A MICROMECHANICAL METHODOLOGY FOR FATIGUE LIFE PREDICTION OF POLYMERIC MATRIX COMPOSITES

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1 General Introduction

Polymeric matrix composites (PMCs) possess superior specific properties to metals, and therefore are widely used in many applications. However, fatigue behavior of composites has been a great concern for years since conventional approaches for fatigue life prediction of metals are not suitable for that of composites due to the existence of anisotropy and the distinction of constituent properties. Despite the fact that so many efforts have been invested into the research on fatigue life prediction of composites [1-10], so far there does not exist a well-established and widely-accepted methodology which can provide satisfactory life prediction for composite structures. Micromechanics is a powerful tool compared with traditional macro-level methods since it provides insight to the micro stress distribution in each constituent, and consequently better understanding of fatigue failure mechanism at the constituent level can be developed, which results in more reasonable explanation of fatigue behavior of PMCs as well as more accurate life prediction of composite structures. In this paper, a micromechanics-based methodology for fatigue life prediction of PMCs was proposed. Theoretical prediction of fatigue life of glass-fiber reinforced laminates which are intended for wind turbine blade application was compared with fatigue test results, and good agreement was obtained.

2 Theory and Approach

2.1 Computation of Micro Stresses

The first step towards fatigue analysis at microscopic level is to obtain micro stresses in each constituent, i.e. fiber, matrix, and fiber-matrix interface, of a composite laminate under external loadings. For a continuous fiber reinforced lamina (UD), a micromechanical model is required to characterize its micro structure such that the micro stresses can be calculated from ply stresses with reasonable accuracy. The micro structure of a UD features longitudinally aligned and transversely randomly distributed fibers embedded in polymeric matrix. Assuming the actual random fiber arrangement on the cross-section of a UD can be replaced by an equivalent regular fiber arrangement, a unit cell consisting of both fiber and matrix can be extracted from the regular fiber array as the basic constructing element. Fig. 1 shows three frequently cited regular fiber arrays: the square (SQR), hexagonal (HEX), and diamond (DIA) arrays, as well as their corresponding unit cells.

In order to correlate ply stresses and micro stresses in each constituent, a concept called Stress Amplification Factor (SAF) was introduced, so that the micro stresses can be calculated with the formula shown below [11]:

$$\boldsymbol{\sigma} = \mathbf{M}_\sigma \bar{\boldsymbol{\sigma}} + \mathbf{A}_\sigma \Delta T \qquad (1)$$

where $\boldsymbol{\sigma}$ is the micro stress at a certain micro point within either fiber or matrix, $\bar{\boldsymbol{\sigma}}$ being the macro (ply-level) stress, ΔT being the temperature increment, $\mathbf{M}_\sigma$ and $\mathbf{A}_\sigma$ being the SAF for macro stress and temperature increment, respectively. The dimension and value of SAF depend on the location of the micro point [11]. If the micro point resides in fiber or matrix, $\boldsymbol{\sigma}$ and $\bar{\boldsymbol{\sigma}}$ in Eq. (1) are 6×1 matrices containing six micro and macro stress components, respectively, while $\mathbf{M}_\sigma$ and $\mathbf{A}_\sigma$ are in the form of a 6×6 matrix and 6×1 matrix, respectively. For the fiber-matrix interface, $\boldsymbol{\sigma}$ becomes a 3×1 matrix containing three interfacial tractions, i.e. the longitudinal traction t_x, the tangential traction t_t, and the normal traction t_n, as indicated by Fig. 2. Accordingly, $\mathbf{M}_\sigma$ and $\mathbf{A}_\sigma$ become 3×6 and 3×1, respectively. By applying appropriate boundary conditions to the finite element model of a unit cell, the SAF for any specific micro point can be determined numerically.

After the introduction of regular fiber arrays, it is necessary to validate the equivalence between the idealized fiber arrangement and the real cases. One example of verification was provided in Ref. [12]. It was concluded that the SQR and HEX arrays were comparatively superior to the DIA array.

2.2 Constituent Fatigue Models

A UD consists of three constituents: fiber, matrix, and fiber-matrix interface. Due to the distinct mechanical properties possessed by each constituent, a UD is microscopically inhomogeneous. Therefore, it is rational to propose a fatigue model for each constituent, rather than treating the UD as a whole and employing one fatigue governing law.

