Abstract
In this paper, a new model is presented for simulating the damage behaviour of laminated composites using a sub-laminate based approach [1-3]. By conducting tests on notched specimens and using a full-field displacement measurement technique, the damage properties of composites are obtained leading to calibration of this model. The performance of the calibrated damage model is validated by comparing its predictions with experimental data obtained from notched specimens.

1. Introduction
Most recently, laminated composites have been used extensively as the main load carrying components in advanced structures. Consequently, there is a need to develop computationally efficient and physically based damage models to simulate the behaviour of these structures under extreme loading conditions. Ply-based models have been developed to simulate the damage behaviour of laminated composites (e.g. [4-7]). Plies connected with cohesive interfaces are the building blocks of such models. To calibrate the damage parameters of composites in such models, uniaxial tests need to be conducted on unidirectional plies. However, it should be noted that since a laminated composite is made by stacking unidirectional layers on each other, the mismatch between the layers creates a complex local stress/strain field. The complexity increases when the damage is introduced to the system and damage modes start to interact at the smaller scales. The interaction between the layers and existence of various failure modes influence the behaviour of a ply within the laminate significantly. Therefore the behaviour observed in the tests performed on unidirectional coupons does not represent the effective behaviour of a layer in a laminated system. For example, the fracture energy of a composite laminate cannot be accurately estimated based on the properties of individual plies. Moreover, by including the complex interaction of all plies in the numerical model, the ply-based approach does not offer a computationally practical method to simulate real-size structures. The sub-laminate-based approach originally introduced by Williams et al. [1] offers an alternative framework to simulate the damage behaviour of composite components. This macro-scale approach takes the sub-laminate as the building block of the structure. The objective of the sub-laminate approach is to provide a model that predicts the effective and overall damage response of large coupons and structures.

Although the formation of micro-cracks, matrix cracking, fibre breakage and delamination, all contribute to the damage behaviour of the laminate, it is not necessary to simulate details of all of these failure modes to predict the overall response of the structure. By using a damage model that represents the overall behaviour of the material in terms of macro-scale parameters, such as fracture energy, the damage response of a large structure can be predicted effectively [8-11]. For example, it has been shown that a model as simple as the scalar damage model can be used to predict the load-displacement response of notched CFRP specimens fairly accurately (see [12] among others). In this paper, a combined experimental and numerical approach based on the sub-laminate damage assumption is presented that can simulate the damage behaviour of laminated composites. In this approach, initially, tests such as the Over-height Compact Tension (OCT) [13] or Compact Compression (CC) [8] are conducted to obtain self-similar damage growth. Using the digital image correlation technique, full-field displacement vectors of the specimen surface are measured during each test. Based on the acquired data and using the basic
principles of mechanics (equilibrium and compatibility), damage properties of the composite laminate such as damage initiation strain or damage height (or height of the damage zone) can be extracted. These properties are then used to calibrate a newly developed sub-laminate based damage model, CODAM2, which is the second generation of the UBC Composite Damage Model ([1], [14]). CODAM2 addresses the material objectivity issues that the original CODAM formulation faced. The proposed damage model is also equipped with a non-local averaging scheme to inhibit the localization problem and improve the predicted damage patterns.

2. Methodology

2.1. Experimental Approach

In this study, the OCT specimen geometry developed by Kongshavn and Poursartip (Fig. 1a) [13] is used to produce stable and self-similar crack growth so that the damage zone and its propagation could be analyzed. Before conducting each test, a random pattern is applied on the surface of the specimen using black and white spray paints. During the test, surface images are captured using high resolution cameras. These images are then analyzed using the image analysis software package, DaVis [15] and the displacement vectors for virtual nodes on the specimen surface are measured (Fig. 1b).

Here we assume that the elastic properties of the composite laminate are known from standard tests. Using these properties and assuming that the laminate is undamaged, stresses and consequently nodal forces in each element can be calculated using standard finite element procedures. The assumption that the elements are not damaged will be violated when damage initiates and starts to propagate. Within the measured field, each node is shared by four elements as shown in Fig. 1b. If all the elements attached to a node remain undamaged, the sum of all nodal forces will be within an acceptable tolerance, \( e \). On the other hand, if the sum of the nodal forces exceeds the tolerance limit, one or more elements attached to that node are deemed to be damaged. By checking equilibrium equations for all nodes, the damage zone corresponding to each image can be detected as follows:

\[
\begin{align*}
\sum_{i=1}^{4} F_{x_i} &< e \quad \text{and} \quad \sum_{i=1}^{4} F_{y_i} < e \rightarrow \text{no damage} \\
\sum_{i=1}^{4} F_{x_i} &> e \quad \text{or} \quad \sum_{i=1}^{4} F_{y_i} > e \rightarrow \text{damage}
\end{align*}
\]

where \( F_{x_i} \) and \( F_{y_i} \) are the nodal forces contributed by all the elements \((i=1,\ldots,4)\) that share the node that is being interrogated.

Consequently, damage properties such as damage height, \( h_d \), damage initiation strain, \( \epsilon_i \), and fracture energy, \( G_f \), can be extracted (see [16] for more details). These damage properties are then used to calibrate the sub-laminate based damage model, CODAM2.

2.2. Numerical Approach

In CODAM2, the in-plane secant stiffness of the damaged laminate is written as the summation of the effective contributions of the layers in the laminate as shown below:

\[
A^d = \sum_{k=1}^{n} T_k^T Q_k^d T_k t_k
\]

where \( A^d \) is the damaged secant stiffness of the laminate, \( T_k \) is the transformation matrix for the strain vector and \( Q_k^d \) is the damaged in-plane stiffness and \( t_k \) is the thickness of the \( k^{th} \) layer.

