NONLINEAR VIBRATION ANALYSIS OF 3D BRAIDED COMPOSITE CYLINDRICAL PANELS RESTING ON ELASTIC FOUNDATIONS IN THERMAL ENVIRONMENTS

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ABSTRACT

3D braided composite in aerospace structural design has attracted a great deal of interest. Nonlinear vibration analysis for a 3D (three-dimensional) braided composite cylindrical panel of finite length subjected to static or periodic axial forces has been presented. Based on a micro-macro-mechanical model, a 3D braided composite may be treated as a cell system combined by the representative unit cell. The components and the geometry of each cell are deeply dependent on their positions in the cross-section of the cylindrical panel. The panel is embedded in an elastic medium which is modeled as a Pasternak elastic foundation. The motion equations are based on a higher order shear deformation shell theory with a von Kármán-Donnell-type of kinematic nonlinearity. A two-step perturbation technique is employed to determine the linear and nonlinear frequency, parametric resonances of the 3D braided cylindrical panels. The numerical illustrations concern the nonlinear vibration behavior of braided composite cylindrical panels with different value of initial stress, of geometric parameter, of fiber volume fraction and of elastic foundation.

1 INTRODUCTION

Composite materials have wide application in the aerospace structure, due to their considerable stiffness-to-weight ratio. In these applications, nonlinear mechanical behavior of plate and shell structures has drawn a great attention. For a new class of composite materials, textile composites are manufactured by fabrication methods derived from the textile industry. Unlike laminated composites, the textile composites are able to eliminate the delamination due to the inter-lacing of the tows in the through-thickness direction.

There are a number of previous studies on the linear and nonlinear vibration of shells about composite shell behavior (i.e., stability and nonlinear vibration) available in literatures. Reissner [1] began to study vibrations of cylindrical shells about fifty years ago, who isolated a single half-wave of the vibration mode and analyzed it for simply supported shells. By using Donnell’s nonlinear shallow-shell theory for thin-walled shells, Reissner found that the nonlinearity could be the hardening or softening type. Dawe and Wang [2] developed a spline finite strip method to predict the postbuckling response of composite laminated panels, with the nonlinearity introduced in enhanced strain-displacement equations in a total Lagrangian approach prismatic panels to a progressive uniform end shortening. Pellicano et al. [3] obtained multimode Galerkin-type solutions of cylindrical shells for simply supported boundary conditions. The Donnell-type nonlinear shell equations (neglecting in-plane inertia) for a general anisotropic material are obtained by introducing an Airy stress function. The modern use of laminated composite moderately thick shells and the inaccuracy of classical thin shell theory have prompted researchers to develop new theories to study linear and nonlinear vibration
characteristics of moderately thick shells. Iu and Chia [4] studied nonlinear free vibrations of cross-ply laminated circular cylindrical shells by using the Airy stress function based on a modified Donnell nonlinear shell theory to take into account rotary inertia and shear deformation with a first-order approximation. Amabili [5] studied the geometrically nonlinear forced vibrations of laminated circular cylindrical shells by using the Amabili-Reddy higher-order shear deformation theory. The numerical results are compared to those obtained by using a higher-order shear deformation theory retaining only nonlinear term of von Kármán type and the classical shell theory. Moreover, combined with their original studies of nonlinear vibrations of shells and panels, including (1) fluid-structure interactions, (2) geometric imperfections, (3) effect of geometry and boundary conditions, (4) thermal loads, (5) electrical loads and (6) reduced-order models and their accuracy including perturbation techniques, proper orthogonal decomposition, nonlinear normal modes and meshless methods, Alijani and Amabili [6] presented an literature review in depth, mainly focused on geometrically nonlinear free and forced vibrations of shells made of traditional and advanced materials. Recently, Li and Shen [7, 8] and Li et al. [9] developed and extended a new micro-macro-mechanical model for the shell buckling to study buckling and postbuckling behavior of 3D braided cylindrical shells subjected to mechanical loads. The nonlinear flexural vibration behavior of isotropic and composite laminated cylindrical panels has received considerable attention (Ye and Soldatos [10]; Liew et al. [11]). Although in a well-known reference case there seems to be a reasonable agreement for a flat plate, there are unresolved discrepancies between the results obtained by different authors for a cylindrical panel (Ye and Soldatos [10]; Liew et al. [11]). Unlike flat plates, the curves of nonlinear frequency as a function of amplitude of curved panels might be hardening or softening type. It is noted that the governing differential equations for a 3D braided cylindrical shell are identical in form to those of unsymmetric cross-ply laminated shells (Shen and Xiang [12]). However, it remains unclear whether the 3D braided cylindrical panels still display a hardening nonlinearity under a low fiber volume fraction and this motivates the current investigation.

