CONSTITUTIVE MODELLING OF THE HYSTERETICAL BEHAVIOR OF TEXTILE COMPOSITES UNDER LARGE STRAIN

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ABSTRACT

The main purpose is to develop a constitutive modelling of the hysteretical behavior of textile composites under a large strain. As we know, a lot of formulations already exist, however the majority of these are not adapted for textile materials, but nothing prevents from using and adapting them for this specific topic. Then it is possible to show through the analysis of the thermodynamics of reversible processes that the additive decomposition of Green-Naghdi (GN) and the multiplicative decomposition of Kröner-Lee (KL) can be used without any problem. The coupling of these two decompositions allows to define the elastic part of the Green-Lagrange tensor which give the possibility to find the yield criterion and then the yield function as well. Therefore, it follows the plastic flow as the derivative of the yield function in the space of the Piola-Kirchhoff II stresses. Concerning the hysteretical aspect, as before, the model of Prager or Mroz are not really adapted. It is shown that multilayer plasticity based on nested surfaces present some inconvenient for textile composites. In fact, the surfaces have to inflate or deflate proportionally of the plastic strain and as usual, they have to move proportionally of the plastic strain ratio. Considering these specifics evolutions, the kinematic and isotropic laws have to be different. Some conditions have to be respected for each nested surfaces but the point of this model is its simplicity. Indeed, the plastic flow, the evolution laws and the hardening functions are just dependent of the plastic strain. This makes the model quiet simple to use. This originality permitted to do some numerical simulations quiet easily, quickly and similar to the experimental acquisition.

1 INTRODUCTION

The field of composite offers a wide range of materials. Depending on the application, these composite materials can be of different natures, be composed of different elements and thus have geometrical and mechanical characteristics peculiar of themselves. Depending of the type of composite, the shaping may vary from simple and controlled processes to much more delicate processes whom are still subject to many stages of research and discussion. The object is to study the dissipative behaviour of a textile reinforcement, which has the particularity of being continuous fiber. The dissipation of the textile is present during its shaping and more particularly during the preforming phase of the process. During this phase, the material will undergo very large deformations in order to match with the matrix shape. The material will therefore deform according to different modes (elongations of the fibers, pure shear …) since this type of material generally consists of fiber that have a very high stiffness (which prevents them from elongating) then the majority of the dissipation is due to shear modes). Moreover, the deformations are often very important, so the elasto plastic model presented here is written under large strains and the plastic dissipation will be exclusively defined by a pure shear kinematic. Indeed, unlike simple shear, where the fibers elongate, pure shear corresponds mush better to the behaviour because of the non-elongation of the fibers. Then, as we can see in black on the [Fig3], the behavior is extremely non-linear. A lot of formulations already exist to describe this evolution [1], [2], [3], however the majority of these are not adapted for textile materials. Thought, after using the thermodynamics of reversible processes [1], [2], it is shown that the additive decomposition of Green – Naghdi and the multiplicative decomposition of Kröner – Lee are usable even under large strains [4], [5], [6]. These to decompositions make possible to use the intermediate configuration [Fig2]. This configuration represents the condition of the woven material once no more loading is applied to it. This coupling make possible to reduce the tensor of the transformation from the intermediate configuration (where nothing is known) to the initial configuration (where everything is known). Thus, all the calculated quantities (transformation, Green-Lagrange, Piola-Kirchoff, …) depend solely on the initial configuration which is completely known. Hence, the constitutive modelling presented here is formulated in total Lagrangian. A Lagrangian formulation offers a great deal of flexibility and give some simplicity to the model. After that, it is shown that the plastic flow is describes as the derivative of the yield function in the space of Piola – Kirchoff II stresses. To do that, the hypothesis of generalized constraints has to be done and it is assume that the constraint is integrated into the thickness. Moreover, the general behavior of the composite dissipation of the composite is hesteretic and some models as Prager, Ziegler [7], Mroz [8] or other [9], [10] are not adapted. In fact, the evolution of the stress describes a very non-linear return with an asymptotic path. (In black in the [Fig3]). This asymptotic way seems difficult to manage, but by controlling the internal (nested) surface it becomes possible to match with this behavior. To do that, the dimension of (nested) surfaces have to increase or decrease proportionally to the plastic strain. Nevertheless, the surfaces can move as the plastic strain increment. Considering this specific evolution, the kinematic and isotropic laws have to be different and no existing models can be applied for these reasons. Furthermore, these laws are not the same if the nested surface is activate or not, then it is very important to check which surfaces are going to be transformed. Even if some conditions have to be respected for each nested surfaces, the point of this model is its simplicity. Another model [11] using fractional derivatives could have been used and adapted but they were not as simple as the nested surfaces. Indeed, the plastic flow, the evolution laws and the hardening functions are just dependent of the plastic strain and this makes the model quite simple to use. This originality permitted to do some numerical simulations quite easily and quickly to get a result close to the experimental acquisition [Fig3]. Concerning this experimental acquisition and considering the textile material and the behavior of the fibers, the best way to have meaningful and satisfactory data was to do a Picture Frame test which imposes a pure shear kinematic [Fig3].

