HYPERELASTIC APPROACH FOR THE SIMULATION OF WOVEN REINFORCEMENTS AT MESOSCALE

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1 Introduction

The high specific properties and the wide range of shapes which can be processed are the principal reasons why fabric-reinforced composites keep developing in state of the art aeronautical industries and are generalising amongst other industries. The so-called RTM (Resin Transfer Moulding) process is a typical process used to form fabric-reinforced parts: the dry reinforcement is first deformed to match the shape of the final part, then the polymer resin is transferred by injection or infusion (different variations of RTM process).

Intense research efforts concentrate on simulating the behaviour of dry woven reinforcements during the first step of RTM process (preforming step): during this step, the yarns get their final shape and fibre density. These properties are of great interest since the final part directly inherits its stiffness properties from them, and since they represent the initial conditions for the simulation of the transfer step.

This information about the shape and density of the yarns lies at the scale of the woven cell. In this paper a model is proposed for the unit cell of plain weave glass fabric reinforcements. A hyperelastic behaviour law is set up to describe the behaviour of the yarn, considered as a continuum at the scale of the unit cell. The results of this model are compared to experimental results of biaxial traction test and picture frame test, and show good overall concordance.

2 Mechanical behaviour of woven reinforcements

2.1 Biaxial traction

Biaxial traction test are non-trivial tests which are presented in [1]. It consists in subjecting warp and weft networks of the fabric to longitudinal deformations. The ratio between the deformations in warp and weft directions is denoted $k=\varepsilon_1/\varepsilon_2$ (where *1* is the direction along which the traction force is measured). The particular case $k=1$ (also referred to as equibiaxial traction) is of much importance for the identification of the behaviour law associated to the compaction and distortion of the yarn in the transverse plane: when both networks are submitted to identical elongation, the yarns cross sections undergo large transformation.

2.2 Picture frame test

Picture frame test [2] is one of the two reference tests used for the characterisation of shear resistance of fabric-reinforced composites. A pin-jointed frame constrains the fabric to undergo pure shear deformation (fig.1). The measured force and displacement data are post processed to obtain an equivalent shear torque and the shear angle. The equivalent shear torque is computed from picture frame experiments by the formula:

$$C_s = F\,\frac{L_{frame}}{L^2_{fabric}}\,\frac{\cos(\gamma)}{2\cos(\pi/4-\gamma/2)} \qquad (1)$$

with F the effort measured on the frame, L_{frame} the side of the frame, L_{fabric} the side of the square of fabric inside the frame and γ the angle variation between warp and weft directions (i.e. the shear angle). On the other hand, an energy-based approach allows the calculation of this torque per unit initial area from the simulation:

$$C_s = \frac{\dot{W}}{S_u\,\dot{\gamma}} \qquad (2)$$

where $\dot{W}$ is the time-derivative of the external work, S_u the initial surface of the unit cell and $\dot{\gamma}$ the time-derivative of the shear angle

During the first phase of the shear deformation, the warp and weft rotate with only small deformation of the yarns. When the so-called jamming angle of the fabric is reached, higher efforts appear as well as finite deformation of the cross section of the yarns.

One aim of the model presented here is to describe precisely the changes of geometry of the unit cell during these two phases of shear deformation, so that permeability calculation can be performed by using the obtained solid skeleton of the woven unit cell.

3 Behaviour law for the yarns

3.1 Hyperelastic materials

A hyperelastic material is a non-dissipative material whose stress tensor derivates from stored energy density function. If $\underline{\underline{S}}$ denotes the second Piola-Kirchhoff tensor, w the stored energy density function and $\underline{\underline{C}}$ the right Cauchy-Green tensor, the general form of the constitutive law of hyperelastic materials is:

$$\underline{\underline{S}} = 2\frac{\partial w\left(\underline{\underline{C}}\right)}{\partial \underline{\underline{C}}} \tag{3}$$

The theory of invariants [3] states that at most five invariants are needed to describe the behaviour of transversally isotropic materials. The stored energy density function can then be defined as a function of the following invariants:

$$w\left(\underline{\underline{C}}\right) = w\left(I_1, I_2, I_3, I_4, I_5\right) \tag{4}$$

where

$$\begin{gathered} I_1 = \mathrm{trace}\left(\underline{\underline{C}}\right), \\ I_2 = \frac{1}{2}\left(\mathrm{trace}\left(\underline{\underline{C}}\right)^2 - \mathrm{trace}\left(\underline{\underline{C}}^2\right)\right), \\ I_3 = \det\left(\underline{\underline{C}}\right),\ I_4 = \underline{\underline{C}} : \underline{\underline{M}},\ I_5 = \underline{\underline{C}}^2 : \underline{\underline{M}} \end{gathered} \tag{5}$$

Amongst these invariants, the first three are the principal invariants of the right Cauchy-Green tensor, and the last two are mixed invariants which characterize the anisotropy of the material. The tensor $\underline{\underline{M}}$ is the structural tensor built from the principal direction $\underline{m}$ of the transversely isotropic material: $\underline{\underline{M}} = \underline{m} \otimes \underline{m}$.