In the present work, we focus our attention on the nonlinear vibration behavior of the 3D braided composite cylindrical panels based on a micro-macro-mechanical model, in which a unit cell has been introduced as a representative cell including four interior cells and two surface cells in different regions by using a simple rule of mixtures idea. In this model, the total effective stiffness of 3D braided composite by using coordinate transformation and the volume averaging formula from different regions can be obtained. The motion equations are based on Reddy's higher order shear deformation shell theory with a von Kármán-type of kinematic nonlinearity. The equations of motion are solved by a two-step perturbation technique to determine the nonlinear transverse frequencies of the 3D braided panels.

2 THEORETICAL DEVELOPMENT

3D braiding technique, in which the braiding yarns interlock through a volume of material, is developed to overcome some inferior mechanical properties of 2D laminates. Based on the movement of carriers, we analyzed the yarn traces systematically in 3D tubular braided preforms using the control volume and control surface method similarly reported by Wang and Wang [13]. A new micro-macro-mechanical model of unit cells was established. The macro-cell of the model is further decomposed into simpler elements, here called unit cells, whose stiffness is calculated from the geometry of the fiber tow and the stiffness of the constituents and yarn cross-section changes into an elliptic shape due to interaction between two yarns. According to the load-sharing relations between the unit cells, the macroscopic stiffness is then calculated. For each unidirectional “lamina”, three-dimensional stress/strain relationship under the global coordinate system (XYZ) in 3D braided composite structures, is given by (see Sun and Qiao [14] in detail)

$$\left[\sigma\right]_k = \left[\tilde{C}\right]_k \left[\varepsilon\right]_k \quad (k = 1-6)$$

where $$\left[\sigma\right]_k = \{\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{zx}, \tau_{xy}\}_k$$, $$\left[\varepsilon\right]_k = \{\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{yz}, \gamma_{zx}, \gamma_{xy}\}_k$$.

In order to simplify the analysis, the assumption of plane stress condition is made. Then, we have

$$\sigma_z = \tau_{yz} = \tau_{zx} = 0.$$ Eq.(1) is reduced to
\[
\begin{align*}
\left\{ \sigma_x \right\}_k &= \left[ \bar{Q}_{11} \bar{Q}_{12} \bar{Q}_{16} \right] \\
\left\{ \sigma_y \right\}_k &= \left[ \bar{Q}_{21} \bar{Q}_{22} \bar{Q}_{26} \right] \\
\left\{ \tau_{XY} \right\}_k &= \left[ \bar{Q}_{61} \bar{Q}_{62} \bar{Q}_{66} \right]
\end{align*}
\]

In this model, a 3D braided composite may be viewed as a cell system and the geometry of each cell is deeply dependent on its position in the cross-section of the cylindrical panel. The unit cell coordinate system is defined as local coordinate system \((X'Y'Z')\). As the setting of coordination system in Figure 1, the elasticity matrix and effective density can be expressed as

\[
\left[ \bar{Q}_{ij} \right]_k = V_f \left[ T \right]_k \left[ C_f \right]_k T^T + V_m \left[ C_m \right]_k , \quad (k = 1 \sim 6)
\]

where \([C_f]\) and \([C_m]\) are elasticity matrix for yarn and matrix, \(V_f\) and \(V_m\) are the yarn and the matrix volume fractions and are related by

\[
V_f + V_m = 1
\]

In Eq.(3), \([T]\)_k is the transform matrix and can be expressed as

\[
[T]_k =
\begin{pmatrix}
1^2 & m_1^2 & n_1^2 & 2m_1 n_1 & 2n_1 l_1 & 2l_1 m_1 \\
m_2^2 & n_2^2 & 2m_2 n_2 & 2n_2 l_2 & 2l_2 m_2 \\
1^3 & m_3^3 & n_3^3 & 2m_3 n_3 & 2n_3 l_3 & 2l_3 m_3 \\
l_1^3 & m_1 n_1 & m_1 n_1 + m_1 n_1 & n_1 l_1 & l_1 m_1 + l_1 m_1 \\
l_2^3 & m_2 n_2 & m_2 n_2 + m_2 n_2 & n_2 l_2 & l_2 m_2 + l_2 m_2 \\
l_3^3 & m_3 n_3 & m_3 n_3 + m_3 n_3 & n_3 l_3 & l_3 m_3 + l_3 m_3
\end{pmatrix}, \quad (k = 1 \sim 6)
\]

where \(l_i, m_i, n_i (i = 1, 2, 3)\) are the direction cosines of a yarn with the surface braiding angle \(\theta\), interior braiding angle \(\gamma\) and inclination angles \(\chi\), where \(\chi\) is the angle between the projection of yarn axis on the \(YOZ\) plane and the \(X\)-axis, and the braiding angle is the angle between the yarn axis and the \(X\)-axis in Figure 1.

