A PSEUDO STRENGTH FUNCTION FOR THE GENERATION OF LOCAL LAMINATE REINFORCEMENT DOUBLERS

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1 Introduction

Due to their superior mass-specific mechanical properties, laminated composite structures are today well established for aircraft and spacecraft applications. The possibility to tailor the properties of laminates to local requirements at different regions of the structure is an additional advantage. The large number of design variables as well as the anisotropic behavior of the layers result in a complex and time-consuming design process. The usage of an automated design procedure including an optimization can reduce these costs and lead to better solutions.

Stress concentrations caused by cut-outs or load introduction points may reduce the strength of a structure. These areas can be reinforced with local laminate doublers. Finding efficient doubler geometries is not trivial and hard to be done intuitively. Automated design processes can help improving solutions quality and reduce time-costs.

In contrast to eigenfrequencies, buckling or compliance, strength is a local phenomenon. The problem of finding optimal, homogeneous laminates for a given loadcase has been considered by many researchers [1-7] (and references therein). Considering non homogeneous stress states, the strength of the structure is restricted by the maximum stresses. Allowing locally varying laminates, a design change may cause a sudden relocation of the critically stressed region what leads to a non-differentiable objective function. Gradient-based optimization algorithms cannot be applied and stochastic algorithms are often used instead [8-10].

Hansel and Becker [11] address the problem of finding weight-minimal solutions considering strength constraints by taking advantage of local reinforcement doublers. Material is removed layer-wise depending on the principal stresses and the failure indices. The heuristic approach is later enhanced with a genetic algorithm [12]. The generation of reinforcement doublers for laminated plates with holes using this method is presented in [13]. The parameterization scheme ensures that the obtained designs are adapted for production.

In this paper, a differentiable pseudo objective function is introduced that unites the failure indices of the structure. This enables the determination of a gradient field that expresses the influence of a layer thickness change on the strength. It is later used to generate reinforcement doublers starting from an unreinforced initial design. Material is added step-wise based on gradient information until a required strength is reached. Since weight is linearly depending on the layer thicknesses, the placement of the reinforcements is very efficient in terms of mass.

2 Pseudo Strength Function

The proposed pseudo strength function is based on the well-known Tsai-Hill criterion [14,15] for first ply failure in laminated composite structures. The Failure Index *FI* can be determined with equation (1), where σ_1, σ_2 and τ_{12} stand for the plane-stress components in material principal coordinates and *X*, *Y* and *S* are the corresponding strength values. Having different strength values for tension and compression, the choice of *X*, *Y* and *S* is depending on the stress state.

$$FI = \left(\frac{\sigma_1}{X}\right)^2 - \frac{\sigma_1\sigma_2}{X^2} + \left(\frac{\sigma_2}{Y}\right)^2 + \left(\frac{\tau_{12}}{S}\right)^2 \qquad (1)$$

By rearranging the stress components into a stress vector, the equation can be transferred to matrix notation (Eq. (2)).

$$FI = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}^T \begin{bmatrix} (X)^{-2} & -(\sqrt{2}X)^{-2} & 0 \\ -(\sqrt{2}X)^{-2} & (Y)^{-2} & 0 \\ 0 & 0 & (S)^{-2} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \quad (2)$$

This equation again can also be written in a short form.

$$FI = \boldsymbol{\sigma}_{12}^T \boldsymbol{S}_{TH} \boldsymbol{\sigma}_{12} \quad (3)$$

The strength matrix $\boldsymbol{S}_{TH}$ differs for the single layers but not for the elements containing a specific layer.