3.2 Physically based invariant formulation

In order to define a physically motivated behaviour law, different invariants $\left(I_{ext}, I_{comp}, I_{dist}, I_{sh}\right)$ obtained as combinations of the invariants (5), are used to describe the transformation of the yarns. The different deformation modes characterised by these new invariants are respectively:

- the extension of the yarn in the direction of fibres (I_{ext}),
- the compaction of the yarn in the plane of isotropy (I_{comp}),
- the distortion of the yarn in the plane of isotropy (I_{dist}),
- the shear along fibres (I_{sh}).

These deformation modes are assumed to be independent, which means that no couplings between them are considered. According to this uncoupling assumption, an energy density function is associated to each invariant: the whole energy density function is written under the form:

$$\begin{aligned} w_{tot}\left(\underline{\underline{C}}\right) &= w_{ext}\left(I_{ext}\right) + w_{comp}\left(I_{comp}\right) \\ &+ w_{dist}\left(I_{dist}\right) + w_{sh}\left(I_{sh}\right) \end{aligned} \tag{6}$$

The second Piola-Kirchhoff stress tensor is then obtained by differentiation of (6) by using (3):

$$\begin{aligned} \underline{\underline{S}} &= \frac{\partial w_{ext}}{\partial I_{ext}}\frac{\partial I_{ext}}{\partial \underline{\underline{C}}} + \frac{\partial w_{comp}}{\partial I_{comp}}\frac{\partial I_{comp}}{\partial \underline{\underline{C}}} \\ &+ \frac{\partial w_{dist}}{\partial I_{dist}}\frac{\partial I_{dist}}{\partial \underline{\underline{C}}} + \frac{\partial w_{sh}}{\partial I_{sh}}\frac{\partial I_{sh}}{\partial \underline{\underline{C}}} \end{aligned} \tag{7}$$

Finally, the Cauchy stress tensor is computed by use of the usual formula:

$$\underline{\underline{\sigma}} = \frac{1}{J}\underline{\underline{F}} \cdot \underline{\underline{S}} \cdot \underline{\underline{F}}^T \tag{7}$$

3.3 Identification of the behavior law

A piecewise nonlinear behaviour of the yarn in extension is defined, as some types of yarn (usually glass fibres yarns) may have a stiffening behaviour at the beginning of an extension test. The parameters of this extension behaviour law are directly identified with an extension test on the yarn alone. As it can be seen on Fig.2, curve “Yarn”, the obtained identified behaviour matches the extension behaviour of the yarn very well. Because direct tests on the yarn, other than extension test, are difficult to perform on the yarn itself, the form of the strain energy density function of the other deformation modes are postulated, and then identified with an inverse method using macroscopic tests on the fabric.

The behaviour law associated to the shear along fibres is assumed to show a linear response of stresses, which only requires a simple, quadratic, strain energy density function. As the shear along fibres have a significant role in the nonlinearity of the extension behaviour of the whole fabric (see Fig.2, “free” curve), the parameter of this energy density can be identified by using such an extension test, as soon as the extension behaviour of the yarn alone is known. As previously mentioned, direct tests on the yarn are difficult to envisage. An inverse method, using Levenberg-Marquardt optimisation, is then set up for the identification of compaction and distortion energy density functions. As these deformation modes are significantly solicited during $k=1$ biaxial tension test, this test is used as the reference test for the optimization of parameters. The curves obtained at each iteration of the Levenberg-Marquardt algorithm are shown on Fig.3.

4 Finite element simulations

The hyperelastic constitutive equation has been implemented in Abaqus FEA software. The finite element model used for the simulations depends on the symmetry of the boundary conditions [4]: the whole unit cell has to be modelled for shear simulations, while only the eighth of the unit cell is required for biaxial tension simulations. After identification, the simulation of biaxial tension shows good accordance with the experimental tests, as regards force-deformation data (fig. 2) and geometry. The result of shear simulation (fig. 4) as regards geometry is also in quite good accordance with experimental observations. As shown in Fig.5, the equivalent shear torques computed from picture frame test and from simulation are also in good accordance.