![Figure 1: Configuration of a braided composite cylindrical panel and its coordinate system and unit cells for the interior (cell A-D) and the surface (cell E-F).](image)

For different regions (interior cells \(j = 1\), and surface cells \(j = 2\)), braiding angle and inclination angles have different meanings. Thus, all direction cosines of the four “laminae” are obtained. It can be shown by calculations that the absolute values of the direction cosines for the four “laminae” are identical and only the signs are different. In actual application, if the unit cells (local coordinate) does not coincide with the structural coordinate system (global coordinate), coordinate transformation need to be performed. The elasticity matrix of the unit cell can be obtained by summing up individual individual...
elasticity matrices of all the “lamina”. For the representative “lamina” of axial tows, no rotation is considered since these tows do not have any crimp and are aligned in direction of the axis. A thickness ratio is attributed to each “lamina” and then multiplied by the corresponding stiffness matrix.

Therefore, the total effective elasticity matrices can be calculated by

\[
\begin{align*}
(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}) &= \frac{1}{2c} \sum_{l=2}^{M+2} \int_{0}^{\theta_l} \int_{z_l}^{\theta_l} (\Omega_{ij})_2 (1, Z, Z^2, Z^3, Z^4, Z^6) dZ dY \\
&+ \frac{1}{2c} \sum_{l=2}^{M+2} \int_{0}^{\theta_l} \int_{z_l}^{\theta_l} (\Omega_{ij})_1 (1, Z, Z^2, Z^3, Z^4, Z^6) dZ dY \\
&+ \frac{1}{2c} \sum_{l=2}^{M+2} \int_{0}^{\theta_l} \int_{z_l}^{\theta_l} (\Omega_{ij})_3 (1, Z, Z^2, Z^3, Z^4, Z^6) dZ dY \\
&+ \frac{1}{2c} \sum_{l=2}^{M+2} \int_{0}^{\theta_l} \int_{z_l}^{\theta_l} (\Omega_{ij})_4 (1, Z, Z^2, Z^3, Z^4, Z^6) dZ dY \\
&+ \frac{1}{2c} \sum_{l=2}^{M+2} \int_{0}^{\theta_l} \int_{z_l}^{\theta_l} (\Omega_{ij})_5 (1, Z, Z^2, Z^3, Z^4, Z^6) dZ dY \\
&+ \frac{1}{2c} \sum_{l=2}^{M+2} \int_{0}^{\theta_l} \int_{z_l}^{\theta_l} (\Omega_{ij})_6 (1, Z, Z^2, Z^3, Z^4, Z^6) dZ dY
\end{align*}
\]

(6a)

\[
\begin{align*}
(A_{ij}, D_{ij}, F_{ij}) &= \frac{1}{2c} \sum_{l=2}^{M+2} \int_{0}^{\theta_l} \int_{z_l}^{\theta_l} (\Omega_{ij})_2 (1, Z^2, Z^4) dZ dY \\
&+ \frac{1}{2c} \sum_{l=2}^{M+2} \int_{0}^{\theta_l} \int_{z_l}^{\theta_l} (\Omega_{ij})_1 (1, Z^2, Z^4) dZ dY \\
&+ \frac{1}{2c} \sum_{l=2}^{M+2} \int_{0}^{\theta_l} \int_{z_l}^{\theta_l} (\Omega_{ij})_3 (1, Z^2, Z^4) dZ dY \\
&+ \frac{1}{2c} \sum_{l=2}^{M+2} \int_{0}^{\theta_l} \int_{z_l}^{\theta_l} (\Omega_{ij})_4 (1, Z^2, Z^4) dZ dY \\
&+ \frac{1}{2c} \sum_{l=2}^{M+2} \int_{0}^{\theta_l} \int_{z_l}^{\theta_l} (\Omega_{ij})_5 (1, Z^2, Z^4) dZ dY \\
&+ \frac{1}{2c} \sum_{l=2}^{M+2} \int_{0}^{\theta_l} \int_{z_l}^{\theta_l} (\Omega_{ij})_6 (1, Z^2, Z^4) dZ dY
\end{align*}
\]

