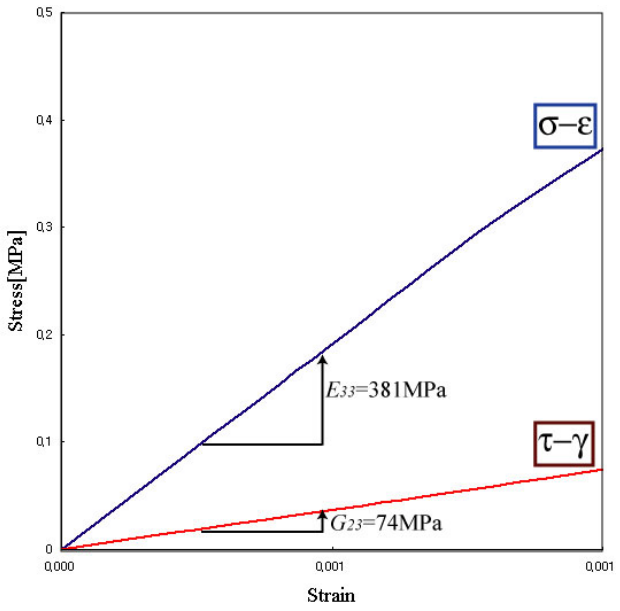

Fig.9 Result of FE analysis

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References

[1] F. W. Zok, S. A. Waltner, Z. Wei, H. J. Rathbun, R. M. McMeeking, A. G. Evans, “A Protocol for Characterizing the Structural Performance of Metallic Sandwich Panels: Application to Pyramidal Truss Cores”. *Int. Journal of the Solids and Structures*, Vol. 41, pp. 6249~6271, 2004.

[2] V. S. Deshpande, N. A. Fleck, M. F. Ashby, "Effective Properties of the Octet-Truss Lattice Material," *Journal of the Mech. Phys. Solids*, Vol. 49, pp. 1747~1769, 2001.

[3] S. Hyun, A. M. Karlsson, S. Torquato, A. G. Evans, "Simulated Properties of Kagome and Tetragonal Truss Core Panel," *Int. Journal of the Solids and Structures*, Vol. 40, pp.6989~6998, 2003.

[4] K. J. Kang, G. P. Jeon, S. J. Nah, B. S. Ju, N. H. Hong, "A New Way to Manufacture Ultra Light Metal Structures," *Journal of the Korean Society of Mechanical Engineers*, Vol. A-28, pp. 296~303, 2004.

[5] Y. H. Lee, B. K. Lee, I. Jeon, K. J. Kang, “Wire-Woven Bulk Kagome (WBK) truss cores,” *Acta Materialia*, Vol. 55 pp. 6084, 2007.

[6] R.E. Goodman, *Introduction to Rock Mechanics*, Wiley, 2nd Edition, 1989

[7] H. K. Hur, J.H Joo and Kang, K.J, 2010, "Geometrical Modeling and Collapse Surfaces of Wire-woven Bulk Kagome Structures," *American Institute of Aeronautics and Astronautics*.