2 MODELE DESCRIPTION

Considering the characteristic of the fiber, the best kinematic to describe the plastic deformation is a shear. However, two main types of shear exists. Here, only the kinematic of pure shear will be used because of the non-elongation of the fibers. As the problem is defined in generalized constraint, a two dimensional analysis is done. Then, the plastic part of the transformation tensor associated to this kinematic is following:

$$\underline{\underline{F}}_p = \mathrm{Cos}\left(\frac{\gamma_p}{2}\right)\cdot\left(\underline{G_1}\otimes\underline{G_1}+\underline{G_2}\otimes\underline{G_2}\right)+\mathrm{Sin}\left(\frac{\gamma_p}{2}\right)\cdot\left(\underline{G_1}\otimes\underline{G_2}+\underline{G_2}\otimes\underline{G_1}\right) \tag{1}$$

For simplicity reasons, the second kinematic [Fig2b] is chosen, indeed this kinematic give a symmetric tensor. This tensor is the basement of all the reasoning. In fact, the user gives the total transformation tensor. As the plastic transformation tensor is known, throw the multiplicative decomposition of Kröner-Lee it becomes possible to get the elastic transformation tensor. Nevertheless, the theory presented here has to follow the objectivity principle and the the orientation of the kinematic do not change the result of the dissipation. Then, to define all the fundamental tensors, the coupling between Kröner-Lee and Green-Neghdi decompositions has to be done. To do that, the hypothesis to write the model in total Lagrangian and to use the Green-Naghdi decomposition under large strain have to be discussed.

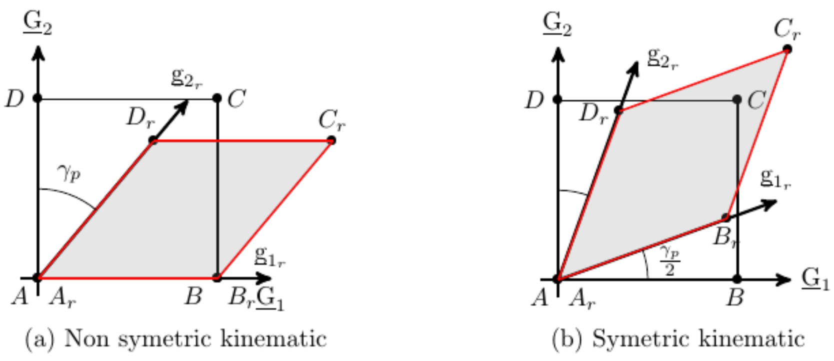

Figure 1 : Two kinematics of pure shear

A thermodynamical analysis of solid mechanic [1], [2] under large strain was done to show the consistency of the GN Decomposition [12] under large strain. Then, the use of the intermediate configuration give the possibility to define every tensor with quantities get from the initial configuration. This give the possibility to manage every quantities and to describe the entire model with only known variables [4], [5], [6].