(6b)

where \((\Omega_{ij})_1, (\Omega_{ij})_2, (\Omega_{ij})_3, (\Omega_{ij})_4, (\Omega_{ij})_5\) and \((\Omega_{ij})_6\) are referred to elasticity matrix of four interior cells and two surface cells in different positions. In Eqs.(5) and (6) (with \(l = 2, 3, \cdots, M+2\))

\[
\begin{align*}
\eta &= H \tan \theta \\
z_l &= Y / \tan \phi + (2l - M - 7) d / 2 \sin \phi \quad (0 \leq Y \leq \eta) \\
\zeta_l &= -Y / \tan \phi + (2l - M - 3) d / 2 \sin \phi \quad (0 \leq Y \leq \eta) \\
t_l &= (2l - M - 7) d / 2 \sin \phi \quad t_l = (2l - M - 3) d / 2 \sin \phi \\
\tau_l &= (t - (M - 1) d) / \sin \phi / 2
\end{align*}
\]

(7)

In view of macro-mechanism, a shear deformable panel model is adopted. Consider a circular cylindrical panel with \(R\) is the radius of curvature, \(h\) is the panel thickness, \(a\) is the length in the X-direction, and \(b\) is the length in the Y-direction. The panel is referred to a coordinate system \((X, Y, Z)\), in which \(X\) and \(Y\) are in the axial and circumferential directions of the panel and \(Z\) is in the direction of the inward normal to the middle surface. The corresponding displacement are designated \(\bar{U}, \bar{V}\) and
\( \dot{W} \), \( \Psi_X \) and \( \Psi_Y \) are the rotations of normals to the middle surface with respect to the \( Y \)- and \( X \)-axes, respectively. The outer surface of the panel is in contact with elastic medium which acts as an elastic foundation represented by the Pasternak model. The foundation is customarily assumed (Paliwal et al. [15]) to be a compliant foundation, which means that no part of the shell panel lifts off the foundation in the large amplitude vibration region. The load-displacement relationship of the foundation is assumed to be \( p_0 = K_0 \ddot{W} - K_2 \nabla^2 \dot{W} \), where \( p_0 \) is the force per unit area, \( K_1 \) is the Winkler foundation stiffness, and \( K_2 \) is the shearing layer stiffness of the foundation, and \( \nabla^2 \) is the Laplace operator in \( X \) or \( Y \). The origin of the coordinate system is located at the end of the panel. The panel is assumed to be relatively thick, let \( \ddot{W}(X,Y) \) be the additional deflection and \( \ddot{F}(X,Y) \) be the stress function for the stress resultants defined by \( \ddot{N}_X = \bar{F}_{YY}, \bar{N}_Y = \bar{F}_{XX} \) and \( \ddot{N}_{XY} = -\bar{F}_{XY} \), where a comma denotes partial differentiation with respect to the corresponding coordinates. Taking the compatibility equation into account, that is

\[
\frac{\partial^2 \epsilon_X}{\partial Y^2} + \frac{\partial^2 \epsilon_Y}{\partial X^2} - \frac{\partial^2 \gamma_{XY}}{\partial X \partial Y} = \left( \frac{\partial^2 \ddot{W}}{\partial X^2} \right)^2 - \frac{\partial^2 \ddot{W}}{\partial X^2} \frac{\partial^2 \ddot{W}}{\partial Y^2} - \frac{1}{\kappa} \frac{\partial^2 \ddot{W}}{\partial X \partial Y}
\]  

(8)

Based on Sander’s shell theory, Reddy and Liu [16] developed a simple higher order shear deformation shell theory, in which the transverse shear strains are assumed to be parabolically distributed across the shell thickness and which contains the same dependent unknowns as in the first order shear deformation theory, and no shear correction factors are required. The theory is capable of accurately predicting the global behavior (computation of deflections, natural frequencies, buckling loads, and so forth) of (Simitses and Anastasiadis [17]), although it fails to fulfill the continuity conditions for the transverse shearing stresses. Based on Reddy’s higher order shear deformation theory, and using von Kármán-type kinematic relations, the governing differential equations for 3D braided composite cylindrical panels are derived and can be expressed in terms of a stress function \( \ddot{F} \), two rotations \( \ddot{\Psi}_X \) and \( \ddot{\Psi}_Y \), and transverse displacement \( \ddot{W} \). They are

\[
\ddot{L}_{11} = \ddot{L}_{22} \ddot{\Psi}_X - \ddot{L}_{23} \ddot{\Psi}_Y + \ddot{L}_{44} \ddot{F} - \frac{1}{\kappa} \frac{\partial^2 \ddot{F}}{\partial X^2} + K_1 \ddot{W} - K_2 \nabla^2 \ddot{W}
\]