Equation (3) can also be expressed with element strains in global coordinates instead of the principal stress components. This enables an efficient evaluation since global element strains arise directly from the displacement vector obtained in the finite element analysis. For that, strains are mapped to stresses with the material stiffness matrix $\boldsymbol{Q}$ which is also unique for a single layer. For simplicity, only plane stresses are considered so that $\boldsymbol{Q}$ becomes the membrane stiffness matrix.

$$\boldsymbol{\sigma}_{12} = \boldsymbol{Q}\, \boldsymbol{\varepsilon}_{12} \quad (4)$$

Subsequently, the element strains in global coordinates are rotated to local coordinates by an angle φ with a transformation matrix $\boldsymbol{T}$ and a so called Reuter-matrix $\boldsymbol{R}$ [16].

$$\boldsymbol{\varepsilon}_{12} = \boldsymbol{RTR}^{-1} \boldsymbol{\varepsilon}_{xy} \quad (5)$$

with

$$T = \begin{bmatrix} \cos^2\varphi & \sin^2\varphi & 2\sin\varphi\cos\varphi \\ \sin^2\varphi & \cos^2\varphi & -2\sin\varphi\cos\varphi \\ -\sin\varphi\cos\varphi & \sin\varphi\cos\varphi & \cos^2\varphi - \sin^2\varphi \end{bmatrix} \quad (6)$$

and

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}. \quad (7)$$

Combining equation (4) and (5) leads to

$$\boldsymbol{\sigma}_{12} = \boldsymbol{QRTR}\boldsymbol{\varepsilon}_{xy}\,. \quad (8)$$

Equation (8) can then be used to transform the stress-based strength matrix to a strain-based strength matrix applying a both-sided multiplication.

$$\overline{\boldsymbol{S}}_{TH} = \left[\boldsymbol{QRTR}^{-1}\right]^T \boldsymbol{S}_{TH}\, \boldsymbol{QRTR}^{-1} \quad (9)$$

Thus the Failure Indices FI can be expressed by

$$FI = \boldsymbol{\varepsilon}_{xy}^T \overline{\boldsymbol{S}}_{TH} \boldsymbol{\varepsilon}_{xy}\,. \quad (10)$$

Also the transformed strength matrix has to be determined only once for every single layer since $\boldsymbol{Q}$, $\boldsymbol{R}$ and $\boldsymbol{T}$ are element independent and thus unique for a single layer.

Based on the strain-based matrix formulation of the failure indices FI, a pseudo strain function is defined. It is inspired by the optimality criterion that a structure has an optimal material distribution when the stress distribution is homogeneous. Kress [17,18] takes the stress standard deviation as objective function to find homogeneous designs considering different side constraints. Here, the objective function is formulated by summarizing the squares of the failure indices of every ply of every finite element, also called element-layers, subtracted by a given value β.

$$f_{TH} = -\sum_{i=1}^{nel}\left(\boldsymbol{\sigma}_{xy,i}^T \boldsymbol{S}_{TH} \boldsymbol{\sigma}_{xy,i} - \beta\right)^2 \quad (11)$$

The minus sign in front converts the minimization problem to a maximization problem, so that areas where the structure should be reinforced show high gradient values. This function becomes maximal (or equal to 0), if all failure indices match the required value β. The choice of β has to be done manually by the designer. Choosing a value of 1 implies that the strength of the optimal design is critical in every part of the structure. To obtain non-critical designs, β should be set to a low value (e.g. equal zero). It is clear that the optimal value of the function may never be reached in a real optimization process. Moreover, it is not guaranteed that all failure indices fall below the critical threshold of 1. However, this formulation unifies the strength which is, as mentioned before, a local phenomenon, into a global expression that can be used as an objective function for optimization.

Referring to equation (10), the pseudo strain function can be expressed in terms of element strains.

$$f_{TH} = -\sum_{i=1}^{nel}\left(\boldsymbol{\varepsilon}_{xy,i}^T \overline{\boldsymbol{S}}_{TH} \boldsymbol{\varepsilon}_{xy,i} - \beta\right)^2 \quad (12)$$

To be able to perform an optimization of the laminate, sensitivities with respect to the thicknesses of the element-layers will be determined. Thicknesses are chosen as design parameters to have the possibility of finding solutions with a good strength-to-mass ratio. Generally, also orientation angles could be taken as design parameters. Doing that, only mass neutral solutions can be found. Additionally, designs may have orientation angle distributions that cannot be manufactured.