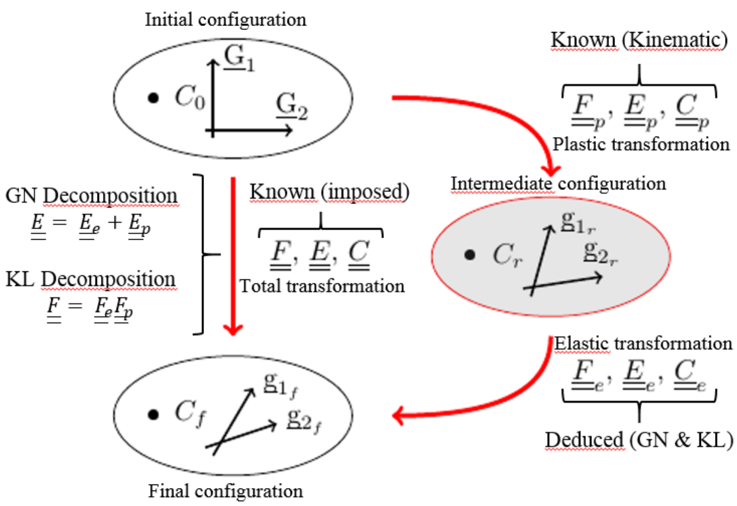

Figure 2 : Intermediate configuration schema

With this theory of intermediate configuration, it is possible to calculate every fundamental tensor and especially the elastic Green – Lagrange tensor of deformation, which is very important since it is useful for the description of the hyper elastic law. Then the elastic part of Green – Lagrange is defined as following :

$$\underline{\underline{E}}_e = \frac{1}{2} \cdot \left(\underline{\underline{F}}_p^t \cdot \left(\underline{\underline{F}}_e^t \cdot \underline{\underline{F}}_e - \underline{\underline{I}}_2 \right) \cdot \underline{\underline{F}}_p \right) \tag{2}$$

With $\underline{\underline{I}}_2$ the second order identity tensor and the terms about the plastic transformation put the tensor back to the initial configuration. After that, every fundamental tensors can be written and it is possible to get the elastoplastic behaviour of the material. To do that, three conditions have to be define:

- A plasticity criterion (called fs), which permits to know if the material is going to plastify or if it is still inside the elastic area. It depends of the stress tensor, a yield (called Sy) and hardening laws (Isotrope called α and Kinematic called q).

- A free energy potential, which give the hyper elastic law after being derivate by the elastic part of the Green – Lagrange tensor.
- A flow rule, which give the direction of the plastic flow.

We can write a basic form of the energy potential considering two terms linked with the elongations of the fibres and one term describing the shear mode. Then, the derivative under the elastic part of Green – Lagrange tensor give us the stress tensor of Piola – Kirchhoff II. This derivation made a relation between PKII and the elastic part of Green – Lagrange tensor. If we are looking at the internal energy, it is shown that this energy is calculated from PKII and its dual, the rate of plastic deformation. Nevertheless, only the off diagonal component of the rate tensor exists. Then in order to write the plasticity criterion, only the off diagonal component of PKII will be used. After what, it is possible to replace the definition of PKII made by the derivative of the potential inside the criterion. At the end, the criterion will depend of the elastic part of Green Lagrange tensor. This is good because through the Green – Naghdi decomposition, it is possible to transform the criterion and make him only dependent of known quantities (total transformation imposed and plastic transformation calculated during radial return). Last point, the flow rule gives us some information concerning the evolution of the plastic deformation and especially the direction of the plastic flow. It will be particularly useful for the description of the elastic prediction and the radial return theories.

3 MODEL ELABORATION

As we know, the only dissipative mode is following a pure shear kinematic. Then the best way to approach this kinematic is to do a Picture Frame Test acquisition. Then this method consist to put a piece of woven material inside an articulated framework. [Fig3]. The main idea is to notice and examine the hysteretic behaviour of the textile. That is why a cyclic growing loading is applied onto it.