(9)

\[
\ddot{L}_{21} = \ddot{L}_{32} \ddot{\Psi}_X + \ddot{L}_{33} \ddot{\Psi}_Y - \ddot{L}_{34} \ddot{F} = \frac{1}{\kappa} \frac{\partial^2 \ddot{W}}{\partial X^2} = -\frac{1}{2} \ddot{L}(\ddot{W},\ddot{W})
\]

(10)

\[
\ddot{L}_{31} = \ddot{L}_{32} \ddot{\Psi}_X - \ddot{L}_{33} \ddot{\Psi}_Y + \ddot{L}_{34} \ddot{F} = I_{31} \ddot{\Psi}_X + I_{32} \ddot{\Psi}_Y + I_{34} \ddot{F}
\]

(11)

\[
\ddot{L}_{41} = \ddot{L}_{42} \ddot{\Psi}_X + \ddot{L}_{43} \ddot{\Psi}_Y + \ddot{L}_{44} \ddot{F} = I_{41} \ddot{\Psi}_X + I_{42} \ddot{\Psi}_Y + I_{44} \ddot{F}
\]

(12)

where linear operators \( \ddot{L}_0() \) and nonlinear operator \( \ddot{L}() \) are defined as in Shen [18]. Note that the geometric nonlinearity in the von Kármán sense is given in terms of \( \ddot{L}() \) in Eqs.(9) and (10).

The four edges of the panel are assumed to be simply supported with or without in-plane displacements, referred to as ‘movable’ and ‘immovable’ in the following, when temperature is increased steadily, so that the boundary conditions are

\[
X = 0, a: \quad \ddot{W} = \ddot{\Psi}_y = \ddot{V} = 0, \quad \ddot{M}_x = \ddot{P}_x = 0,
\]

(13a)

\[
\int_0^b \ddot{N}_X \dd{Y} + \sigma_X hb = 0, \quad \ddot{U} = 0 \text{ (immovable edges)},
\]

(13b)

\[
Y = 0, b:
\]
\[ \ddot{W} = \Psi_x = 0, \quad \dot{N}_{xy} = 0, \quad (13c) \]
\[ \int_0^a \dot{N}_y dX = 0 \quad \text{(movable edges)}, \quad \ddot{V} = 0 \quad \text{(immovable edges)} \quad (13d) \]

where \( \ddot{M}_x \) is the bending moment and \( \dddot{P}_x \) is higher order moment, as defined in Shen [18]. The conditions expressing the immovability condition in Eq.(13) may be fulfilled on the average sense as,
\[
\int_0^b \frac{\partial \dddot{U}}{\partial X} dXdY = 0, \quad \int_0^b \frac{\partial \dddot{V}}{\partial Y} dXdY = 0,
\]

or
\[
\int_0^b \left[ \left( A_{11}^* \frac{\partial^2 \dddot{F}}{\partial Y^2} + A_{12}^* \frac{\partial^2 \dddot{F}}{\partial X^2} \right) + \left( B_{11}^* - \frac{4}{3h^2} E_{11}^* \right) \frac{\partial \dddot{W}}{\partial X} + \left( B_{12}^* - \frac{4}{3h^2} E_{12}^* \right) \frac{\partial \dddot{W}}{\partial Y} \right] dXdY = 0,
\]

\[
\int_0^b \left[ \left( A_{22}^* \frac{\partial^2 \dddot{F}}{\partial X^2} + A_{12}^* \frac{\partial^2 \dddot{F}}{\partial Y^2} \right) + \left( B_{22}^* - \frac{4}{3h^2} E_{22}^* \right) \frac{\partial \dddot{W}}{\partial X} + \left( B_{22}^* - \frac{4}{3h^2} E_{22}^* \right) \frac{\partial \dddot{W}}{\partial Y} \right] dYdX = 0
\]

In Eq.(15) the reduced stiffness matrices \([A_{ij}^*], [B_{ij}^*], [D_{ij}^*], [E_{ij}^*], [F_{ij}^*], [H_{ij}^*] (i, j=1, 2, 3, 4, 5, 7). \]