Two reduction coefficients, \( R_f \) and \( R_m \), that represent the reduction of stiffness in the longitudinal (fibre) and transverse (matrix) directions are employed. The shear modulus is also reduced by the same ratio as the matrix reduction factor. To achieve a symmetric secant stiffness matrix, the major and minor Poisson’s ratios are reduced by \( R_f \) and \( R_m \) respectively. This leads to the following damaged stiffness matrix:

\[
Q^d_f = \begin{bmatrix}
R_f E_1 & \frac{R_f R_s E_2}{1 - R_f R_s V_{12} V_{21}} & 0 \\
\frac{R_f R_s E_2}{1 - R_f R_s V_{12} V_{21}} & R_f E_2 & 0 \\
0 & 0 & R_m G_{12}
\end{bmatrix}
\]

where

\[ \begin{align*}
\sum_{i=1}^{4} F_{x_i} &< e \quad \text{and} \quad \sum_{i=1}^{4} F_{y_i} < e \rightarrow \text{no damage} \\
\sum_{i=1}^{4} F_{x_i} &> e \quad \text{or} \quad \sum_{i=1}^{4} F_{y_i} > e \rightarrow \text{damage}
\end{align*} \]
The damage parameters are explicitly defined in terms of the maximum experienced non-local equivalent strain functions. The equivalent strain function that governs the fibre stiffness reduction parameter is taken to be equal to the longitudinal normal strain as shown in Equation 4.

\[ \varepsilon_{eq}^f = \varepsilon_{11} \]  

(4)

The equivalent strain function that governs the matrix stiffness reduction parameter is written in an interactive form in terms of the transverse and shear components of the local strains as shown in Equation 5.

\[ \varepsilon_{eq}^m = \text{sign}(\varepsilon_{22}) \sqrt{\frac{\varepsilon_{22}^2 + \gamma_{12}^2}{4}} \]  

(5)

The local equivalent strain parameters are then subjected to an averaging procedure to calculate the non-local equivalent strain parameters. The non-local averaging is employed to address the mesh size and orientation dependency problems that local damage models suffer from. This damage model, CODAM2, has been implemented as a material model in the open-source object-oriented finite element code, OOFEM [17] (see [14] for more details).

3. Experimental Results

3.1. Test Specimens

The OCT test geometry was used in this study (Fig. 1) with the material being an IM7/8552 carbon-epoxy laminate with a [45/90/-45/0]s lay-up and nominal thickness of 4 mm. A MTS hydraulic uniaxial testing machine with an Instron controller was used. A testing jig, designed and built in-house, was used for all of the tests [13]. The specimens were loaded in tension through pins located above and below the notch. An Instron extensometer was used to measure the pin opening displacement (POD) during each test. The displacement rate during loading and unloading was set at 0.5 mm/min. Before conducting the tests, the surface of the specimens was treated with a speckle pattern using black and white spray paints to measure the full-field displacement vectors. A 12 bit LaVision Imager QE camera was used to capture images of the specimen surface during the tests. Two representative load-displacement curves obtained from these tests are shown in Figure 2.

3.2. Damage Detection and Quantification

The images obtained from the surface of the OCT specimens were analyzed to measure the full-field displacement vectors and thereby identify the damage zone during each test. Damage properties of the composite laminate were then extracted [8, 16]. These damage properties which include the values of the damage initiation strain, damage height and fracture energy are listed in Table 1.

4. Numerical Results

Using the experimentally determined damage properties, the input parameters for the damage model were calibrated. The model was then employed to simulate the behaviour of OCT specimens using OOFEM. Fig. 2 shows the predicted load-POD curve and its comparison with the corresponding measured data in terms of both the peak load and post-peak behaviour. Fig. 3 shows the predicted damage pattern in front of the initial notch at the end of the simulation. The comparison between the numerically predicted and experimentally identified damage patterns shows a good agreement in terms of height and length of the damaged zone.

5. Conclusion

In this paper, a sub-laminate based damage model, CODAM2, is presented that can simulate the essence of the damage response of laminated composites. The experimental method described briefly in this paper, offers a non-destructive approach for measuring the damage properties of composites. By conducting special tests, such as over-height compact tension (OCT) tests, that produce stable crack growth, and using a full-field displacement measurement technique, the parameters of CODAM2 can be calibrated.
It has been shown that CODAM2 can successfully predict the damage response of OCT specimens both in terms of the peak load and post-peak behaviour. By virtue of its sub-laminate basis, which takes into account the damage interactions among the plies, CODAM2 is more physically based than ply-based damage models. Also compared to its ply-based counterpart, CODAM2 reduces the computational cost significantly and therefore is a promising tool for simulating the damage response of realistic size composite components.

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Table 1. Damage properties of the IM7/8552 carbon-epoxy laminate with a [45/90/-45/0],s lay-up obtained from analysis of the surface images of OCT specimens.

<table>
<thead>
<tr>
<th>Damage Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damage initiation strain</td>
<td>$\varepsilon$</td>
<td>0.011</td>
</tr>
<tr>
<td>Fracture energy (kJ/m$^2$)</td>
<td>$G_f$</td>
<td>90</td>
</tr>
<tr>
<td>Damage height (mm)</td>
<td>$h_c$</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Fig. 1. (a) The OCT specimen geometry; and, (b) the displacement vectors on the specimen’s surface obtained using image analysis technique.

Fig. 2. Comparison between the Load-POD curves obtained from tests on OCT specimens and the corresponding numerical simulation.
APPLICATION OF A COMBINED EXPERIMENTAL AND NUMERICAL DAMAGE MODEL TO SIMULATE THE DAMAGE BEHAVIOUR OF NOTCHED COMPOSITE SPECIMENS

Fig.3. Comparison of the numerically predicted damage zone and the corresponding damage zone obtained from surface image analysis of the specimen.

References