Considering equation (12), only element strains are sensitive to a thickness change. Applying the chain rule, the derivatives form to

$$\frac{\partial f_{TH}}{\partial \boldsymbol{t}} = -\sum_{i=1}^{nel} 2\left(\boldsymbol{\varepsilon}_{xy,i}^{T}\overline{\boldsymbol{S}}_{TH}\boldsymbol{\varepsilon}_{xy,i} - \beta\right)\cdot 2\boldsymbol{\varepsilon}_{xy,i}^{T}\overline{\boldsymbol{S}}_{TH}\frac{\partial \boldsymbol{\varepsilon}_{xy,i}}{\partial \boldsymbol{t}} \quad (12)$$

The element strains $\boldsymbol{\varepsilon}_{xy}$ are the product of the spacial derivatives of the shape functions $\boldsymbol{B}_i$ and the nodal displacements $\tilde{\boldsymbol{u}}$.

$$\boldsymbol{\varepsilon}_{xy,i} = \boldsymbol{B}_i\tilde{\boldsymbol{u}} \quad (13)$$

$\boldsymbol{B}_i$ contains the derivatives of the element shape function $\boldsymbol{\Phi}$ of the i^{th} element with respect to the reference coordinates and is not depending on the layer thicknesses $\boldsymbol{t}$. The element strain derivatives can be determined with equation (14).

$$\frac{\partial \boldsymbol{\varepsilon}_{xy,i}}{\partial \boldsymbol{t}} = \boldsymbol{B}_i\frac{\partial \tilde{\boldsymbol{u}}}{\partial \boldsymbol{t}} \quad (14)$$

The displacement derivatives again can be determined using the sensitivity equation (15).

$$\frac{\partial \tilde{\boldsymbol{u}}}{\partial \boldsymbol{t}} = \boldsymbol{K}^{-1}\left\{\frac{\partial \boldsymbol{r}}{\partial \boldsymbol{t}} - \frac{\partial \boldsymbol{K}}{\partial \boldsymbol{t}}\tilde{\boldsymbol{u}}\right\} \quad (15)$$

There, $\boldsymbol{K}$ stands for the stiffness matrix and $\boldsymbol{r}$ for the load vector. The evaluation of the sensitivities can be done efficiently since they are analytically derived. The numerical costs arise from the displacement sensitivities that are necessary for the calculation of the strain derivatives. The embraced term of equation (15) is simplified by assuming that loads are not depending on the design. The determination of the stiffness matrix derivatives is straight forward considering that the stiffness of an element is only sensitive to a change of it is own thickness. Thus, it can be done on an element level since the contributions of all elements except the currently considered one are zero. The displacement derivatives are then the result of a linear system of equations with multiple right hand sides. It is a matrix with the size of number of degrees of freedom times the number of element-layers. However, computational capacities of today's workstations are able to handle this amount of data if the size of the model is not too big. Using the sensitivities in an iterative optimization, the process can be accelerated by assuming that the changes of the displacement derivatives during the optimization process are small enough to be neglected. Thus, equation (15) is only evaluated once at the beginning of the process.

The sensitivities of the pseudo strain function give an assumption how and where the current design can be reinforced most efficiently in terms of additional mass. They consider the effect of changing the thickness of a layer in one element to the failure indices of all parts of the structure. Thus, a possible strength increase in one element by changing the thickness of a layer in another element can be localized. The strength problem with local characteristics becomes a global problem.

An analogous pseudo strength function can be formulated for the Tsai-Wu [19] criterion by applying slight modifications to the strength matrix.

3 Reinforcement Patch Generation Process

The obtained sensitivities can be used for an iterative generation of local laminate reinforcement doublers. Sensitivities can also be calculated for layers with predefined material and orientation but with a thickness specified to zero, also called Ghost Layers [20]. Adding an arbitrary number of Ghost Layers at the interfaces of the laminate stack does not affect the structural response of the design. However, an infinitesimal thickness change of these virtual layers would change the behaviour so that sensitivities differ from zero.

For the generation of the laminate reinforcement doublers, a predefined set of Ghost Layers is added to the existing laminate. Subsequently, a real thickness is assigned to the layers having the highest sensitivity values. The reinforcement layers start to grow on the structure applying the evaluation of the sensitivities and the thickening in an iterative procedure. The scheme of the patch generation process is illustrated in Figure 1.

