A CRITICAL ASSESSMENT OF COMPUTER TOOLS FOR CALCULATING PROPERTIES OF BRIDGE GIRDERS WITH DIFFERENT CROSS-SECTION

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ABSTRACT

The purpose of this paper is to critically assess several computer tools for calculating properties of bridge girders. The basic ides of each tool are briefly summarized and the advantages and disadvantages of each tool are pointed out. Several benchmark examples, including open sections, such as "π" beams, and closed sections, such as box-beams are used to evaluate the performance of different tools. Very different predictions have been found for different tools. Such a systematic and critical assessment should provide some guidance for engineering to choose the right tool to effectively design and analyze cross section of girder with confidence.

1 INTRODUCTION

The great majority of new bridge have been constructed in order to cope with the increasing volume of highway traffic. Bridge engineering is becoming one of the most feasible and affordable renewable area. To further reduce the weight of cross section of bridge girders and resist the dynamic loads, such as wind and earthquake, engineers are actively pursuing cross section with less conservative and more optimized design. Now, cross sections of bridge girders up to 30m width (such as decks in cable-stayed or suspension bridge) or 11m height (such as in rigid frame bridge) are routinely produced, and several designers have prototyped cross section with more width and height to meet the ever-increasing demand. These huge and sophisticated cross section pose a significant challenge for engineering design and analysis. Furthermore, composite construction, using a reinforced concrete slab on top of steel girders, is used over a wide range of span size because of its economical and popular form of construction for highway bridges. The goal of simulation in finite element analysis is to accurately model the bridge girders quickly with reasonable accuracy. However, due to the scale and cost of the girder under development, preliminary design must be accurate as building prototypes and testing to validate models is costly. The increasing size and complex of the girder has always challenged designers and simulation methods usually are quickly developed to take into account any new considerations.

The bridge girders are critical components of the bridge system, and how to design and build better girder is an active field of research and development in the bridge engineering. Better designed girder will not only increase its own cost effectiveness, but also could result in substantial saving for several major components such as the towers and stayed cables or suspensions, and ultimately reduce the initial cost of the whole system and increase the competitiveness of bridge. Large composite beams are now rendering the early try-and-error intuition-based approach outdated and engineers are relying on more reliable computer tools to analyze the girder in the early design process. To confidently design composite girder, one must integrate both aerodynamic and structural concerns based on a rigorous treatment of the aeroelastic nature of the system. With the recent advances of computational hardware and software, it is possible to tackle this aeroelastic problem using the finite element analysis coupled with the Computational Fluid Dynamics (CFD).

The most brute-force approach is to use the three-dimensional (3D) FEA based on brick elements. If everything is done right, this approach should provide the most accurate prediction. However, this approach required detailed geometric and other information of the girder, making the modeling and computational costs too prohibitive for it to be an efficient approach for the first several design stages including but not limited to both construction design and preliminary design, not to mention that many structural details necessary for 3D modeling are not available until late stages of the design process after many design and analysis iterations.

Because the height of beams are usually small when compared to its length, it is possible to use FEA based on two-dimensional (2D) shell elements to simplify the analysis with a model of a size two orders of magnitude less than of FEA using brick elements. However, it has been found that FEM using shell elements with offset nodes may results in very poor prediction for the shear stress, see, for example, the finite element analysis of a composite box girder [7]. The laminated shell elements are usually based on the classical laminate theory invoking the Kirchhoff-love assumption that effectively ignores the through-thickness effect of materials due to the transverse interaction between adjacent layers. Although shell elements with reference surface at the middle can provide acceptable results for the simple cylinder with very thin walls, this approach leads to model discontinuity to ply-drops and different thickness, making it difficult to setup the model, interpret results, and evaluate the accuracy of this method.

