1 Introduction
Discontinuous fiber-reinforced plastics are used in many engineering fields, due to their excellent formability into complex shapes. However, the mechanical strength of discontinuous fiber-reinforced plastics is much lower than that of continuous fiber-reinforced plastics. Therefore, they haven’t been applied to structural components. To overcome this shortage of strength, this study proposes a new compression molding sheet, composed of carbon fibers and thermoplastic resin. This sheet enables in-plane random orientation and dispersion of carbon fibers in composites. Composites are made by stacking the sheets and integrating them with hot pressing. In Fig.1, carbon fibers are well-dispersed. This contributes to its superior formability. A complex-shaped component, such as a rib structure, is easily fabricated (Fig.2). Even in the rib structure, a small matrix-rich region is maintained. This implies the components made of the sheet have superior shape stability. The most important feature of the composite is its strength. If the length of fibers in the composite is appropriately determined, we can obtain superior composite strength, almost equivalent to that of continuous fiber-reinforced plastics. This study focused on the effects of fiber length on the strength of composites made of the sheet, using tensile tests and micromechanical analyses. To investigate the influential damage for the fracture mode, our analysis utilized Duva-Curtin model [1] for fiber breakage. We incorporated this damage model into an equivalent inclusion model [2] combined with the Mori-Tanaka theory [3] to predict the tensile strength of the composites [4]. The fiber length range and the fracture mode to optimize the
composite strength are discussed by comparing the predicted results and those of the experiments.

2 Effect of fiber length on composite strength
We can easily control the fiber orientation and the fiber length in the composite by controlling those in the carbon fiber mat (CF mat), which is a precursor of the sheet. In this study, we prepared six tensile test specimens with different fiber lengths. The fiber orientation in each specimen was controlled to be in-plane random as showed in Fig.3. The composites were made of carbon fiber T700S (TORAY Industries, Inc.) and poly-propylene matrix resin. The $V_f$ of each composite was 20%.

Fig. 4 plots the experiment data for fiber length distribution in each composite. Fiber length was distributed with the upper limit $l_{ini}$, which is the initial fiber length in the CF mat. For simplicity, we utilized the average fiber length ($l_{ave}$) to discuss the fiber length effect on composite strength.

Fig. 5 plots the relationship between average fiber length in composite $l_{ave}$ and tensile strength of the composite. The results indicated that tensile strength increased with an increase in fiber length. When $l_{ave}$ reached 3.1mm, the tensile strength became almost constant at 270MPa. Thomason reported that GMT, composed of randomly oriented glass fiber and polypropylene matrix, has a tensile strength of 70MPa at a $V_f$ of 20% [5]. Thus, the composite is much stronger than conventional discontinuous fiber-reinforced plastics.

The fracture patterns for the case of $l_{ave}$ of 1.3mm and 4.7mm are depicted in Fig. 6. In the enlarged views of fibers parallel to the loading direction, we observe that the fiber surface is covered with matrix resin in both cases. This indicates that the interface of the fiber and the matrix was maintained until the composite reached final fracture. In short, the final fracture was dominated by fiber breakage or matrix
3 Micromechanical analyses

3.1 Stress capacity of fiber in composite

To determine the stress capacity of fiber in composite, we utilized Duva-Curtin model based on shear-lag analysis [1]. Suppose that the fibers with length $L_f$ are aligned parallel to $x_3$ axis. Considering strength distribution of fibers, Duva-Curtin model gives the relationship between applied strain of fiber $\varepsilon_{33,f}$ and averaged fiber stress $\sigma_{33,f}$ as below.

$$\sigma_{33,f} = \frac{E_f \varepsilon_{33,f}}{\Phi} (1 - e^{-\Phi})$$  

(1)

$$\Phi = \frac{\sqrt{3} E_f \varepsilon_{33,f} r_f}{\sigma_Y} \left\{ \frac{1}{L_f} + \frac{1}{L_0} \left( \frac{E_f \varepsilon}{\sigma_0} \right)^\rho \right\}$$  

(2)

where $E_f$ and $r_f$ are Young’s modulus and radius of fiber, respectively. $\sigma_Y$ is tensile yielding stress of matrix. $\rho$ is Weibull modulus, and $\sigma_0$ is the characteristic strength of the fiber with length $L_0$.