3. ANALYTICAL METHOD AND ASYMPTOTIC SOLUTIONS

We are now in a position to solve Eqs.(9)-(12) with boundary condition (13). The nonlinear Eqs.(9)-(12) may then be written in dimensionless form as
\[
L_{11}(W) - L_{12}(\Psi_x) - L_{13}(\Psi_y) + \alpha_{14} L_{14}(F) + \frac{1}{2} \alpha_{14}^2 F_{,xx} + K_1 W - K_2 \dddot{W} = 0
\]
\[
= \gamma_{15} \beta^2 L(W, F) + L_{17}(\dot{W}) + \gamma_{18} \frac{\partial \dddot{W}}{\partial X} + \gamma_{19} \frac{\partial \dddot{W}}{\partial Y} + \gamma_{20}
\]
\[
L_{21}(F) + \gamma_{21} L_{22}(\Psi_x) + \gamma_{22} L_{23}(\Psi_y) + \gamma_{23} L_{24}(W) + \gamma_{24} \dddot{W}_{,xx}, = -\frac{1}{2} \gamma_{25} \beta^2 L(W, W)
\]
\[
L_{31}(W) + L_{32}(\Psi_x) - L_{33}(\Psi_y) + \gamma_{44} L_{34}(F) = \gamma_{31} \dot{W} + \gamma_{32} \frac{\partial \dddot{W}}{\partial X} + \gamma_{33} \frac{\partial \dddot{W}}{\partial Y}
\]
\[
L_{41}(W) - L_{42}(\Psi_x) + L_{43}(\Psi_y) + \gamma_{44} L_{44}(F) = \gamma_{41} \dot{W} + \gamma_{42} \frac{\partial \dddot{W}}{\partial X} + \gamma_{43} \frac{\partial \dddot{W}}{\partial Y}
\]

where \( L_{ij}() \) and \( L() \) are defined as in Shen [18].

The boundary conditions of Eq.(13) become
\[ x = 0, \pi: \quad W = \Psi_x = 0, F_{,xy} = 0, \int_0^r \beta^2 \frac{\partial^2 F}{\partial y^2} dy = 0, \text{(movable edges)} \quad (20a) \]
$$\int_0^r \int_0^s \left[ \gamma_{22} \beta \varepsilon^2 F - \gamma_{11} \varepsilon^2 F + \gamma_{24} \left( \gamma_{11} \varepsilon^2 F + \gamma_{23} \beta \varepsilon^2 F \right) - \gamma_{24} \gamma_{244} \varepsilon^2 F - \gamma_{244} \beta \varepsilon^2 F \right] - \frac{1}{2} \gamma_{24} \left( \frac{\partial W}{\partial x} \right)^2$$

$$+ \gamma_{24} \left( \gamma_{24} \gamma_{244} - \gamma_{5} \gamma_{T2} \right) \Delta T \right] dxdy = 0, \text{ (immovable)} \quad (20b)$$

$$y = 0, \pi:$$

$$W = \Psi_x = 0, \quad F_{xy} = 0, \quad \int_0^r \int_0^s \varepsilon^2 F \partial^2 dxdy = 0, \text{ (movable edges)} \quad (20c)$$

$$\int_0^r \int_0^s \left[ \frac{\partial^2 F}{\partial x^2} - \gamma_{24} \beta \varepsilon^2 F + \gamma_{24} \left( \gamma_{11} \varepsilon^2 F + \gamma_{22} \beta \varepsilon^2 F \right) \right] - \gamma_{24} \gamma_{243} \varepsilon^2 F - \gamma_{24} \gamma_{32} \beta \varepsilon^2 F$$

$$+ \gamma_{24} \left( \gamma_{24} \gamma_{244} - \gamma_{5} \gamma_{T2} \right) \Delta T \right] dydx = 0, \text{ (immovable edges)} \quad (21)$$

And the initial conditions are assumed to be $W \big|_{t=0} = \frac{\partial W}{\partial t} \big|_{t=0} = 0, \quad \Psi_x \big|_{t=0} = \frac{\partial \Psi_x}{\partial t} \big|_{t=0} = 0.$

$$\Psi_y \big|_{t=0} = \frac{\partial \Psi_y}{\partial t} \big|_{t=0} = 0.$$

Applying Eqs.(16)-(19), the nonlinear transverse vibration response of shear deformable 3D braided cylindrical panels now is determined by a two-step singular perturbation technique, for which the small perturbation parameter has no physical meaning at the first step, and is then replaced by a dimensionless vibration amplitude at the second step. The essence of this procedure, in the present case, is to assume that

$$W(x, y, \varepsilon) = \sum_{j=0}^\infty \varepsilon^j W_j(x, y) \cdot F(x, y, \varepsilon) = \sum_{j=0}^\infty \varepsilon^j F_j(x, y) \cdot F(x, y, \varepsilon) = \sum_{j=0}^\infty \varepsilon^j F(x, y)$$

$$\Psi_x(x, y, \varepsilon) = \sum_{j=0}^\infty \varepsilon^j \Psi_{x,j}(x, y) \cdot \Psi_y(x, y, \varepsilon) = \sum_{j=0}^\infty \varepsilon^j \Psi_{y,j}(x, y) \cdot \lambda_{x,y}(x, y, \varepsilon) = \sum_{j=0}^\infty \varepsilon^j \lambda_{j,x}$$