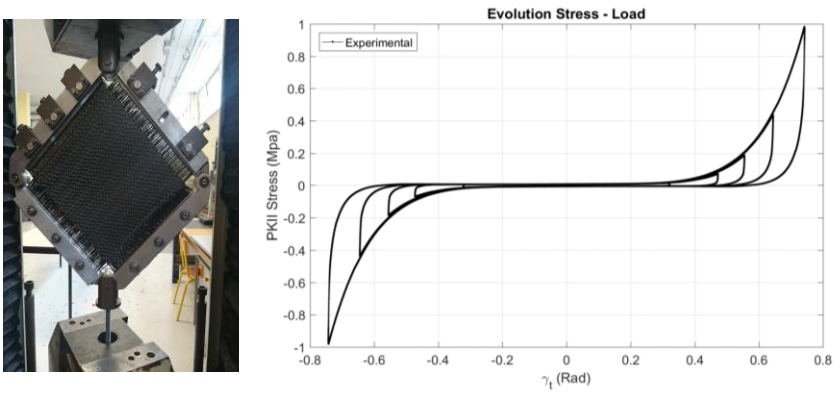

Figure 3 : Experimental PF acquisition

It is easy to notice that the behaviour is non-linear and asymptotic. Another point to notice is the small elastic yield of this woven. In fact, after a very light load, it is possible to see that the material is starting to dissipate energy. That is why writing a plasticity criterion without any hardening laws is not the best way to do right simulations. Then, the plasticity criterion will include mixed hardening laws which whom will only be dependent of the plastic shear angle. After that, it becomes possible to write the domains in which the constraint can be located after an elastic prediction. The goal of this elastic prediction is to check if the material is still in the elastic area or if it is going to plastify. There is three distinguish domains [1], [2], [3]:

- Elastic domain: The criterion give a negative result. It means that the material is still elastic. In this case, no radial return has to be done.
- Limit of the elastic domain: The criterion give a null result. It means that the material is at the boundary between the elastic and plastic domain. Nothing needs to be done is this case. Loading can continue.

- Plastic domain: The criterion give a positive result. It means that the material is under plasticity. In that case, a radial return has to be done through the plastic law, which give the direction and the quantity of plastic flow.

If the last domain is activated, the goal is to find the plastic deformation (here, the plastic shear angle) in order to go back on the boundary of the elastic domain. In other words, going back until the criterion give a null result. This problem is not so simple since the hardening laws are here. Then, to make the resolution simple, the idea was to get these laws only depended of the plastic shear angle. Then in that case, the entire criterion only depends of this plastic angle. (The total angle is imposed throw the load).

$$f_s(\gamma_p) = \left|\rho \cdot K_{sh} \cdot \left(\underline{\underline{E}}_e : \underline{G_1} \otimes \underline{G_2} + \underline{\underline{E}}_e : \underline{G_2} \otimes \underline{G_1}\right) - q(\gamma_p)\right| - \left(S_y + \alpha(\gamma_p)\right) \tag{3}$$

Now, we know that the entire criterion only depends of this plastic angle shear, then it becomes possible to use usual algorithms to find the solution in order to get the criterion equal to zero [Fig4]. Two very famous algorithms can do that, Newton – Raphson or Dichotomy. As soon as we know the value of this plastic angle shear, it is possible to update all variables, the hardening laws and the plastic flow. If we know the plastic laws, it becomes possible to know the behaviour of the surface and the evolution of the material. Then, if we manage the hardening laws coefficients, it is possible to match with the experimental acquisition [Fig5].

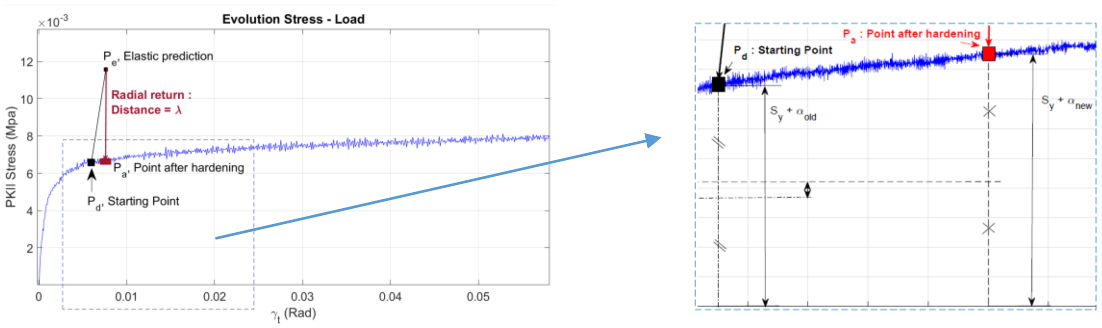

Figure 4: Elastic Prediction & Radial Return

Obviously, the elastic part of Green – Lagrange tensor, through the Green – Naghdi decomposition depends of the load and the dissipative (plastic) behaviour. Then, to calibrate the model, some Picture Frame Test were done. In order to fit the numerical response and the experimental acquisition, a kinematic of pure shear was imposed. Then the deformation tensor is known and the plastic one too. Then the criterion fs can be written by following expression:

$$f_s(\gamma_p) = \left|\rho \cdot K_{sh} \cdot \left(\sin(\gamma_t) - \sin(\gamma_p)\right) - q(\gamma_p)\right| - \left(S_y + \alpha(\gamma_p)\right) \tag{4}$$

This first simulation permitted to find every hardening laws coefficients and fit the model and the experimentation. The result is shown [Fig5].