It is well known that we cannot design and analyze the girders by themselves, and to accurately estimate the dynamic behavior of the beams, we have to perform an aerodynamic analysis of the multibody beam system. Even if the aerodynamic part can be simplified, the multibody dynamic behavior must be simulated. The current practice is to obtain the frequencies and mode shapes for the first several elastic modes and replace the girders using model elements in a multibody dynamics code, such as ADAMS. The elastic modes can be directly obtained from FEA using brick elements or shell elements. However, because the girders are very slender with one dimension much larger than the other two, the first several elastic modes will demonstrate the so-called girder behavior including vertical bending, lateral bending, and torsion. For this reason, the FEA based on brick elements or shell elements, although valuable for obtained detailed stress distribution, are believed to be overkill for aeroelastic analysis of the multibody system.

An alternative approach is to model girders as one-dimensional (1D) beams. This approach is of particular value for introducing aeroelastic analysis and multi-flexible dynamic analysis into the early design phase. Using this approach, structural optimization can be achieved through numerous iterations between the design modifications and comprehensive dynamic and aeroelastic simulations. Serious aeromethanical problems can be timely discovered before the design space is significantly narrowed. It has also been shown that beam models of composite girders, if constructed appropriately, can achieve almost the same accuracy as 3D FEA using brick elements, for both global behavior and pointwise 3D stress/strain distribution, at a cost two to three orders of magnitude less.

Beam models such as the Eular-Bernoulli model and the Timoshenko model have been well established for a long time. However, how to evaluate sectional properties, including both structural and inertial properties, for composite beams with complex geometry has been an active research field in recent years. Particularly relevant to girders, various approaches have been proposed in the literature and several tools are commonly used including CrossX (cross-sectional analysis), Sigma-X (Sigma-X section analysis),and VABS (Variational Asymptotic Beam Sectional Analysis). As the accuracy of girder properties directly affects the simulation of the static and dynamics, and ultimately the performance and failure of the whole bridge system, it is crucial for us to have confidence in the calculated girder properties before we proceed to other calculations necessary for the design and analysis of this system. Hence, accurately predict the girder structural properties, designer can better engineer the better girders.

To this end, we are going to provide a critical assessment of the computer tools currently used for calculating girder properties. First, we will describe the properties necessary for modeling girders. Then we will describe several tools commonly used to obtain the girder properties. The basic ideas of each tool are briefly summarized along with its advantages and disadvantages. Finally, we will use several benchmark examples, including a homogeneous and/or highly heterogeneous open section, such as ”π” beams, and a closed section, such as box-beams, to evaluate the performance of different tools. We believe that such a systematic and critical assessment will provide some guidance for engineers to choose the right tool to effectively design and analyze girder with confidence.

2 INERTIAL AND STRUCTURAL PROPERTIES OF COMPOSITE GIRDERS

The girder in bridge engineering is a complex flexible structure. To accurately predict the behavior of the complex girder structure using a beam model, we need to find a way to reproduce, as accurately as possible, the energies including both the kinetic energy and strain energy stored in the original 3D structure in a 1D format. Suppose *V*1, *V*2 and *V*3 represent three linear velocity components and $\Omega_1, \Omega_2, \Omega_3$ represents three angular velocity components of any point in the beam reference, the kinetic energy density of the beam can be written as

$$\kappa = \frac{1}{2}\begin{Bmatrix} V_1 \\ V_2 \\ V_3 \\ \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{Bmatrix}^T \begin{bmatrix} \mu & 0 & 0 & 0 & \mu x_{m3} & -\mu x_{m2} \\ & \mu & 0 & -\mu x_{m3} & 0 & 0 \\ & & \mu & \mu x_{m2} & 0 & 0 \\ & & & i_{22}+i_{33} & 0 & 0 \\ & sym. & & & i_{22} & -i_{23} \\ & & & & & i_{33} \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \\ \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{Bmatrix} \tag{1}$$

where μ_1 is mass per unit length, (x_{m2}, x_{m3}) is the location of mass center measured in the user defined reference coordinate system, i22,i33 are the mass moment of inertia about x_2 and x_3 axis, respectively. i_{23} is the product of inertia. Here, we choose x_1 along the beam reference line, and x_2 and x_3 for the coordinates in the cross-sectional plane, as sketched in Figure 1.