(22)

where $\varepsilon$ is a small perturbation parameter and first term of $W_j(x, y, t)$ is assumed to have the form

$$w_j(x, y, t) = A_{11}^{(i)}(t) \sin mx \sin ny$$

(23)

Introduced a parameter $\varepsilon$, we have $\tilde{t} = \varepsilon t$ to improve perturbation procedure for solving nonlinear vibration problem. Substituting Eq.(22) into Eqs.(16)-(19) and collecting the terms of the same order of $\varepsilon$, a set of perturbation equations is obtained. By using Eq.(22) to solve these equations step by step, we obtain asymptotic solutions of $W, F, \Psi_x, \Psi_y$ and $\lambda_{x,y}$. In fact, $\lambda_{x}$ may be divided into the static item $\lambda_{x}$ and the oscillating item $\lambda_{d}$, respectively. For the case of free vibration with initial stress $\lambda_{x}$, in the absence of $\lambda_{d}$, one has $\lambda_{y} = 0$. Let,

$$\int_0^{2\pi} \int_0^{\pi} \lambda_{x,y}(x, y, t) \sin mx \sin n y dxdy = 0$$

(24)

From which one has,

$$g_{40} \frac{d^2 (eA_{11}^{(i)})}{dt^2} + g_{41} (eA_{11}^{(i)}) + g_{42} (eA_{11}^{(i)})^2 + g_{43} (eA_{11}^{(i)})^3 + g_{45} (eA_{11}^{(i)})^5 = 0$$

(25)

The solution of which may be written as,

$$\omega_{NL} = \omega_{L} \left( 1 + \frac{3 g_{41}}{4 g_{40}} A^2 + \frac{5 g_{45}}{8 g_{40}} A^4 \right)^{1/2}$$

(26)
where \( \omega_L = (g_{41}/g_{40})^{1/2} \) is the dimensionless linear frequency, \( A=W_{\text{max}} \) is the dimensionless amplitude of the panel, and corresponding linear frequency is expressed as \( \Omega_L = \omega_L(\pi/L)(E_{22}/\rho)^{1/2} \).

When the periodic coefficient is taken into consideration, applying the Galerkin procedure to the expression of \( q \) over the panel area, Eq. (25) is in the form of a second order differential equation with periodic coefficient of the Mathieu-Hill type, which representing the dynamic stability problem of 3D braided panels under a periodic in-plane force.

\[
g_{40} \frac{d^2(eA_1^{(1)})}{dt^2} + g_{41}(eA_1^{(1)}) + g_{42}(eA_1^{(1)})^2 + g_{43}(eA_1^{(1)})^3 + g_{45}(eA_1^{(1)})^5 - \gamma_4 \beta^2 \lambda_4 \cos \theta t = 0 \quad (27)
\]

where \( \theta \) is the frequency of the external excitation, the harmonic parameter \( f = \gamma_4 \beta^2 \lambda_4 / g_{40} \).

It is worth noting that \( \varepsilon \) is no longer a small perturbation parameter in the large amplitude vibration region when the panel deflection amplitude is sufficiently large, i.e. \( \bar{W}/h > 1 \), and by a two-step perturbation scheme where \( \varepsilon \) is definitely a small perturbation parameter in the first step and in the second step \( (A_1^{(1)} \varepsilon) \) may be large in the large-amplitude vibration region.

4 NUMERICAL RESULTS AND COMMENT

Validations

Numerical results of large-amplitude vibration are presented in this section for simply supported 3D braided composite cylindrical panels. As a part of the validation of the present method, the dimensionless linear frequencies \( \tilde{\omega} = \Omega b \sqrt{\rho/E_{22}} \) for \( (0/90)_T \) cross-ply laminated cylindrical panels are calculated and compared in Table 1 with the classical shell theory solutions of Leissa and Qatu [19], the first order shear deformation shell theory results of Wu et al. [20], and the 3-D solutions of Ye and Soldatos [10]. The computing data adopted are: \( a/b = 1 \), \( R/h = 10 \), \( h/b = 0.05 \), 0.1 and 0.15, \( E_{11}/E_{22} = 25 \), \( G_{12} = G_{13} = 0.5E_{22} \), \( G_{23} = 0.2E_{22} \), \( v_{12} = 0.25 \) and \( \rho = 1 \). In addition, the nonlinear to linear frequency ratios \( \omega_{NL}/\omega_L \) for an isotropic cylindrical panel are calculated and compared in Figure 2 with the Galerkin method results of Shin [21]. The computing data adopted are: \( b/a = 1 \), \( b/h = 10 \), \( R/h = 25 \), \( E = 68.95 \text{ GPa} \), \( v = 0.3 \) and the vibration mode is taken to be \( (m, n) = (1, 1) \). These comparison studies show that the results from the present method are in good agreement with the existing results.