This optimization process does not guarantee that the obtained solutions are globally optimal. Moreover, the probability that the process steps into a local optima is high. However, the process is fast and it is shown in Section 4 (and it has already been shown in [20]) that the reinforced designs have superior properties compared to the initial designs.

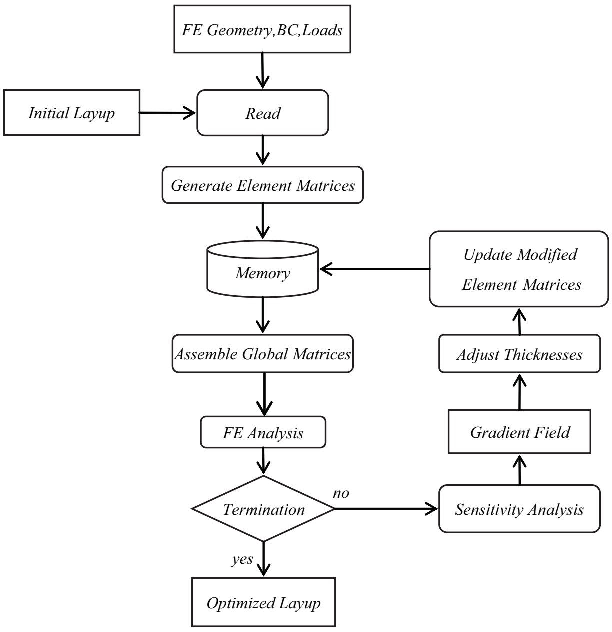

Fig. 1: Patch generation process scheme

4 Numerical Experiment

The potential of the patch generation process taking advantage of the pseudo strain function and the resulting gradients is illustrated on the common example of a uniaxial tension specimen made of carbon reinforced plastics with a centered hole. The rectangular plate has an extension of 200*mm* times 50*mm* and the hole has a diameter of 20*mm*. The plate is made of uniaxial reinforced carbon/epoxy plies (see Table 1) with a layup of $(0,45,-45)_S$. Each ply has a thickness of 0.25*mm* what results in a total thickness of 1.5*mm*.

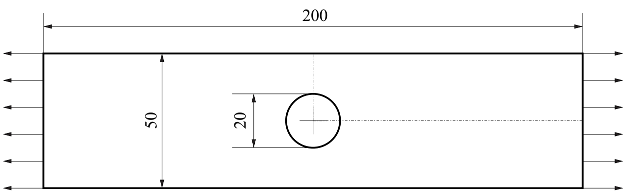

Fig. 2: Geometry of the notched plate

The tension load for the optimization is set to 19500*N*. This is exactly the first-ply-failure load (with Tsai-Hill criterion) of a plate having the double laminate layup of the basic plate ($(0,45,-45)_{2S}$).

For time-savings, only a quarter of the model will be considered. The boundary conditions have to be modified in order to have an equivalent load case. Displacements in x-direction on the left cutting edge as well as in y-direction on the lower cutting edge are locked. Additionally, out of plane rotational degrees of freedom on both cutting edges are locked. The finite element model is illustrated in Figure 3.

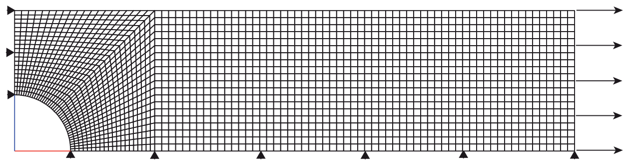

Fig. 3: Finite element model (quarter model)

To predefine the material and the orientation angles of the reinforcement doublers, three Ghost Layers with a layup of 0, 45 and -45 degrees are stacked to each side of the laminate. Applying the iterative process proposed in Section 3, the reinforcement doublers are generated. The process will be stopped as soon as the maximal failure indices drop below the critical value of one. Consequently, the goal of the optimization is to reach the same strength, as a design with double layup, with a significant lower mass.

