

A NEW REGULARIZED VIRTUAL FIELDS METHOD FOR COMPOSITE MATERIAL PARAMETERS IDENTIFICATION

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1 Introduction

Accurate identification of composite materials constitutive parameters is a complex problem. Full-field measurement techniques, such as Digital Image Correlation (DIC) and Digital Volume Correlation (DVC), have progressively spread in the field of mechanical properties identification of composites constituents. These techniques can provide displacement or strain fields on the surface or even inside opaque materials, subjected to external loadings. Thanks to the experimental availability of such rich information, several identification techniques, either in the form of an inverse problem like Finite Element Model Updating (FEMU) [1] or direct methods like Virtual Fields Method (VFM) [2] have been developed. The FEMU method has been widely exploited to determine mechanical constants of composites [3,4]. The method aims at iterative updating of input mechanical parameters into a Finite Element (FE) model so as to minimize the discrepancy between measured and numerically (FE) predicted displacement fields.

The VFM, as its name implies, is developed based on the principle of virtual work [5]. Assuming that the deformation field over the surface of material is known, the aim in the virtual fields method is writing the equilibrium equation of internal and external work using a set of admissible virtual displacement and strain fields. In the case of linearly elastic materials this equilibrium leads to a linear system of equations, through which the unknown parameters are directly identified. The identification procedure was first developed by Grédiac [6] and has been successfully applied for determining material constants from elastic as well as elastoplastic constitutive models [7,8]. The method has been applied to identify bending [9,10] and in-plane [11,12] properties of composite materials. Through-thickness characterization of composites either with

a linear elastic or a nonlinear behavior, have also been studied [13]. The VFM is favored over the other identification methods due to some advantages. Unlike inverse methods, the VFM does not rely on the finite element calculations and allows direct identification (no updating) of constitutive parameters. Furthermore, in contrast to the FEMU method, the VFM does not necessarily require an iterative optimization procedure and no initial values are required for identifying sought parameters. Moreover, the sensitivity to noise in the measured data is less than the other identification methods, as investigated for instance by Avril et al. [14]. This method exhibits however some drawbacks when dealing with composites whose constituents stiffnesses possess significant contrast, such as E-glass-epoxy or carbon-epoxy composites, especially in the presence of noisy strain fields. These composites have higher-order heterogeneity of strain fields and the high contrast phenomenon might result in inappropriate solutions for the stiffer phase that undergoes much less deformation.

An improved FEMU strategy, namely Regularized Model Updating (RMU) [15] was recently introduced by the present authors and was successfully validated by conducting several virtual experiments. Regularization constraints based on a micromechanical homogenization model were added to the optimization algorithm in order to nullify noise effects that may adversely influence the quality of identified parameters. It was indicated that adding regularization terms enhances significantly the efficiency of the identification procedure in comparison with the FEMU method.

In this study a new Regularized Virtual Field (RVF) methodology for the accurate mechanical parameters identification of composite materials constituents is proposed. The novelty of this approach is that mechanical constraints, consisting of a

micromechanical homogenization model, are used in an optimization problem to regularize solving the related system of linear equations of the virtual fields method. For the purpose of evaluating the performance of the proposed approach, it was applied to identify elastic properties of a 2D artificial aligned long fiber composite from noisy full-field measured strain fields.

2 The Virtual Field Method

The VFM is basically developed through writing the global equilibrium of a body subjected to a given load, with the principal of virtual work (PVW) [2]. The idea is to expand the PVW with a set of independent Kinematically Admissible (KA) virtual fields, and to build up a linear system of equations involving the unknown parameters. By introducing the stress-strain relation in homogeneous material as

$$\sigma_i = Q_{ij} \varepsilon_j \quad (1)$$

for plane stress problems the principle of virtual work can be written as

$$\int_S Q_{ij} \varepsilon_j \varepsilon_i^* ds = \int_L \bar{\mathbf{T}}_i \mathbf{u}_i^* dl \quad \forall \mathbf{u}^* \text{ KA} \quad (2)$$

where Q_{ij} 's are the components of the in-plane stiffness matrix, \mathbf{u}^* is the virtual displacement field, ε is the full-field measured strain field, ε^* is the virtual strain field, S is overall surface of specimen and \mathbf{T} is applied forces acting on its boundary. In the case of linear homogenous materials the matrix form of stress-strain constitutive equation is as below