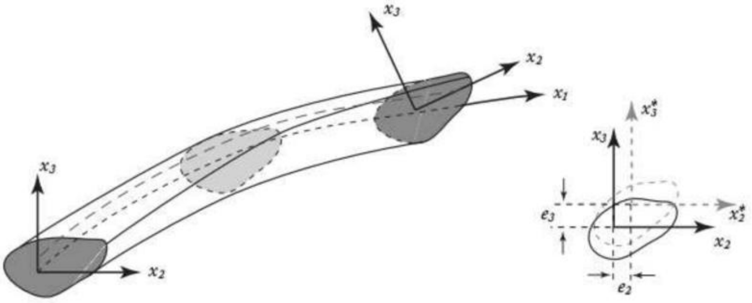

Figure 1: Coordinate system and sketch of a beam

It is noted here that it is common practice that the rotary inertia terms associated bending are discarded in beam analysis using the Euler-Bernoulli model. If we choose the coordinate in such a way that x_1 is the locus of mass centers and x_2 and x_3 along the principal inertial axis, the 6×6 inertia matrix in Eq. (1) will become a diagonal matrix with μ, i_{22}, i_{33} to characterize the inertial properties of the cross-section. Hence, for an arbitrarily chosen coordinate system, we can also use these three values (μ, i_{22}, i_{33}) along with mass center location (x_{m2}, x_{m3}) and the angle between the principal inertial axis and x_2 to replace the inertia matrix in Eq. (1). Often times, the beam reference line is chosen based on engineering convenience. If we choose a different set of coordinates as the reference to express the kinetic energy, such as x_1^*, x_2^* and x_3^* parallel to x_1, x_2, x_3 in Figure 1, we need to carry out a proper transformation of the mass matrix in Eq. (1) which was originally calculated based on the unstarred coordinate system. Based on the definition of the linear and angular velocities [4], we can derive the following relations

$$\begin{gathered} V_1 = V_1^* - e_3\Omega_2 + e_2\Omega_3, V_2 = V_2^* + e_3\Omega_1 \\ V_3 = V_3^* - e_2\Omega_1, \Omega_1 = \Omega_1^*, \Omega_2 = \Omega_2^*, \Omega_3 = \Omega_3^* \end{gathered} \tag{2}$$

with the starred quantities denoting the velocity components in the starred coordinate system. This can also be written in the following matrix form as

$$\begin{Bmatrix} V_1 \\ V_2 \\ V_3 \\ \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & -e_3 & e_2 \\ 0 & 1 & 0 & e_3 & 0 & 0 \\ 0 & 0 & 1 & -e_2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} V_1^* \\ V_2^* \\ V_3^* \\ \Omega_1^* \\ \Omega_2^* \\ \Omega_3^* \end{Bmatrix} \tag{3}$$

We know that the kinetic energy density of the beam, as a scalar, will remain invariant with respect to the choice of the coordinate systems. Substituting Eq. (3) into Eq. (1), we can express the kinetic energy in the starred coordinate system as Eq. 4. with $x_{m3}^* = x_{m3} - e_3$, $x_{m2}^* = x_{m2} - e_2$, and $i_{11}^* = i_{22} + i_{33} + \mu\left(e_2^2 - 2x_{m2}e_2 + e_3^2 - 2e_2 x_{m3}\right)$.