Despite the microscopic inhomogeneity of the UD, each constituent can be assumed homogeneous. So it would be desired to introduce an equivalent micro stress for each constituent such that the overall effect of multi-axial micro stresses is considered. Since fibers undertake most longitudinal loads, the micro longitudinal stress at a point within fiber $\sigma_{\mathrm{x,f}}$ is used as the equivalent stress at that point:

$$\sigma_{\mathrm{eq,f}} = \sigma_{\mathrm{x,f}} \tag{2}$$

For the matrix, it is regarded as isotropic with dissimilar tensile and compressive strengths, and the equivalent stress is derived from the matrix failure criterion presented in Ref. [13]:

$$\sigma_{\mathrm{eq,m}} = \frac{(\beta-1)I_{1,\mathrm{m}} + \sqrt{(\beta-1)^2 I_{1,\mathrm{m}}^2 + 4\beta\sigma_{\mathrm{VM,m}}^2}}{2\beta} \tag{3}$$

where β is the ratio between the matrix static compressive strength C_{m} and static tensile strength T_{m}. $I_{1,\mathrm{m}}$ and $\sigma_{\mathrm{VM,m}}$ are respectively the first stress invariant and the von Mises stress of micro stresses at a point within the matrix.

The failure occurring at the interface is usually in the form of debonding due to normal and/or shear tractions. The equivalent stress for the interface is defined following a critical plane model:

$$\sigma_{\mathrm{eq,i}} = \mathrm{sign}(\sigma_{\mathrm{n}}, \tau)\sqrt{\sigma_{\mathrm{n}}^2 + (k\tau)^2} \tag{4}$$

where σ_{n} and τ are the normal and shear stresses on a plane passing through an interfacial point, while k is a material constant. Attention should be paid that the aforementioned plane does not only refer to the plane which passes through the given interfacial point and tangential to the cylindrical outer surface of the fiber. Rather, the equivalent stress should be calculated for all planes passing through the given interfacial point, and the one on which the equivalent stress attains the maximum is defined as the “critical plane”. The function sign(σ_{n}, τ) yields the sign of the one between σ_{n} and τ which has the greater absolute value.

Since the constituent micro stresses vary with time, the constituent equivalent stresses are also time-varying. Thus, the mean value and amplitude of constituent equivalent stresses are readily calculated. Based on those values, the constituent effective stresses are obtained considering mean stress effect. A modified Goodman formulation has been selected for the Constant Life Diagram (CLD), and the effective stress at a micro point within a constituent is defined as

$$\sigma_{\mathrm{eff}} = \frac{\sigma_{\mathrm{eq}}^{\mathrm{amp}} T}{\frac{T+C}{2} - \left| \sigma_{\mathrm{eq}}^{\mathrm{mean}} - \frac{T-C}{2} \right|} \tag{5}$$

where T and C are static tensile and compressive strengths of the constituent, respectively. $\sigma_{\mathrm{eq}}^{\mathrm{amp}}$ and $\sigma_{\mathrm{eq}}^{\mathrm{mean}}$ symbolize the amplitude and mean values of the constituent equivalent stress. With Eq. (5), experimentally acquired fatigue test data can be fitted with Basquin’s equation to obtain the S-N curve:

$$\log \sigma_{\mathrm{eff}} = A \log N_{\mathrm{f}} + B \tag{6}$$

Where N_{f} is the number of cycles to failure, A, B being constants to be defined by test data. For a given constituent effective stress, the number of cycles to failure is calculated. The damage caused by that effective stress throughout its duration is obtained following Miner’s rule, so the linear cumulative fatigue damage variable D for each constituent was calculated as follows:

$$D = \sum_j \frac{n_j}{N_{\mathrm{f},j}} \tag{7}$$

where n_j is the number of cycles of j-th loading, and $N_{\mathrm{f},j}$ is the number of cycles to failure under j-th loading. The failure of the constituent is achieved when the cumulative damage variable *D* reaches 1. It can be seen from the preceding description that the newly proposed methodology holds an advantage over traditional macroscopic fatigue theories in that it has the capability to predict the fatigue life of each individual constituent.

3 Experimental Verification

A series of tests were performed to confirm the validity of the foregoing micromechanical approach for fatigue life prediction of composites. Two verification cases will be presented in the remaining part of this paper: one case is the comparison between predicted S-N curve and fatigue test data for off-axis GFRP UDs of three different fiber orientations, i.e. 15°, 30°, and 60°; the other case is the comparison of the same contents for multi-axial GFRP laminates of three different layups, i.e. UDT [90°], BX$[\pm 45°]_S$, and TX$[0°_2/\pm 45°]_S$.