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{Bmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} & 0 \\ Q_{yx} & Q_{yy} & 0 \\ 0 & 0 & Q_{ss} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_s \end{Bmatrix} \quad (3)$$

For the case of linear elasticity, having the following relations between stiffness matrix components

$$\begin{aligned} Q_{xx} &= Q_{yy}, \quad Q_{xy} = Q_{yx} \\ Q_{ss} &= \frac{(Q_{xy} - Q_{yx})}{2} \end{aligned} \quad (4)$$

the principal of virtual works can be written as

$$\begin{aligned} Q_{xx} \int_S (\varepsilon_x \varepsilon_x^* + \varepsilon_y \varepsilon_y^* + \frac{1}{2} \varepsilon_s \varepsilon_s^*) dS + \\ Q_{xy} \int_S (\varepsilon_x \varepsilon_y^* + \varepsilon_y \varepsilon_x^* - \frac{1}{2} \varepsilon_s \varepsilon_s^*) dS = \\ \int_{L_f} \bar{\mathbf{T}}_i \mathbf{u}_i^* dl \quad \forall \mathbf{u}^* \text{ KA} \end{aligned} \quad (5)$$

If as many different virtual fields as there are unknown parameters be chosen, Equation (1) leads to the following linear system of equations (general form):

$$\mathbf{A}\mathbf{Q} = \mathbf{B} \quad (6)$$

where \mathbf{A} is a square matrix and \mathbf{B} is a vector whose components are the virtual work of the applied forces. The vector of unknown parameters \mathbf{Q} can be determined by solving the above linear system.

The choice of an appropriate set of virtual fields is one of the key-points for obtaining satisfactory identified parameters. The related virtual fields are chosen so that the resulting equations be linearly independent. Depending on the geometry of the specimen and its actual boundary condition they can be chosen intuitively but satisfy some requirements. Firstly, they have to be differentiable and have C^0 continuity. In addition, they must be Kinematically Admissible i.e. be null over a portion of specimen where the displacement boundary condition is applied for supporting and therefore distribution of external loading is unknown.

3 Homogenization model

The micromechanical homogenization models relate the composite constituents properties to its overall properties. The Mori-Tanaka [16] homogenization, as one of the most accurate models has been chosen to be used in this study. According to this model, the stiffness tensor C of a two-phase linearly elastic composite can be expressed as

$$C = C^0 + c(C^1 - C^0) : D \quad (7)$$

where C^0 and C^1 are the stiffness tensors of the matrix and the fibers, respectively, c denotes the volume fraction of the fibers and D is the strain

localization tensor given by

$$D = T^1 : \left((1-c)T^0 + cT^1 \right)^{-1} \quad (8)$$

where $T^0 = I$ (identity matrix), and T^l can be expressed as follows:

$$T^1 = \left[I + S^1 : C^{0-1} : (C^1 - C^0) \right]^{-1} \quad (9)$$

Note that S^l is Eshelby's tensor and, for the aligned and continuous fiber composites, is only a function of the matrix Poisson's ratio.

4 The Regularized Virtual Fields (RVF) method

The linear system of Eq.(6) in VFM is often solved by matrix inversion method. The other alternative is to solve the system of equation through an optimization procedure, i.e. minimization of the following least square objective function:

$$R(Q) = (AQ^{(k)} - B^{(k)})^T \cdot (AQ^{(k)} - B^{(k)}) \quad (k = 1, 2, \dots, M) \quad (10)$$

where the value of M is equal to the number of virtual fields (unknown parameters). The key idea herein is to add a set of mechanically relevant constraints to the minimization of $R(Q)$ so that the predicted effective properties from Mori-Tanaka homogenization model match those from experimental measurements. Hence, the RVF method aims at minimizing $R(Q)$ subject to the following equality constraints:

$$(\lambda_i^{MT}(Q) - \lambda_i^{\exp}) = 0 \quad (i = 1, 2, \dots, N) \quad (11)$$

Where λ^{MT} and λ^{\exp} are the effective mechanical properties obtained from the Mori-Tanaka model and from the experimental procedure, respectively, and N is number of constraints. The imposed constraints define a feasible promising region for the updating parameters relying on the effective properties. Assuming that the composite's effective stiffness in the out-of-plane, i.e. z direction, is known and also depends on the fiber and matrix properties, the imposing constraints restricts the algorithm to follow some rational search directions during optimization. Therefore, adding a constraint

for the out-of-plane stiffness when minimizing $R(Q)$ can improve the accuracy of the identified parameters since additional physical information is added to the problem.