Fig. 7 presents the simulated results for averaged stress-strain curve of fiber. $\sigma_{33,f}$ takes maximum value by increasing $\varepsilon_{33,f}$. We utilized the maximum value $\sigma_{cr}$ as stress capacity of fibers in composite.

3.2 Constitutive law for composite

An equivalent inclusion model [2] combined with the Mori-Tanaka theory [3] was utilized to predict the tensile strength of the composite. Focusing on the fiber in the composite as illustrated in Fig. 8, the equivalent condition of stress for the fiber region is given by

$$C_{ijkl}^f \left( \varepsilon_{ijkl}^A + \varepsilon_{ijkl}^m \right) = C_{ijkl}^m \left( \varepsilon_{ijkl}^A + \varepsilon_{ijkl}^m \right)$$

(3)

where $C_{ijkl}^f$ and $C_{ijkl}^m$ are the elastic constants of fiber and matrix, $S_{ijkl}$ is the Eshelby tensor, $\varepsilon_{ij}$ with $\sim$ are the strains at the local coordinate system $\tilde{x}_i$. $\left\langle \varepsilon_{ijkl} \right\rangle$ is the averaged value. (Details of each strain are in Ref. [6].) The effect of the fiber orientation is considered by

<table>
<thead>
<tr>
<th>Average fiber length $l_{av}$ (mm)</th>
<th>1.3</th>
<th>4.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile strength (MPa)</td>
<td>149</td>
<td>282</td>
</tr>
</tbody>
</table>

![Macroscopic view](image1)

![Enlarged view](image2)

Fig. 6. Comparison of fracture patterns of the composite when the fiber length is varied.

damage. To investigate the influential damage on composite strength, we utilized micromechanical analyses considering fiber breakage, which we discuss in the following section.
Fig. 7 Simulated result of average fiber stress as a function of the applied strain

\[
\langle \varepsilon_{ij}^* \rangle = \int f(\theta) \psi_{ij}(\theta) d\theta
\]

where \( \theta \) is the in-plane fiber orientation angle and \( f(\theta) \) is the frequency function of fiber orientation.

After some algebra, the elastic constants \( C_{ijkl}^c \) of the composite is given by

\[
C_{ijkl}^c = \left\{ \left( C_{ijkl}^m \right)^{-1} + V_f R_{ijrs} \left( C_{ijkl}^f \right)^{-1} \right\}^{-1}
\]

(5)

where \( R_{ijrs} \) is the constants include \( f(\theta) \).

When strain increment \( \Delta \varepsilon_{ij}^c \) is applied to composite, the stress-strain relation of composite, matrix and fiber are given by

\[
\Delta \sigma_{ij}^c = C_{ijkl}^c \Delta \varepsilon_{kl}^c
\]

\[
\Delta \sigma_{ij}^m = C_{ijkl}^m \left\{ \left( T_{klmn} - R_{klmn} \right) \left( C_{mnrs}^m \right)^{-1} \Delta \sigma_{rs}^c \right\}
\]

\[
\Delta \sigma_{ij}^f = C_{ijkl}^f \left\{ I_{klst} + S_{klst} \right\} \left( C_{mnuv}^m \right)^{-1} \Delta \sigma_{uv}^c
\]

(6)

where \( T_{ijkl} \) is the constants, and \( I_{ijkl} \) is the unit tensor. Note that \( \Delta \sigma_{ij}^f \) is the stress increment of the fiber parallel to the loading direction.

Moreover, the hardening of Polypropylene matrix is considered with the hardening function \( H \).

\[
H = \frac{\frac{d\bar{\sigma}}{d\bar{\varepsilon}}} = 0.8211 \sigma_r \times
\]

\[
\left( \frac{1}{\cosh^2 \left( \frac{\varepsilon_{ij}^* - \varepsilon_{ij}^*}{l} \right)} L + H(\bar{\varepsilon}^p - \varepsilon_r) l \right) \exp \bar{\varepsilon}^p
\]

(7)

Fig. 8 Schematics of present model for predicting the composite strength

where \( \bar{\sigma} \) is equivalent stress, \( \bar{\varepsilon}^p \) is equivalent plastic strain, \( \sigma_r \) is representative strain, \( l_1, l_2 \) are material constants, \( H(\cdot) \) is step function. Under the incremental calculation, \( C_{ijkl}^c \) is a function of the tangent modulus \( E \) based on hardening function \( H \) of matrix, and \( E \) is given as follows:

\[
E = \frac{E_0}{1 + \frac{E_0}{H}} \]

(8)

where \( E_0 \) is the initial Young’s modulus of matrix.