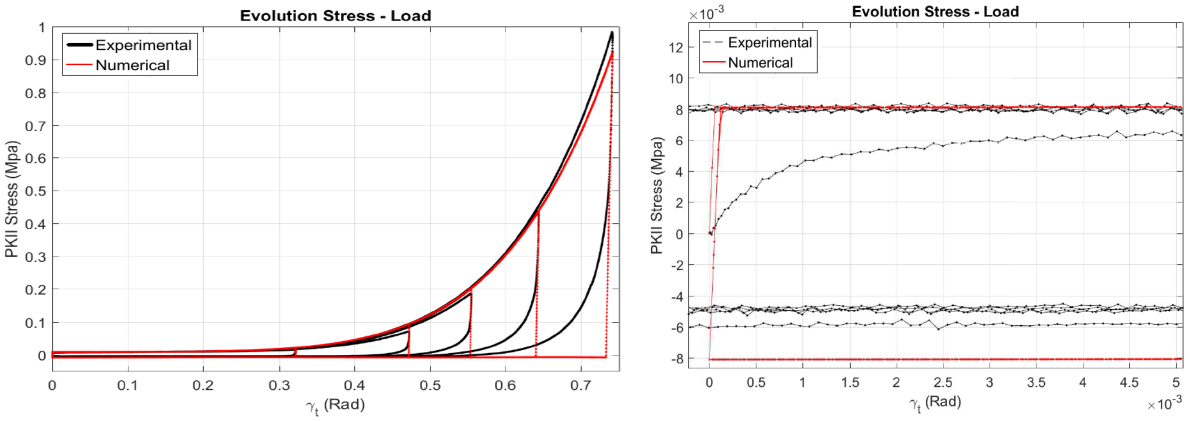

Figure 5: Result without asymptotic return

As we can see, the numerical simulation does not fit with the experimental acquisition during the return (the relaxation of the loading) [Fig5]. The behaviour is asymptotic and few models already exist to approach this path. For example, there is models using fractional derivative [11]. These models are very consistent but they cannot be easily applied under large strain. The model presented here is based on the model of Mroz about nested surfaces [8].

4 APPORACH THE ASYMPTOTIC RETURN

As we said, the model presented here is based on the Mroz model [8]. It means, we define some nested surface and see how to manage them. The difference with model of Mroz, Prager, Ziegler, Popov or other [7], [8], [9], [10], [11] is about the evolution of the surfaces. In fact, the idea here to approach this asymptotic behaviour is to make the size of the surface increase and decrease. In models, which already exist, the size of the surface just increase but never decrease.

In this model, we define i nested surfaces (which are internal) and each surfaces is describes by a criterion. The external surface called n has the hardening law and the plasticity criterion defined above. n is the largest surface of the model. Each nested surfaces i cannot increase or move outside the first surface larger (i+1) than i. This make the first condition:

- C1: The internal surface i must swell less quickly than the surface i+1

Nevertheless, concerning the kinematic of the nested surface, the condition applied by Simo or other is enough to describe the behaviour. This make the second and third conditions:

- C2: The internal surface i has to move faster than the surface i+1
- C3: These internal surfaces kinematic laws are defined by the rate of plastic shear angle.