5 Elastic properties identification of a 2D virtual composite using RVF method

The methodology consisted first in generating a finite element model of an uni-directional long fiber composite, subjected to a set of boundary conditions. The computer-generated composite consisted of an isotropic matrix reinforced with randomly distributed infinite isotropic cylindrical fibers. Fig. 1

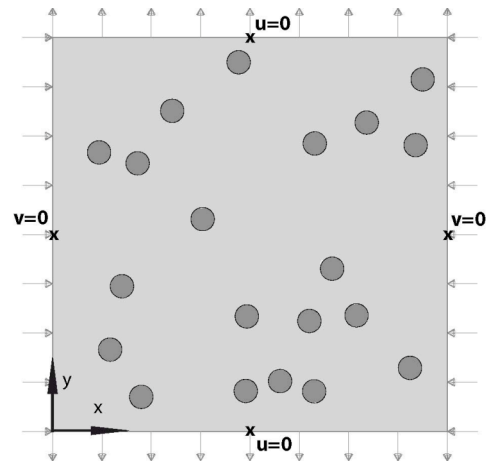


Figure 1. 2D virtual composite (volume fraction=5.5%)

illustrates a cross section of this composite (volume fraction of 5.5%). Two different contrasts of elastic modulus were studied. The four reference elastic properties (elastic modulus and Poisson's ratio of fibers and matrix), being closely related to E-glass-epoxy ($E_f/E_m=21$) and carbon-epoxy ($E_f/E_m=100$) composites, are presented in Table 1.

Table 1. Reference elastic properties

Composite	E_f (GPa)	ν_f	E_m (GPa)	ν_m
A	74	0.2	3.5	0.35
B	350	0.2	3.5	0.35

Subscripts f and m in Table 1 refer to fibers and matrix, respectively.

The resulting displacement fields from simulated composites were used to deform an artificial image

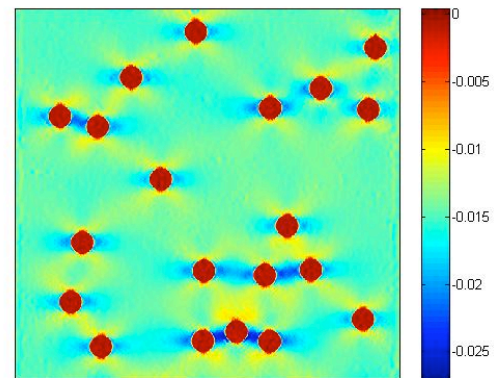
to obtain a pair of deformed and underformed images. The images were generated using analytical function of discrete Fourier transform. This allowed defining the deformed image by calculating the discrete values, hence avoiding any interpolations and the related errors.

A digital image correlation algorithm [17] was then used to measure the strain fields from artificial images. The applied DIC algorithm was based on an Improved Spectral Approach (ISA) that reconstructs continuous displacement fields from their Fourier basis functions. Thanks to the Fourier-based formulation, the algorithm leads to fast and accurate measurements using Fast Fourier Transform (FFT). Furthermore, the continuum-based framework on which the algorithm is developed enables more reliable measurements than those obtained by the subset-based algorithms. The strain fields were derived from measured displacements, which were then considered as the "measured" strain fields in this study.

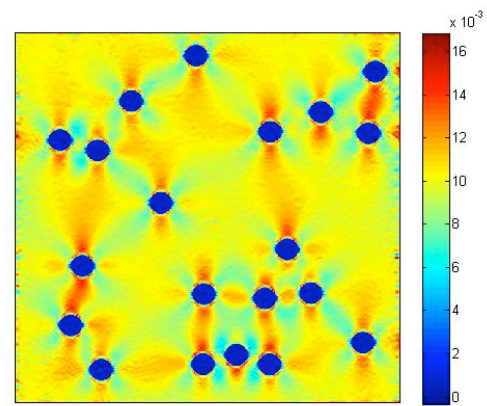
For the purpose of evaluating the robustness of the RVF approach, a set of noisy measured strain fields representative of real experiment conditions was simulated. The standard deviation of measurement error was used as an estimation of uncertainty between measured and FE strain values. For generating noisy strain fields Gaussian white noise with two different noise levels was added to the input image prior to strain measurements, and consequently a set of noisy measured data with standard deviations of 2% and 10% of mean strain values were generated. Fig. 2 shows the measured strain fields in both x and y directions (ϵ_x and ϵ_y) with the noise level of 2%.