Fig. 3(a) and 3(b) show fatigue test data of an epoxy resin (Epon 826) and E-glass fiber, which were retrieved from Ref. [14] and [1], respectively, with fitted constituent S-N curves overlapped. Several things need to be clarified before we proceed further: some of the [0°] UD fatigue test data presented in Ref. [1] was treated as the fatigue test data of pure E-glass, since fiber played a predominant role in tension-tension fatigue of [0°] UD; the static tensile and compressive strengths of the E-glass were taken from Ref. [15]; in the following fatigue failure prediction of off-axis UD, it was postulated that the fiber-matrix interface did not fail under predetermined test conditions. Fig. 3(c) to 3(e) are predictions of S-N curves of off-axis UD from two regular arrays as well as a Multi-Continuum theory (MCT) [16], together with test data from Ref. [1]. It was noticed that both regular fiber arrays gave fairly good predictions: theoretical S-N curves pass vicinities of most test data points. In all three cases shown in Fig. 3(c) to 3(e), regular arrays outperformed the MCT.

Fig. 4(a) and 4(b) show fatigue test data of an epoxy resin (Hexion L135i) and E-glass fiber, respectively. The matrix static and fatigue tests were performed by the author, while the static and fatigue test data of E-glass came from the same sources mentioned above. The fatigue tests of three different multi-directional laminates were also conducted by the author, and the test results were presented together with theoretical predictions in Fig. 4(c) to 4(e) for comparison. Fatigue failure of UDT was due to matrix failure, and so was BX. In the case of TX, the initial failure was due to the matrix, but the final failure was dominated by [0°] plies, i.e. fiber breakage. The test results of TX also match well with the prediction. For all three laminates, predictions from the proposed methodology had good agreement with test data. This shows that the micromechanical approach for fatigue life prediction of composites works well under given conditions: it not only can prediction fatigue life of composite laminate, but also can distinguish the critical constituent.

4 Conclusion

A micromechanical approach for fatigue life prediction was presented in this paper. Micro stresses in each constituent of a UD were calculated from ply stresses under the help of SAF. Three different constituent fatigue models were proposed for fiber, matrix, and interface, respectively. A modified Goodman formulation for CLD was employed to obtain the effective stress for each constituent. Basquin’s equation and Miner’s rule were used to take care of S-N curve formulation and damage accumulation. The proposed approach was verified by two cases: prediction of S-N curves for off-axis GFRP UD and multi-directional GFRP laminates. In both cases the test data were well-matched by theoretical predictions, which demonstrated the capability of the micromechanics-based fatigue life prediction methodology.

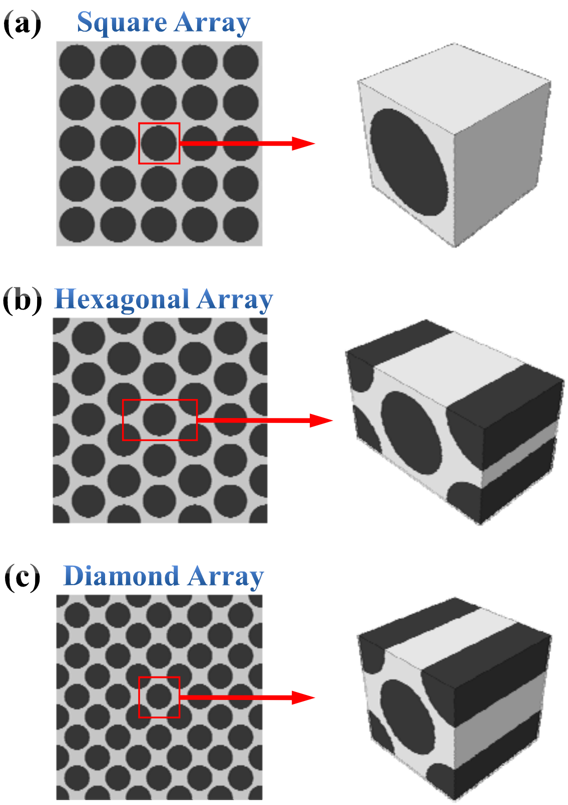


Fig.1. Regular fiber arrays their corresponding unit cells: (a) square array, (b) hexagonal array, (c) diamond array.