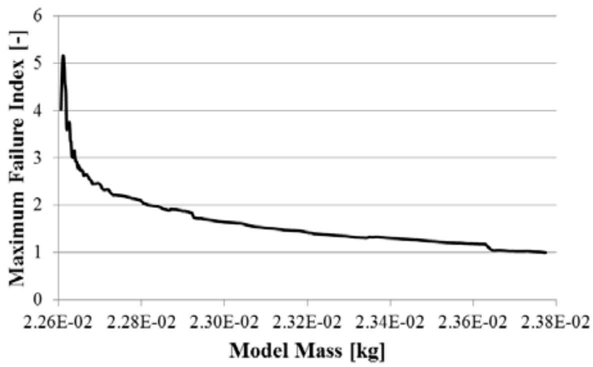

Fig. 4: Characteristics of the patch generation process

In Figure 4, the maximal failure index is plotted over the mass for the patch generation process. Initially, the maximal failure index starts to increase significantly. There, only a few elements build a reinforcement doubler in the area with highest

strains. The resulting discontinuity of stiffness causes an additional stress concentration. Later on, the maximal failure index drops until the required value of 1 has been reached. The unsteadiness of the characteristics is based on the choice of the finite element mesh. Taking a finer finite element mesh will lead to a smoother characteristics since the stress change within one step is smaller.
The terminal solution has been reached after 244 evaluations. The geometries of the reinforcement doublers for the allowed orientation angles are shown in Figure 5 and 6.

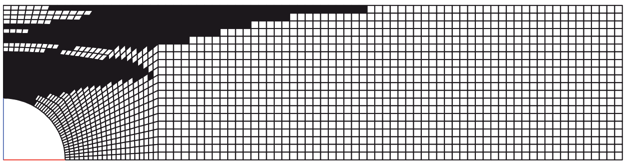

Fig. 5: 0 degree reinforcement doubler

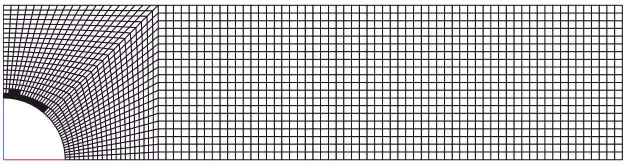

Fig. 6: -45 degree reinforcement doubler

Obviously, the main strength increase arises from the 0°-layer. The -45°-layer has only developed in a small region tangential to the hole and the 45°-layer has not developed at all. The connectivity of the doublers is high so that they can be manufactured without significant modifications. As required, the maximal failure index is below the critical value of 1. However, the increase of mass is only 5.17%. This is remarkable keeping in mind, that the strength of the solution with double layup (which has a mass increase of 100%) is not higher.

Design	Max FI	Mass [kg]	Mass [%]
Initial	4.02	0.022606	100.00
Optimized	0.99	0.023774	105.17
Double	1.00	0.045212	200.00

Tab. 1: Design summary

The proposed patch reinforcement process provides a locally reinforced design that is superior to a design with fully covering layers in terms of strength-to-mass ratio.

4 Conclusion

The usage of the proposed pseudo strength function does not guarantee that all failure indices fall below a required value. Nevertheless, it can be shown that the strength of a thin plate with hole can be increased significantly. The reinforcement doublers keep the weight low since mass is respected in the sensitivities due to the linear dependence on the thickness. The sensitivities consider the effect of a thickness change in one element to the whole structure. Thus, regions can be localized where a change in thickness affects the strength in other regions. This is not possible when only local stresses are taken as decision parameters.
Locally reinforced designs have superior strength-to-mass ratios compared to designs with fully covering layers. The proposed process has high potential to find extreme light-weight structures. Location, geometry, as well as orientation of the reinforcement doublers are result of the iterative generation process. The process is effective and the created designs can hardly be found intuitively.

Appendix: Material Properties

E_{11}	138	GPa
E_{22}	9	GPa
G_{12}	4.5	GPa
G_{23}	3.5	GPa
G_{13}	4.5	GPa
ν_{12}	0.34	-
ρ	1556	kg/m^3
X_t	1500	MPa
X_c	1250	MPa
Y_t	40	MPa
Y_c	200	MPa
S	25	MPa

Tab. 2: Material properties CFRP/Epoxy

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