

RELIABILITY ANALYSIS OF ADHESIVE BONDED STEPPED LAP COMPOSITE JOINTS BASED ON DIFFERENT FAILURE CRITERIA

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1 General Introduction

Adhesive bonded joints are being extensively used for a large variety of applications in the automotive, aerospace, civil engineering, marine and wind turbine industries to mention a few [1]. Adhesive bonded joints are gaining preference over mechanical fastening techniques because of their almost negligible weight penalty [2], while mechanical fastening employs screws, nuts, bolts and rivets, etc., which adds significantly to the weight of the structures and reduces the load-bearing capacity. Furthermore, mechanical fastening requires cut outs and holes in structures leading to severe stress concentrations.

Among the commonly used adhesive bonded joint configurations, scarf and stepped joints have been found to exhibit the highest structural efficiency because significant joint eccentricities (which ultimately act as stress raisers) are eliminated along the loading paths when compared with simple lap joints. In addition, a more uniform stress distribution is obtained across the joint [2]. Large variations in joint strength occur in adhesive bonded joints, and it is therefore necessary and important to investigate the stress transfer and to assess the reliability of adhesive joints.

In the design of stepped lap adhesive joints, scattering and physical as well as subjective uncertainties including neglect, mistakes, incorrect modelling and manufacturing errors must be considered when designing for materials, stacking sequence, dimensions, etc. Accordingly, the development and implementation of a reliability-based design methodology is of vital importance in rational design [3].

In this paper a probabilistic model for the reliability analysis of a stepped lap adhesive composite joint subjected to external loading relevant for wind turbine blades is presented using a 3D FEA modelling. After validation of the FEA model, sensitivity analyses are carried out with respect to the influence of various geometrical and material property parameters on the maximum bond line stress and different failure criteria. Partial safety factors are introduced together with characteristic values. The von Mises, a modified von Mises and the maximum stress failure criteria are applied for the adhesive bond line. The failure criteria are applied to assess the reliability modelling of the uncertain parameters by stochastic variables. Further, calibration of partial safety factors is investigated.

2 Stepped Lap Composite Joint

Fig. 1 shows a model of the considered stepped lap composite joint. Three different materials are used, epoxy adhesive, graphite epoxy and glass epoxy. Each layer includes 8 lamina and the thickness is the same of all lamina. Table (1) shows a stochastic model for the geometrical properties. The geometrical properties are typically assumed to be Normal distributed. No information or measurements are available at present for the coefficients of variation (COV). These are chosen to 10%, but should be verified by measurements on real stepped lap joints. The material properties for epoxy adhesive, graphite-epoxy and glass-epoxy are shown in Tables 2-4, respectively [4, 5, 10].

Fig. 2 shows the FE model and the adopted FE meshing. A macro is used to generate a parametric model where the size of elements through the adhesive thickness is chosen to $t_l/4$ where t

(laminare thickness) is obtained as realisations of a stochastic variable modelling of the thickness. The loading is applied through prescribed displacements, and solid shell elements are used for the composite part and solid elements for the adhesive layers. The analyses are performed assuming linear elastic material behaviour and small displacements. The commercial FE code ANSYS version 12.1 has been used for all the FE simulations.

3 Failure criteria

Previous studies have shown that the assumption of linear elasticity of the adhesive is not realistic [6]. Thus, the response of most polymeric structural adhesives is inelastic in the sense that plastic residual strains are induced even at low levels of loading. One approach to address this could be the concept of effective stress/strain. It assumes, in a ductile material, that plastic residual strains are large compared with the creep strains at normal loading rates [6]. Accordingly, a plastic yield hypothesis can be applied, and the multi-directional state of stress can be related to a simple unidirectional stress state through a function similar to that of von Mises. However, the yield behaviour of polymeric structural adhesives is generally dependent on both deviatoric and hydrostatic stress components. A consequence of this is a difference between the yield stresses in uniaxial tension and compression [6]. Gali et al [7] investigated this behaviour by proposing a modified von Mises criterion:

$$S_e = C_s \sqrt{J_{2D}}^{\frac{1}{2}} + C_v J_1 \quad (1)$$

where

$$C_s = \frac{\sqrt{3}(1+\lambda)}{2\lambda}, \quad C_v = \frac{(\lambda-1)}{2\lambda}; \quad \lambda = \frac{\sigma_c}{\sigma_t} \quad (2)$$

and

$$J_{2D} = \frac{1}{6} \left[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 \right] \quad (3)$$

$$J_1 = \sigma_1 + \sigma_2 + \sigma_3$$

It should be noted that by choosing $\lambda = 1$ the modified von Mises criterion reduces to the von Mises criterion.