The regularization constraints presented in Eq.(11) requires the constant values of effective properties (λ^{exp}) during optimization process. The experimental effective properties of the composite were computed from FE model. Real composite specimens are usually large enough to be considered as Representative Volume Element (RVE) of a larger structure. In the finite element model of a rather small volume however, a RVE must be determined so that the model contains minimum heterogeneity. A RVE study was therefore performed where the evolution of the composites effective properties was evaluated as a function of the number of meshed fibers and for a number of realizations. The RVE

size was determined by convergence of elastic modulus value.



a) strain in x direction (ϵ_x)



b) strain in y direction (ϵ_y)

Figure 2. Measured strain fields with noise level of 2%; a) ϵ_x and b) ϵ_y

Characterization of the long fiber composite was performed by applying both VFM and RVF methods. The aim of both algorithms was to retrieve the reference elastic properties of composite constituents that were initially input to generate the "measured" strain fields. The constitutive materials for both phases are assumed to be isotropic. Knowing that the whole material is not homogeneous, for factorizing the sought parameters out of the surface integrals (see Eq.(5)), the overall surface of composite (S) is split into $S-S'$ and S' i.e. matrix and fibers subregions, respectively. Hence, denoting Q_{xx} , Q_{xy} and Q'_{xx} , Q'_{xy} the stiffness components over $S-S'$ and S' , respectively, Eq. (5) can be rewritten as

$$\begin{aligned}
& Q_{xx} \int_{S-S'} (\epsilon_x \epsilon_x^* + \epsilon_y \epsilon_y^* + \frac{1}{2} \epsilon_s \epsilon_s^*) dS + \\
& Q_{xy} \int_{S-S'} (\epsilon_x \epsilon_y^* + \epsilon_y \epsilon_x^* - \frac{1}{2} \epsilon_s \epsilon_s^*) dS + \\
& Q'_{xx} \int_{S'} (\epsilon_x \epsilon_x^* + \epsilon_y \epsilon_y^* + \frac{1}{2} \epsilon_s \epsilon_s^*) dS + \\
& Q'_{xy} \int_{S'} (\epsilon_x \epsilon_y^* + \epsilon_y \epsilon_x^* - \frac{1}{2} \epsilon_s \epsilon_s^*) dS = \\
& \int_{L_f} \bar{\mathbf{T}}_i \mathbf{u}_i^* dl \quad \forall \mathbf{u}^* \text{ KA}
\end{aligned} \quad (12)$$

Four KA virtual fields independent from each other and compatible with the presented boundary condition must now be chosen. Regarding the biaxial loading condition, where the distribution of loading is known throughout the boundary of material, the virtual displacements $\mathbf{u}^{*(i)}$ and the corresponding strain fields $\epsilon^{*(i)}$ are defined by

$$\begin{cases} u_x^{*(1)} = x \\ u_y^{*(1)} = 0 \end{cases} \quad \begin{cases} \epsilon_x^{*(1)} = 1 \\ \epsilon_y^{*(1)} = 0 \\ \epsilon_s^{*(1)} = 0 \end{cases} \quad (13)$$

$$\begin{cases} u_x^{*(2)} = 0 \\ u_y^{*(2)} = y \end{cases} \quad \begin{cases} \epsilon_x^{*(2)} = 0 \\ \epsilon_y^{*(2)} = 1 \\ \epsilon_s^{*(2)} = 0 \end{cases} \quad (14)$$

$$\begin{cases} u_x^{*(3)} = \frac{x^2}{2} \\ u_y^{*(3)} = 0 \end{cases} \quad \begin{cases} \epsilon_x^{*(3)} = x \\ \epsilon_y^{*(3)} = 0 \\ \epsilon_s^{*(3)} = 0 \end{cases} \quad (15)$$

$$\begin{cases} u_x^{*(4)} = 0 \\ u_y^{*(4)} = \frac{y^2}{2} \end{cases} \quad \begin{cases} \epsilon_x^{*(4)} = 0 \\ \epsilon_y^{*(4)} = y \\ \epsilon_s^{*(4)} = 0 \end{cases} \quad (16)$$

As it can be seen, all of the virtual displacement fields indicated above possess the required conditions, i.e. they are differentiable and also have