The form of 1D strain energy depends on which model the beam theory is based on. For example, for the Euler-Bernoulli model which is capable of dealing with extension, torsion and bending in two directions, the strain energy can be written as

$$\kappa = \frac{1}{2}\begin{Bmatrix} V_1^* \\ V_2^* \\ V_3^* \\ \Omega_1^* \\ \Omega_2^* \\ \Omega_3^* \end{Bmatrix}^T \begin{bmatrix} \mu & 0 & 0 & 0 & \mu x_{m3}^* & -\mu x_{m2}^* \\ & \mu & 0 & -\mu x_{m3}^* & 0 & 0 \\ & & \mu & \mu x_{m2}^* & 0 & 0 \\ & & & i_{11}^* & 0 & 0 \\ & sym. & & & i_{22} + e_3\left(e_3 - 2x_{m3}\right) & \left(e_2 x_{m2} + e_2 x_{m3}^*\right)\mu - i_{23} \\ & & & & & i_{33} + e_2\left(e_2 - 2x_{m2}\right)\mu \end{bmatrix} \begin{Bmatrix} V_1^* \\ V_2^* \\ V_3^* \\ \Omega_1^* \\ \Omega_2^* \\ \Omega_3^* \end{Bmatrix} \tag{4}$$

$$U = \frac{1}{2}\begin{Bmatrix} \gamma_{11} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{Bmatrix}^T \begin{bmatrix} EA & S_{12} & S_{13} & S_{14} \\ & GJ & S_{23} & S_{24} \\ sym. & & EI_{22} & S_{34} \\ & & & EI_{33} \end{bmatrix} \begin{Bmatrix} \gamma_{11} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{Bmatrix} \tag{5}$$

where $\gamma_{11}, \kappa_1, \kappa_2, \kappa_3$ are the extensional strain, twist, bending curvatures about x_2 and x_3, respectively. S_{12} is the extension−twist coupling term, and S_{13}, S_{14} the extension −bending coupling terms. The relations between the couplings and corresponding diagonal stiffness terms are the same as those for initially twisted and curved isotropic solid beams obtained in Berdichevsky and Starosel′skii (1985) [2] and Berdichevsky and Starosel′skii (1983) [1], which are listed here for completeness.

$$\begin{aligned} S_{12} &= \left[S_{33} + S_{44} - 2\left(v+1\right)S_{22}\right]\kappa_1 \\ S_{13} &= -\left(1+v\right)S_{33}\kappa_2 \\ S_{14} &= -\left(1+v\right)S_{44}\kappa_3 \end{aligned} \tag{6}$$

These relations are also verified in other works, including Cesnik et al. (1996) [3] and Hodges (1999) [6]. The fact that our model can reproduce such relations clearly demonstrates the validity of the model for isotropic open sections in general.

The components of the 4×4 stiffness matrix in Eq. (5) depend on the choice of the beam coordinate system, initial curvatures/twist, as well as the geometry and material of the cross-section [12]. The diagonal terms EA, GJ, EI_{22}, EI_{33} are the extensional stiffness, the torsional stiffness, and bending stiffness about x2 and x3, respectively. The off-diagonal terms represent the elastic couplings between different deformation modes.

We could also have the so-called tension center (xt2, xt3) defined such that an axial force applied at this point will not introduce any bending. We can also find the so-called principal bending axis so that there is no coupling between two bending directions. For prismatic beams made of isotropic materials with the reference line located at the tension centers and x2 and x3 aligned with the principal bending axes, the stiffness matrix is diagonal and the four deformation modes are completely decoupled.

$$U=\frac{1}{2}\begin{Bmatrix}\gamma_{11}\\ \kappa_1\\ \kappa_2\\ \kappa_3\end{Bmatrix}^T\begin{bmatrix}EA & 0 & 0 & 0\\ & GJ & 0 & 0\\ sym. & & EI_{22} & 0\\ & & & EI_{33}\end{bmatrix}\begin{Bmatrix}\gamma_{11}\\ \kappa_1\\ \kappa_2\\ \kappa_3\end{Bmatrix} \tag{7}$$