Condition C1, C2 & C3 give the possibility to write the hardening laws (Qi and Ai) of each nested surfaces. However the hardening laws of nested surfaces are different if the external surfaces n is activate. First, the isotropic laws (Ai) do not change but must be update proportionally of the isotropic law of the external surface n. After that, the idea is to calculate the distance between the borders of the nested surface i and the border of the external surface n. This distance is useful to update the translation of the nested surface (Qi). The interest of this method is its simplicity because every hardening variable is just dependant of the plastic shear angle. Then we can keep a Newton-Raphson algorithm to find the value of this angle. The only thing to be careful of is the elastic prediction. We have to check the number of surface exceeded by the prediction. If the number is superior of one, then it is very important to check if after the first radial return, we are still outside the border of a surface i+1. A simulation was made with two internal nested surfaces and one external surface. The result is following [Fig6]:

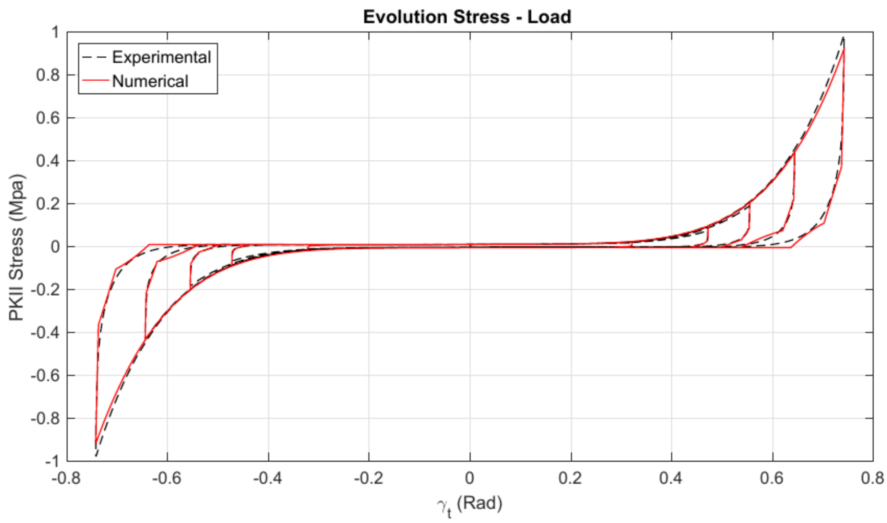

Figure 6: result after asymptotic return model

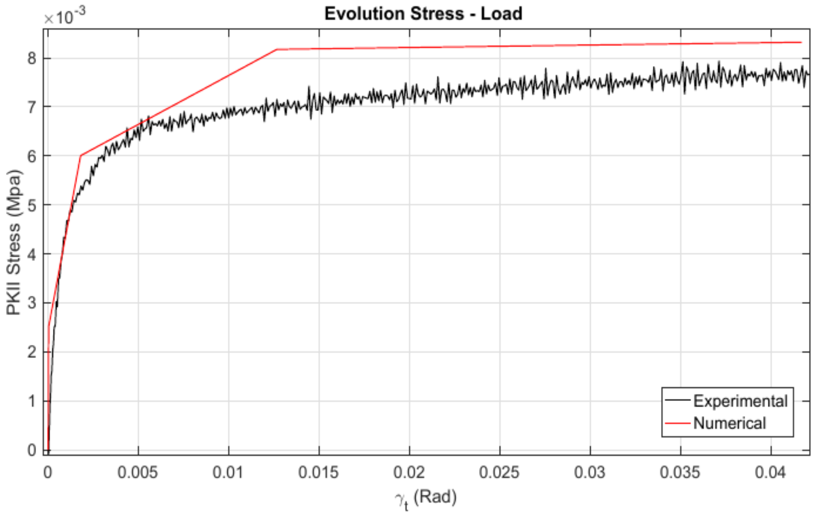

Figure 7: result at the beginning of the load

The results are very interesting and are showing that the model of nested surfaces can be used even under large strain. The numerical responses is very close to the experimental acquisition. The differences at the beginning is not important because the material start to dissipate very quickly. Then the elastic part may be neglected.

5 CONCLUSION

The model presented here permits to describe the hysteretic behaviour of a woven material under large strains. This model is based of models which already exist, and it is shown that the Green Naghdi decomposition and the Mroz nested surfaces theory can be applied for large strains. The biggest interest of this model is its simplicity because everything is known or depend of only one variable, which is the plastic shear angle. The weak point of this model is the number of constant. In fact, to describe properly the evolution and the return, the coefficient of each internal surfaces hardening laws must evolve in proportion to the shearing angles. In other words, depending of the plastic shear angle interval, the coefficient must change. This make many variables but if these variables are well managed, it is possible to fit properly the behaviour. Finally, the more surfaces there are, the more it will be possible to fit to the curve properly.

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