C^0 continuity. Using these virtual fields and assuming identical dimensions of composite in x and y directions ($L_x = L_y = L$), the following system of equation is established from Eq.(12)

$$\begin{bmatrix} \int_{S-S'} \epsilon_x dS & \int_{S-S'} \epsilon_y dS & \int_{S'} \epsilon_x dS & \int_{S'} \epsilon_y dS \\ \int_{S-S'} \epsilon_y dS & \int_{S-S'} \epsilon_x dS & \int_{S'} \epsilon_y dS & \int_{S'} \epsilon_x dS \\ \int_{S-S'} x \epsilon_x dS & \int_{S-S'} x \epsilon_y dS & \int_{S'} x \epsilon_x dS & \int_{S'} x \epsilon_y dS \\ \int_{S-S'} y \epsilon_y dS & \int_{S-S'} y \epsilon_x dS & \int_{S'} y \epsilon_y dS & \int_{S'} y \epsilon_x dS \end{bmatrix} \begin{Bmatrix} Q_{xx} \\ Q_{xy} \\ Q'_{xx} \\ Q'_{xy} \end{Bmatrix} = \begin{Bmatrix} F_x L^2 \\ F_y L^2 \\ \frac{F_x L^3}{2} \\ \frac{F_y L^3}{2} \end{Bmatrix} \quad (17)$$

The linear system indicated in Eq.(17) was solved through matrix inversion (VFM) as well as using a constrained optimization procedures (RVF method) in order to determine the stiffness components.

With respect to Eq.(10), the least square objective function of the problem was developed from the four constitutive equations of linear system Eq.(17). Hence, the following constrained optimization problem was solved in the RVF method to identify the stiffness components

$$\min R(Q) = (AQ^{(k)} - B^{(k)})^T \cdot (AQ^{(k)} - B^{(k)})$$

Subject to

$$\begin{cases} (E_l^{MT}(Q) - E_l^{\exp}) = 0 \\ (v_{lt}^{MT}(Q) - v_{lt}^{\exp}) = 0 \end{cases} \quad (18)$$

where subscripts l and t indicate the out-of-plane and in-plane directions, respectively and the values of E_l^{\exp} and v_{lt}^{\exp} were obtained from RVE size of the composite using the related FE mesh.

The RVF identification was applied to the full-field measured data of the composite using Mesh Adaptive Direct Search (MADS) optimization [18] method. MADS is a frame-based global optimization algorithm for solving nonlinear problems without having any derivative information. The method is

known to be quite robust for optimization problems with nonsmooth objective functions subjected to nonsmooth constraints.

The stiffness components obtained from VFM was considered as initial solution at the beginning of optimization process in the RVF identification. At each iteration the numerical values of the constraints were evaluated after substituting new material parameters into the Mori-Tanaka model, and the feasibility of trial points was checked by equality constraints.

The following relations directly relate the elastic mechanical properties of constitutive phases to the stiffness components

$$\begin{aligned} v_m &= \frac{Q_{xy}}{Q_{xx}}, E_m = Q_{xx}(1 - v_m^2) \\ v_f &= \frac{Q'_{xy}}{Q'_{xx}}, E_f = Q'_{xx}(1 - v_f^2) \end{aligned} \quad (19)$$

For numerical validation, the first sets of strain data processed in the identification were those obtained directly from finite element simulations of the composite. The whole algorithm involved a blackbox that consisted of a main Matlab code and several subroutines associated to Mori-Tanaka homogenization model. The stopping criterion for the MADS algorithm was reached when a maximum number of 300 objective function evaluations was performed.

4 Results and discussion

Table 2 shows the obtained elastic properties of composite A with both VFM and RVF identification methods in different noise levels and also the corresponding errors resulted from each method.

As it can be seen, when dealing with the strain fields of zero noise (FE data) the VFM results in a relatively accurate set of parameters except for the poisson's ratio of fibers which has the maximum value of error. By applying the RVF algorithm the mentioned error is decreased and the same quality of results are acquired for the other parameters. It is clear that the accuracy of matrix parameters is generally higher than that of fibers. When dealing with the strain fields with noise level of 2%, among the parameters identified by VFM the fibers parameters are more influenced by noise effects than

Table 2. The identified parameters of composite A with VFM and RVF identification methods