For more general cases such as initially curved or twisted composite beams, such decoupling is not possible and providing information regarding tension center and principal bending axes is not as meaningful for composite beams as for isotropic beams. The strain energy can be written as

$$U=\frac{1}{2}\begin{Bmatrix}\gamma_{11}\\ \kappa_1\\ \kappa_2\\ \kappa_3\end{Bmatrix}^T\begin{bmatrix}EA & 0 & S_3 & -S_2\\ & GJ & 0 & 0\\ sym. & & EI_{22} & -EI_{23}\\ & & & EI_{33}\end{bmatrix}\begin{Bmatrix}\gamma_{11}\\ \kappa_1\\ \kappa_2\\ \kappa_3\end{Bmatrix} \tag{8}$$

It is emphasized here that for general beams, the mass center might not be the same as the tension center, and the principal inertial axes might not be the same as the principal bending axes. In other words, it is impossible for us to choose a single coordinate system with the origin located both at the mass center and the tension center, x_2 and x_3 aligned with both the principal inertial axes and the principal bending axes. Also it is important to repeat what has been pointed out in Ref. [5] that for accurate prediction using the Euler-Bernoulli beam model, it is necessary for the analyst to choose the reference along the locus of shear centers (x_{s2}, x_{s3}), particularly for torsional behavior. For this very reason, providing the stiffness values for the Euler-Bernoulli model without providing the location of shear center is not sufficient. For general composite beams, only the so-called generalized shear center as defined in Ref. [5] will always exist.

One can also use the Timoshenko model which is capable of dealing with extension, torsion, bending in two directions, and transverse shear in two directions to analyze the blade, the strain energy of which can be written as

$$U=\frac{1}{2}\begin{Bmatrix}\gamma_{11}\\ 2\gamma_{12}\\ 2\gamma_{13}\\ \kappa_1\\ \kappa_2\\ \kappa_3\end{Bmatrix}^T\begin{bmatrix}S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16}\\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26}\\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36}\\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46}\\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56}\\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66}\end{bmatrix}\begin{Bmatrix}\gamma_{11}\\ 2\gamma_{12}\\ 2\gamma_{13}\\ \kappa_1\\ \kappa_2\\ \kappa_3\end{Bmatrix} \tag{9}$$

where $2\gamma_{12}$ and $2\gamma_{13}$ are two transverse shear strains. The Timoshenko model provides better predictions for relatively shorter beams, particularly the dynamic behavior, although it requires more degrees of freedom than the Euler-Bernoulli model. More significantly, the equations of Timoshenko model are hyperbolic, unlike the Euler-Bernoulli model which is partially parabolic, for which wave velocity is unbounded. Also, if one uses the Timoshenko model for the 1D beam analysis, [13] the analyst is free to choose an arbitrary reference line. Of course, when the coordinate system chosen for the 1D beam analysis is different from the coordinate system one used to calculate the stiffness properties, we need to transform the stiffness matrix properly. It can be shown the relationship between the strain measures in the two coordinate systems as sketched in Figure 1 is exactly the same as those in Eq. (3),[4].

Using this relationship in Eq.(3), we can straightforwardly write out the stiffness matrix in the starred coordinate system. To transform the Euler-Bernoulli model into a different coordinate system, we just need to neglect the transverse shear strains in Eq. (3). As mentioned previously, to use the Euler-Bernoulli model, one must choose the shear center as the reference. If a tool outputs a Euler-Bernoulli model at a different reference, we have to do the transformation. For example, if the stiffness coefficients for the Euler-Bernoulli model are given in terms of the tension center, that is S_{13} and S_{14} are zero in Eq. (5), then the Euler-Bernoulli model with the reference at the shear center will be

$$U=\frac{1}{2}\begin{Bmatrix}\gamma_{11}\\ \kappa_1\\ \kappa_2\\ \kappa_3\end{Bmatrix}^T\begin{bmatrix}EA & S_{12} & -e_3EA & e_2EA\\ & GJ & S_{23}-e_3S_{12} & S_{24}+e_2S_{12}\\ sym. & & EI_{22}+e_3^2EA & S_{34}-e_2e_3EA\\ & & & EI_{33}+e_2^2EA\end{bmatrix}\begin{Bmatrix}\gamma_{11}\\ \kappa_1\\ \kappa_2\\ \kappa_3\end{Bmatrix} \tag{10}$$

with e_2 and e_3 denoting offsets from the tension center, positive along x_2 and x_3 directions, respectively