For the failure prediction of composite laminates subjected to a complex stress state a number of failure models and criteria have been proposed [8, 9]. In this study the first ply failure (FPF) concept is applied. Thus it is assumed that the laminated composite has failed when failure has occurred in any of the layers [8]. To simplify the analyses, and without loss of generality, it is usually assumed that the failure probability of the laminate can be approximated by the maximum failure probability estimated in any layer of the lamination sequence [8]. Therefore, the probability of failure of the laminate, P_f , is estimated by:

$$P_f = \max P_f^i \quad (4)$$

where P_f^i is the probability of failure of layer no i . For the adhesive layers, the equivalent stress S_e , which is obtained from the failure criteria, is compared with the ultimate strength. Thus, the probability of failure for the adhesive is estimated by:

$$P_f = P(X_R S_{ultimate} - S_e \leq 0) \quad (5)$$

where the model uncertainty related to the load carrying capacity is given by the stochastic variable X_R . It should be noted that only adhesive layer failure is considered in this study. The formulation can be extended such that a deterministic design equation can be derived (i.e. calculation of the load by using a load multiplier) where partial safety factors are introduced together with characteristic values. Further, a load model relevant for wind turbine blades can be applied, i.e. a model for the load S being described by a number of stochastic variables. Both stochastic models for standstill (parked) and for operation can be applied.

Here a design equation is considered as follows. It is assumed that the wind turbine is parked (not producing power) and only flapwise loads from the wind are taken into account. The design equation is expressed as:

$$G = \frac{1}{\gamma_n} \frac{S_{ultimate}}{\gamma_m} - \gamma_f S_e(L_c) = 0 \quad (6)$$

where γ_n , γ_m and γ_f are partial safety factors for the consequences of failure, material properties and load, respectively, see Table (5). It is assumed that the characteristic load carrying capacity $S_{ultimate}$ can be obtained by inserting characteristic material properties. The characteristic material properties are determined as 50% / 5% quantiles [10]. Usually, the characteristic wind pressure is determined as a 98% quantile in the distribution function for the annual maximum wind pressure, corresponding to a return period of 50 years.

To obtain the characteristic load, first the 5% quantile of the adhesive strength is calculated based on the Weibull distribution in the adhesive layer where failure occurs. The maximum allowable characteristic load is then obtained through FE analysis by calculating $S_e(L_c)$, the average stress over the bond lines. When L_c is obtained, it is used that it is a 98% quantile in the distribution for the annual maximum load, L_s that is assumed to be modelled by a Weibull distribution with COV = 15%. This procedure is used to obtain the stochastic load model for each λ parameter. Further, a load model relevant for wind turbine blades is applied. The load is described in terms of a number of stochastic variables:

$$L = L_s X_{dyn} X_{exp} X_{st} X_{aero} \quad (7)$$

where the load and the model uncertainties are split into their respective components. X_{dyn} is the uncertainty related to the modelling of the dynamic response for the wind turbine, such as damping ratios and eigenfrequencies. X_{exp} is the uncertainty related to the modelling of the exposure, such as the terrain roughness and the landscape topography. X_{st} is taking the statistical uncertainty related to the limited amount of wind data into account, and X_{aero} is related to the uncertainty in assessment of lift and drag coefficients. The stochastic variables used in the limit state function are given in Table (6) [4, 5].

The limit state equation corresponding to the design equation (6) is written as:

$$g = X_R S_{ultimate} - S_e(L) \quad (8)$$

4 Methodology and approach

To calculate the probability of failure and the corresponding reliability index, the Crude Monte Carlo simulation technique is used. The main reason why other reliability methods, such as the First Order Reliability Method, are not used is that some of the failure criteria are discontinuous functions of the stochastic variables.

The FE code ANSYS is run in batch mode from Matlab using geometric parameters, material properties and loads simulated from the distribution functions describing the stochastic variables. Each simulated parameter is read by ANSYS using a macro file, and after the numerical processing a post processing is carried out. The stresses and strains are selected and imported to Matlab. The average stresses are calculated over the 8 adhesive bond lines and finally, based on the chosen failure criterion, the number of failures is calculated. This procedure is conducted for 10000 simulated realisations, and finally the probability of failure (P_f) is obtained by dividing the total number of failures by the number of simulations. The reliability index, β is obtained from:

$$P_f = \Phi(-\beta) \quad (9)$$

where Φ is the standard Normal distribution function.