Properties	E_f (GPa)	v_f	E_m (GPa)	v_m
Targets	74	0.2	3.5	0.35
Method/Noise level				
VFM / FE data (rel. error)	75.9 (2.6%)	0.223 (11.5%)	3.49 (0.3%)	0.350 (0%)
RVF / FE data (rel. error)	71.6 (3.3%)	0.185 (7.5%)	3.47 (0.6%)	0.352 (0.6%)
VFM / 2% (rel. error)	69 (6.7%)	0.219 (9.5%)	3.45 (1.4%)	0.350 (0%)
RVF / 2% (rel. error)	72 (2.7%)	0.193 (3.5%)	3.43 (2%)	0.351 (0.3%)
VFM / 10% (rel. error)	82.3 (11.2%)	0.125 (38%)	3.40 (2.8%)	0.360 (2.8%)
RVF / 10% (rel. error)	72.18 (2.4%)	0.208 (4%)	3.43 (2%)	0.350 (0%)

the matrix parameters. This is because of high strain field heterogeneity between two phases, which makes the less compliant phase, i.e. the fibers, to undergo much less deformation. The RVF identification however leads to more accurate results. It should be noted that due to the biases caused by the applied constraints, the accuracy of matrix parameters might be slightly degraded in RVF identification comparing with VFM, which is negligible. By increasing the noise level to 10%, the relative errors of VFM solutions are increased. As it can be observed, in comparison with the matrix phase parameters that are only slightly affected by noise effects, the noise level has much more influence on the fibers phase and less accurate parameters are obtained. In contrast, by applying the RVF method, thanks to the mechanical constraints effects, the accuracy of the identified properties is considerably improved, especially for the fiber

mechanical properties. It can be seen that the relative errors for all parameters are negligible.

The identified elastic properties of composite B with higher contrast of properties using both identification methods are also presented in Table 3.

Apart from the first set of results obtained from exact FE strain fields, in the presence of different levels of noises the VFM yields appropriate set of matrix parameters such that they are relatively close to the target values. The fibers properties however are significantly affected by increasing the noise level. As it can be seen, the relative errors of fiber properties associated to composite B is grater than the similar case in composite A. This is because of the fact that the stiffer phase herein undergoes much less deformation than in composite A.

Table 3. The identified parameters of composite B with VFM and RVF identification methods

Properties	E_f (GPa)	ν_f	E_m (GPa)	ν_m
Targets	350	0.2	3.5	0.350
Method/Noise level				
VFM / FE data	332.1	0.197	3.47	0.351
(rel. error)	(5%)	(1.5%)	(0.9%)	(0.3%)
RVF / FE data	342.96	0.173	3.56	0.353
(rel. error)	(2%)	(13.5%)	(1.7%)	(0.9%)
VFM / 2%	310	0.165	3.48	0.349
(rel. error)	(11.5%)	(17.5%)	(0.6%)	(0.3%)
RVF / 2%	350.1	0.235	3.54	0.347
(rel. error)	(0%)	(17.5%)	(1.1%)	(0.9%)
VFM / 10%	221.6	0.342	3.4	0.336
(rel. error)	(36.7%)	(71%)	(2.8%)	(4%)
RVF / 10%	347.5	0.255	3.45	0.343
(rel. error)	(0.7%)	(27%)	(1.4%)	(2%)

It can also be observed that the RVF method is

significantly less sensitive to noise effects and also yields solutions with much lower uncertainties when compared with VFM method.

The same undesirable bias created by imposed constraints exists again herein. The matrix properties accuracy degradation however is negligible.

In comparison with Regularized Model Updating (RMU) method [15], despite both methodologies leads to a rather same quality of results, the RVF identification is accomplished in a significantly lower calculation time.

Conclusions

In this study, a new Regularized Virtual Field (RVF) methodology was developed with the aim of improving constitutive parameters identification of composites in terms of accuracy and computing time. The novelty of the approach was that the VFM system of equations was solved in an optimization framework so that a set of mechanical constraint consisting of a micro-mechanical model was included as regularization scheme.

For the purpose of evaluating the performance of the proposed algorithm, it was applied to the noisy measured strain fields of 2D virtual composites with different contrasts of constitutive phases mechanical properties. The efficiency of the identification method was compared to that of VFM in the presence of noise. Validation on numerical test cases indicates that adding mechanical constraints enhances the efficiency of VFM, when dealing with high contrast composites which inherit high-order heterogeneity of strain fields. Results demonstrate that the average accuracy of the constitutive parameters identified by RVF method is much higher than the VFM especially regarding fiber properties. In addition, its low sensitivity to noise effects directly characterizes the robustness of this new technique.

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