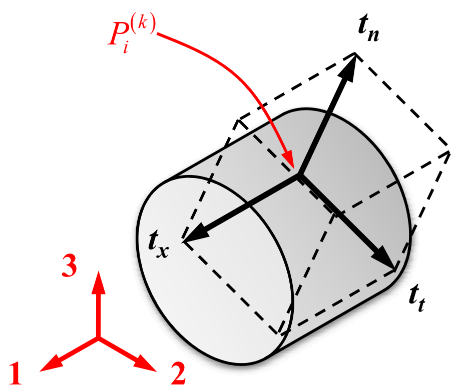
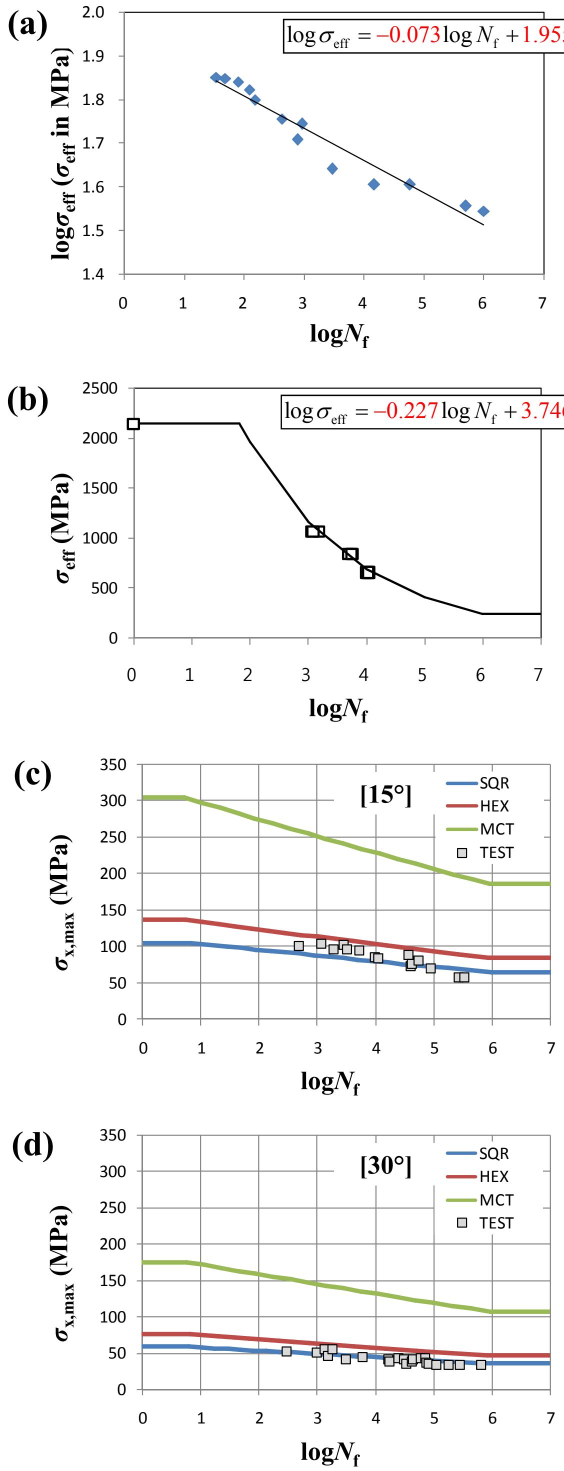


Fig. 2. The illustration of interfacial tractions at an arbitrary point $P_i^{(k)}$ located at the fiber-matrix interface.