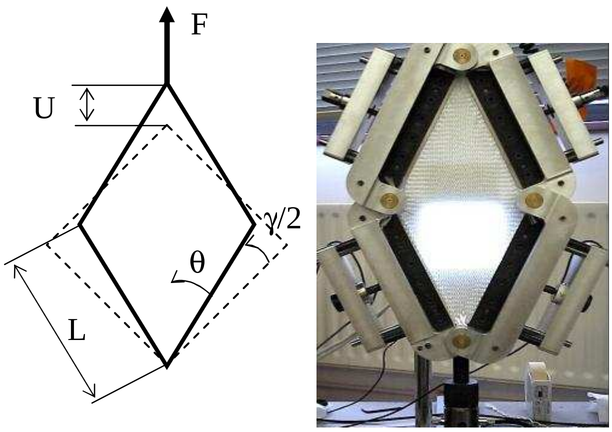

Fig.1. Picture frame test on a plain weave glass fabric

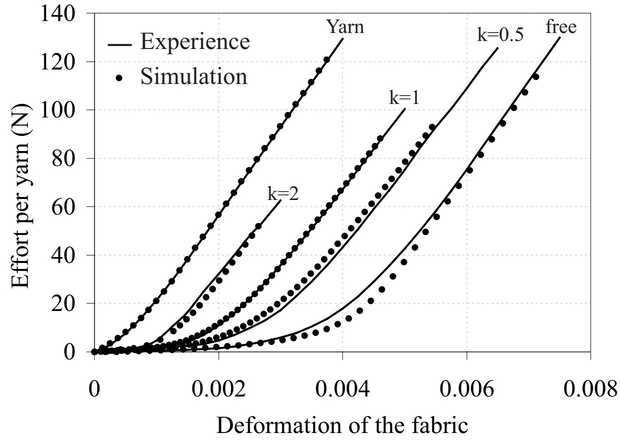

Fig.2. Comparison of experiment and simulation data for biaxial traction

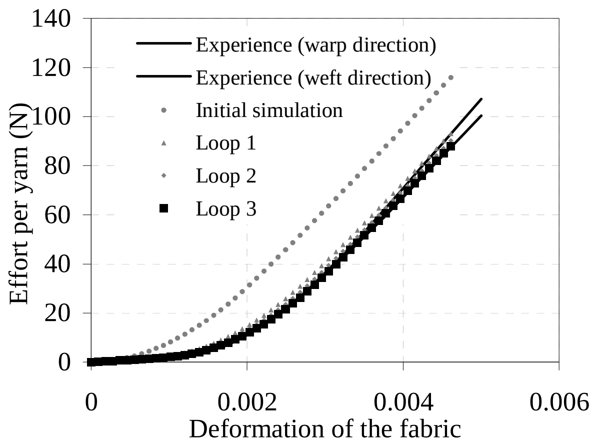

Fig.3. Equibiaxial tension curves obtained during the optimisation of compaction and distortion behaviour

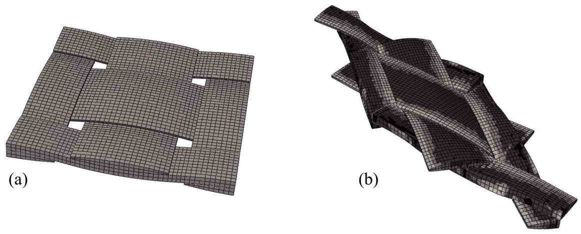

Fig.4. Simulation of the shear of the unit cell

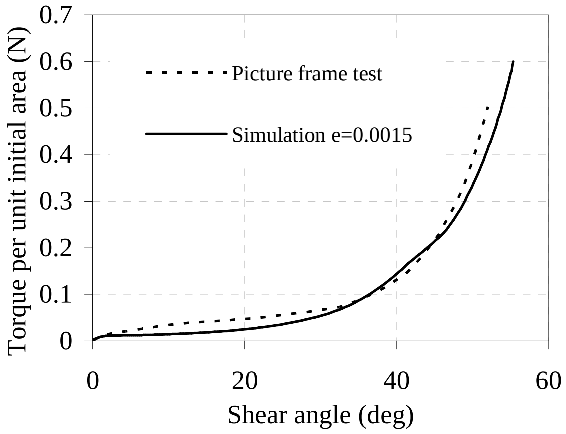

Fig.5. Shear torque by unit initial area computed from simulation and from experimental picture frame test

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