5 Sensitivity analysis

To show the influence of material properties on the maximum stress and failure criteria a sensitivity analysis is performed. The influence of various geometrical parameters and material properties on the maximum stress, as well as the influence of the adhesive thickness, Young modulus, fibre angles, loading etc. is investigated. As shown in Table (7), loading, E_I Glass/Epoxy, E adhesive and fibre angles are the most important parameters. Approximate measures of the importance of the stochastic variables on the reliability index can be obtained by assuming a linear approximation and Normal distributed variables. If $\partial g / \partial x_i$ is the derivative of the limit state equation with respect to the stochastic variable x_i and σ_i is the standard deviation of the variable X_i , then $\partial g / \partial x_i \sigma_i$ is a measure of the importance on the reliability index of

the variable X_i . Approximate α -values are obtained from:

$$\alpha_i \approx \frac{-\frac{\partial g}{\partial x_i} \sigma_i}{\sqrt{\sum_i \left(-\frac{\partial g}{\partial x_i} \sigma_i\right)^2}} \quad (10)$$

Also a sensitivity analysis has been performed to show the influence of the parameter λ in the modified von Mises criterion on the output equivalent stress. Variation of the ratio between equivalent stress according to the modified von Mises criterion and the equivalent stress according to the classical von Mises criterion for varying values of the parameter λ are presented in Fig. 3.

6 Results and discussions

Initially a convergence test for the FE model is performed to ensure that the FE model has an appropriate number of elements as shown in Table (8). During this test, the number of elements in the FE model is increased until the point at which the obtained results converge, thus yielding the proper number of elements for the FE model.

Based on the random variables and the limit-state equation, the reliability for the three different failure criteria considered is calculated. The influence of increasing the proportion of deviatoric and hydrostatic stresses on the probability of failure is investigated expressed in terms of the factor λ , which is the ratio of the compressive to tensile yield stress ($\lambda=1$ for the von Mises criterion). The results show that for a single applied (tensile) load, the influence of the λ parameter on the probability of failure is almost negligible. The results show that the choice of failure criteria is very important for assessment of the probability of failure and thereby also for calibration of the partial safety factors.

Since the number of simulations is limited, the estimate of the probability of failure will be subjected to statistical uncertainty. Therefore the probability of failure, P_F , and the corresponding reliability index, $\beta = -\Phi^{-1}(\sqrt{P_F})$, are expressed by a confidence interval. The standard error for the

probability of failure using Crude Monte Carlo simulation is obtained from:

$$s = \sqrt{\frac{P_f(1-P_f)}{N}} \quad (11)$$

Confidence intervals for the estimate of the probability of failure can be determined using that P_f becomes asymptotically Normal distributed for $N \rightarrow \infty$. Eq. (11) can also be used to determine the required number of simulations to obtain a required accuracy of the estimate for the probability of failure. The obtained results for the probability of failure, the reliability index and the confidence interval based on different failure criteria are presented in Table (9). Implicitly the IEC-61400-1 [11] standard requires a minimum reliability index for structural wind turbine component equal to 3.1. Therefore in this example the reliability level is satisfactory, and further indicates that the partial safety factors could be decreased slightly.

6 Conclusions

A probabilistic model for the reliability analysis of adhesive bonded stepped lap joints has been presented. The defects in the adhesive stepped lap joints are an outcome from the production process and will influence the reliability of the component. The influence of variations in the material strength over the joint has been studied. The reliability function for the adhesive joints was derived using stochastic distributions, and calculation of the reliability of the adhesive joints was based on the First Ply Failure (FPF) criterion. The influence of failure criteria on the probability of failure has been discussed, and the reliability index for each criterion has been calculated. In addition, the influence of λ , which is the proportion of the deviatoric and hydrostatic stresses, on the reliability has been analyzed.

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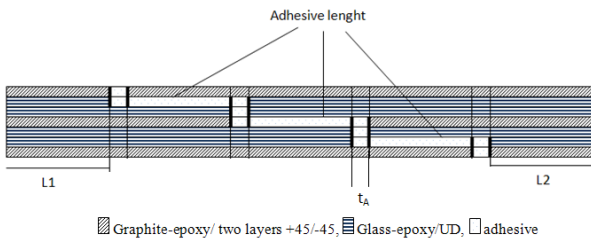


Fig. 1. Geometry of adhesive stepped lap joint.

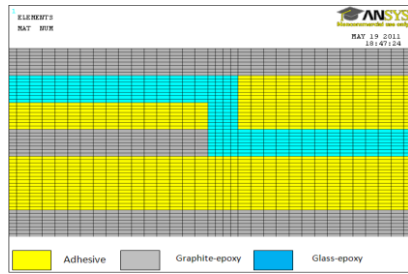


Fig. 2. FE mesh for adhesive stepped lap joint.