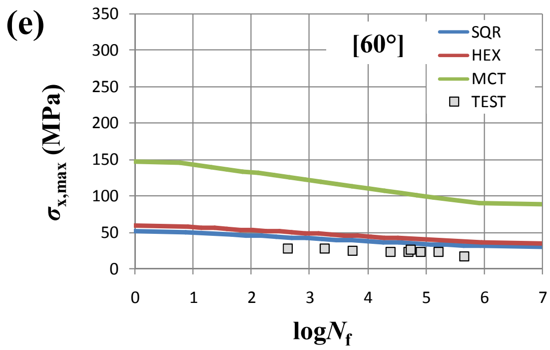
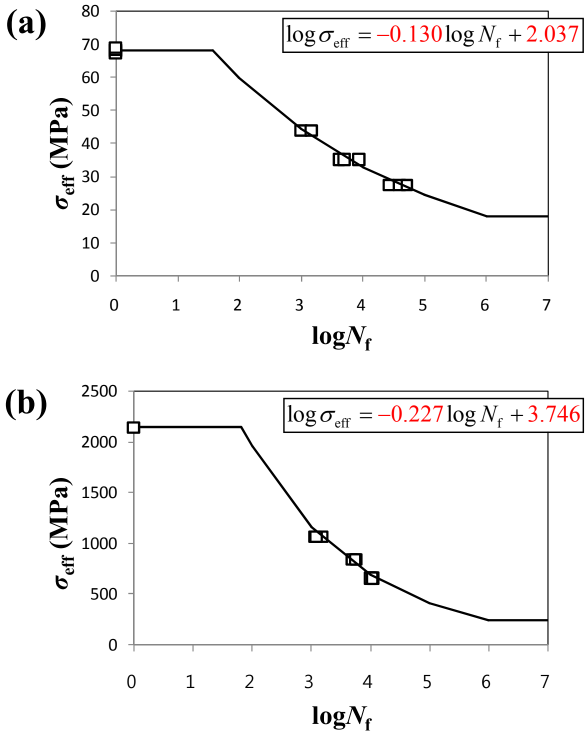


Fig. 3. Comparison between theoretical prediction and experimental results of fatigue life of three types of off-axis GFRP UDs: fatigue test data and fitted S-N curve of (a) matrix, and (b) fiber; predicted S-N curves and fatigue test data of (c) [15°] off-axis UD, (d) [30°] off-axis UD, and (e) [60°] off-axis UD.



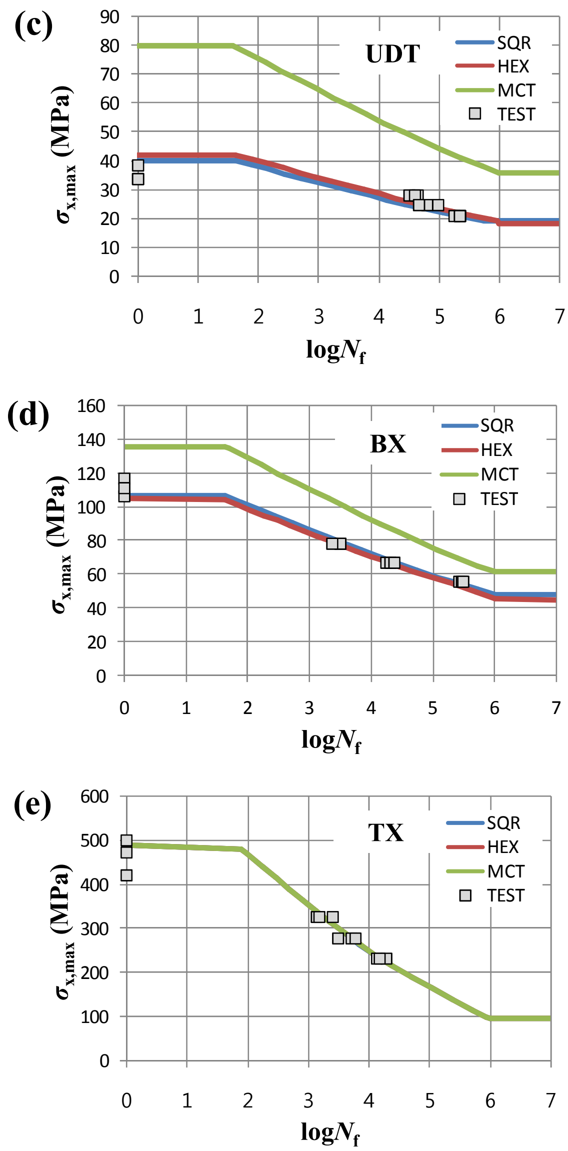


Fig. 4. Comparison between theoretical prediction and experimental results of fatigue life of three types of multi-directional GFRP laminates: fatigue test data and fitted S-N curve of (a) matrix, (b) fiber; predicted S-N curves and fatigue test data of (c) UDT [90°] laminate, (d) [±45°]$_S$ laminate, and (e) [0°$_2$/45°]$_S$ laminate.

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