Using eq.6, strain increment analysis was carried out to predict tensile strength of composite. During tensile loading, fibers parallel to the loading direction are easily broken and assumed to determine the final failure of the composite. Therefore, we assumed that composite fracture dominated by fiber breakage occurs when the axial stress of these fibers reaches \( \sigma_c \) obtained from Duva-Curtin model. Thus, present model gives maximum tensile strength of composite by using fracture criterion of fiber breakage.

4 Analytical result

The transition of tensile strength of the composite was analyzed by gradating the fiber length under the condition of in-plane random fiber orientation and \( V_f \)
of 20%. The material properties used in the simulation are shown in Table 1 and 2. The predicted tensile strength is presented in Fig. 5 with the experiment data. The predicted results agreed well with those of the experiments when \( l_{\text{ave}} \) exceeded 3mm. This agreement suggests that the strength of composites made of the sheet is almost equivalent to that of continuous fiber-reinforced plastics. On the other hand, the predicted result exhibited higher value compared with experiments when \( l_{\text{ave}} \) was shorter than 3 mm. This indicated the final fracture of composite would be dominated by matrix damage in the fiber length range.

As mentioned above, we can easily control the fiber orientation in the composite to improve the mechanical properties in a specific direction. For this application, we investigated the strength properties of the composite when the fiber orientation was localized in the plane (Fig. 9). The strength analysis was carried out by considering the measured fiber orientation ratio \( f(\theta)\Delta\theta \) into eq. 2. Average fiber length and fiber volume fraction \( V_f \) in the composite were 3.1mm and 20%, respectively. Table 3 presents experiment and predicted results for the 0° and 90° directions. In both cases, the predicted results agreed well with the experiment results; thus, the present model can predict composite strength even if the fiber orientation is nonuniform.

### 5 Conclusions

We proposed a new compression molding sheet, composed of randomly oriented and dispersed carbon fibers. We focused on the effects of fiber length on tensile strength of composite made of the sheet. (1)Experiments demonstrated that the tensile strength of the composite increased with increasing average fiber length \( l_{\text{ave}} \) and became

| Table 1 Material properties of fiber (Toray T700S). |
|-----------------|----------|
| Young's modulus \( E_f \) | 230GPa |
| Fiber radius \( r_f \) | 3.5\( \mu \)m |
| Weibull scale factor \( \sigma_0 \) | 4316MPa |
| Gage length of the single fiber specimen \( L_0 \) | 25mm |
| Weibull modulus \( \rho \) | 5.55 |

| Table 2 Material properties of Polypropylene matrix. |
|-----------------|----------|
| Young's modulus \( E_m \) | 1.8GPa |
| Poisson's ratio \( \nu_m \) | 0.3 |
| Initial yield stress \( \sigma_r \) | 11.1MPa |
| Reference strain at rehardening \( \varepsilon_r \) | 0.175 |
| Hardening rule \( l_1 \) | 100 |
| \( l_2 \) | 2.37 |

Fig. 9 Fiber orientation distribution of CF mat with local fiber orientation \( l_{ini} = 6.4\)mm.

| Table 3 Comparison of experiments and analytical result when \( l_{ini} = 6.4\)mm. |
|-----------------|----------|
| Loading direction | Tensile strength (MPa) |
|                 | 0° | 90° |
| Experiment     | 432 | 183 |
| Analysis       | 492 | 190 |
Almost constant at 270MPa when $l_{ave}$ exceeded 3.1mm. This value was much higher than that of conventional discontinuous fiber reinforced plastics.

(2) A micromechanical model to predict tensile strength was presented. The predicted results agreed well with experiments when $l_{ave}$ exceeded 3mm. This suggests the composite has superior strength property almost equivalent to that of continuous fiber-reinforced plastics.

(3) The present model gives tensile strength of composites with various patterns of fiber orientation. Thus, the present model is useful in designing composites.

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References