Figure 2: Comparison of frequency-amplitude curves for an isotropic cylindrical panel
Table 1: Comparison of dimensionless frequencies $\tilde{\omega} = \Omega b \sqrt{\rho / E_{22}}$ for or $(0/90)_t$ cross-ply laminated cylindrical panels

<table>
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<td>1.23707</td>
<td>1.273611</td>
</tr>
</tbody>
</table>

**Parametric study**

A parametric study has been carried out and typical results are shown in Table 2, and Figure 3-Figure 5. For these examples $R/h = 40$ and the total thickness of the panel is $h = 4$ mm, pitch length $H = 2.8$ mm and $M = 9$ and for the case of ‘movable’ end condition. The carbon fiber tows are used as braiding material and the material properties of graphite fiber and epoxy adopted, as given in Shapery [22] and Adams and Crane [23], are: $E_1 = 233.13$ GPa, $E_{12} = 23.11$ GPa, $G_{12} = 8.97$ GPa, $v_{12} = 0.2$ GPa, $v_{23} = 0.4$, $\rho_f = 1.8 \times 10^3$ kgm$^{-3}$, $E_m = (3.51 - 0.03\Delta T)$ GPa and $\nu_m = 0.34$ and $\alpha_m = 45.0 \times 10^{-6}$ K$^{-1}$, in which $T = T_0 + \Delta T$ and $T_0 = 300$ K (room temperature) and $\rho_m = 45.0 \times 10^{-6}$ kgm$^{-3}$.

Figure 3 shows effect of temperature dependency on the frequency-amplitude curves of braided composite cylindrical panels under thermal environmental condition $\Delta T = 100$ K for two cases of thermo-elastic material properties, i.e. T-ID and T-D. It can be seen that the frequency-amplitude curve becomes slightly lower when the temperature-dependent properties are taken into account.

![Figure 3: Effect of temperature dependency on the frequency-amplitude curves of braided composite cylindrical panel with 'immovable' end condition](image)

Table 2 gives the natural frequency $\Omega$ (Hz) for braided composite cylindrical panels under axial tensile load $\lambda_x = -0.5\lambda_{cr}$ and compressive load $\lambda_x = -0.5\lambda_{cr}$, where $\lambda_{cr}$ is the critical buckling load of the panels. Furthermore, Figure 4 presents effect of initial stress on the frequency-amplitude curves of braided composite cylindrical panels with 'movable' end condition. It is observed that, initial axial compression will result in a decrease in the natural frequency. In contrast, an initial axial tension will help increase the natural frequency.
Figure 4: Effects of initial stress on the frequency-amplitude curves of braided composite cylindrical panels with ‘movable’ end condition

Table 2: Comparison of natural frequency $\Omega$ (Hz) for pre-stressed braided composite cylindrical panels with different values of panel geometric parameter and fiber volume fraction and for the case of ‘immovable’ end condition
To investigate the significance of the 3D braided composite cylindrical panels on the nonlinear vibration behavior, the linear and nonlinear frequencies of 3D braided composite panels with different fiber volume fraction, geometric properties of panels, end boundary conditions and elastic foundations are plotted in Table 2 and Figure 3-5. It can be seen from these tables and figures that initial stress, geometric properties of panels, and magnitude of excitation are all the crucial components factors in determining the linear and nonlinear frequencies and dynamic responses of 3D braided composite panels.

5 CONCLUSIONS

To assess the linear and nonlinear transverse vibration behavior of 3D braided composite cylindrical panel, a fully nonlinear vibration analysis is presented based on a micro-macro-mechanical model. Numerical calculations have been made for braided composite cylindrical panels with different values of geometric parameter and of fiber volume fraction in different cases of end conditions, elastic foundation parameters. The results reveal that the panel geometric parameter have a significant effect on fundamental frequency and the nonlinear to linear frequency ratios of 3D braided cylindrical panels.

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