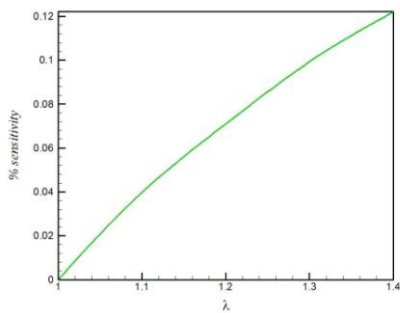


Fig. 3. Sensitivity analysis for λ to show the variation of von Mises criterion with respect to $\lambda=1$

Table (1). Stochastic variables for the geometry.

Parameter	Symbol	Mean value	COV	Distribution
Lamina thickness (mm)	t_l	0.125	10%	Normal
Layer thickness (mm)	t_L	1	10%	Normal
Adhesive thickness (mm)	t_A	1	10%	Normal
Initial length (mm)	L_1	40	10%	Normal
Lateral length (mm)	L_2	40	10%	Normal
Fibre angles	-	-	10%	Normal

Table (2). Stochastic variables for epoxy adhesive.

Parameter	Mean	COV	Distribution	Characteristic value
E (GPa)	2.21	10.0%	Lognormal	2.21
ν	0.4	18.0%	Lognormal	0.4
S (MPa)	45	10.6%	Weibull	37 (5% quantile)

Table (3), Material properties for Graphite-epoxy

Parameter	Mean	COV	Distribution
E_1 (GPa)	131	10.60%	Lognormal
E_2 (GPa)	8	13.60%	Lognormal
E_3 (GPa)	8	13.60%	Lognormal
ν_{12}	0.3	18.00%	Lognormal
ν_{13}	0.3	18.00%	Lognormal
ν_{23}	0.07	18.00%	Lognormal
G_{12} (GPa)	5	10.00 %	Lognormal
G_{13} (GPa)	5	10.00%	Lognormal
G_{23} (GPa)	4	10.00%	Lognormal

Table (4), Material properties for Glass-epoxy

Parameter	Mean	COV	Distribution
E_1 (GPa)	39	10.6%	Lognormal
E_2 (GPa)	14.5	13.6%	Lognormal
E_3 (GPa)	9.8	13.6%	Lognormal
ν_{12}	0.29	18.0%	Lognormal
ν_{13}	0.07	18.0%	Lognormal
ν_{23}	0.29	18.0%	Lognormal
G_{12} (GPa)	4.2	10.7%	Lognormal
G_{13} (GPa)	4.2	10.7%	Lognormal
G_{23} (GPa)	2.7	10.7%	Lognormal

Table (5): Partial safety factors according to IEC 61400-1 [11].

Partial Safety Factor	Ultimate
γ_n – Consequences of failure	1.00
γ_m – Material properties	1.30
γ_f – Load	1.35

Table (6). Stochastic variables for the model and physical uncertainty related to the loading.

Variable	Description	Distribution	Mean	COV
X_R	Load carrying capacity	Lognormal	1	5%
X_{st}	Limited wind data	Lognormal	1	10%
X_{dyn}	Dynamic response	Lognormal	1	5%
X_{exp}	Exposure	Lognormal	1	10%
X_{aero}	Lift/Drag coefficients	Gumbel	1	10%

Table (7), Sensitivity analysis for material properties and geometry.

Parameter	α -values
Load	0.817
E1 Glass/Epoxy	0.422
E adhesive	-0.363
Glass/Epoxy fibre angle	-0.100
Adhesive thickness	0.092
Graphite/Epoxy fibre angle	-0.086
v adhesive	-0.005
E1 Graphite/Epoxy	0.004
E2 Graphite/Epoxy	0.001
E2 Glass/Epoxy	0.001

Table (8), Convergence study of the FE model.

Average Stress	Number of elements
2.25E+07	4843
2.28E+07	8192
2.28E+07	12288

Table (9). Probability of failure for different failure criterion based on 10000 simulations.

Failure criteria	Von Mises ($\lambda=1$)	Modified von Mises $\lambda=1.4$	Max stress
Probability of failure	$8 \cdot 10^{-4}$	$8 \cdot 10^{-4}$	$6 \cdot 10^{-4}$
Reliability index	3.15	3.15	3.23
Standard error	$2.82 \cdot 10^{-4}$	$2.82 \cdot 10^{-4}$	$2.44 \cdot 10^{-4}$
Probability bounds (95% confidence)	$[5 \cdot 10^{-4}, 11 \cdot 10^{-4}]$	$[5 \cdot 10^{-4}, 11 \cdot 10^{-4}]$	$[3 \cdot 10^{-4}, 8 \cdot 10^{-4}]$
Reliability index bounds (95% confidence)	[3.06, 3.28]	[3.06, 3.28]	[3.14, 3.38]

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