The task of calculating the inertia properties in Eq. (1) and the structural properties in either Eq. (5) or Eq. (7) belongs to the domain of a cross-sectional analysis. Accurate evaluation of these properties is extremely important for success of modeling the beams for design and analysis purpose. It is emphasized here that most current beam simulation tools only require a subset of the inertial properties and structural properties, such as mass per unit length, bending stiffness (EI_{22}, EI_{33}), and torsional stiffness GIt. Currently the torsional stiffness is not used for beam design in bridge engineering and analysis mainly because the most beams in operation are torsionally stiff and beams have low coefficient of moment. However, as the beam becomes larger, more flexible and more anisotropic, other inertial and structural properties, such as the twist-bending coupling will be critically needed for better prediction.

3 DIFFERENT APPROACHES FOR CALCULATING BEAMS

In recent years, several approaches have been proposed for calculating the inertial and structural properties for beams including CrossX, VABS, Sigma-X and ANSYS. It is pointed out that one can also use these tools to calculate the beam properties for other slender components in the bridge engineering if the engineer chooses to model such components as beams. Without repeating the details of each tool, which can be found in their relevant publications, we will only briefly summarize the theoretical foundation of each approach and point out the advantages and disadvantages of each approach in this section. A more extensive evaluation of each tool will be presented in the next section using some benchmark examples.

3.1 VABS

Under nearly two decades of support from the US army and the national rotorcraft technology center, Hodges and his co-workers have developed VABS (variational asymptotic beams analysis), a unique cross-sectional analysis tool capable of realistic modelling of initially curved and pre-twisted anisotropic beams with arbitrary sectional topology and material construction. The salient features of VABS are:

- *Use the variational asymptotic method to avoid apriori assumptions*, which are commonly invoked in other approaches, providing the most mathematical rigor and the best engineering generality and simplicity.
- *Decouple a 3D nonlinear problem into two sets of analysis*: a linear cross-sectional analysis over the crosssection and a geometrically exact beam analysis over the reference line. The allows the 1D beam analysis to be formulated exactly as a general continuum and confines all approximations to the cross-sectional analysis, the accuracy of which is guaranteed to be the

best by the variational asymptotic method. That is, it can calculate the geometric properties requiring neither the costly use of 3-D finite element discretization nor the loss of accuracy inherent in the simplified representations of the cross section [11].

- *Maintain the engineering simplicity and legacy* by repacking the refined asymptotically correct functionals into common engineering models such as the Euler-Bernoulli model, the Timoshenko model, or the Vlasov model. It can perform a classical analysis for initially twisted and curved inhomogeneous, anisotropic beams with arbitrary geometry, material properties, and reference cross sections.

VABS not only calculates the sectional properties compatible with linear and nonlinear beam analysis, but can also recover the pointwise distribution of the 3D displacement/stress/strain field [8]-[9]. When compared to the FEA using 3D brick elements, two to three orders of magnitude in computing time can be saved by using VABS, with little loss of the accuracy. The advantages of VABS over other technologies are demonstrated by its virtues of being general-purposed, accurate, and robust. However, VABS requires a finite element discretization of the cross section, and it is very tedious to generate VABS input files for realistic beams. Very recently, a design driven pre-processing computer program, PreVABS, has been developed for efficiently generating VABS inputs for realistic girders by directly using the design parameters such as CAD geometric outputs and the spanwise and chordwise varying crosssectional laminate lay-up schema. PreVABS can handle complex beam configurations, including both symmetric and asymmetric profiles, both spanwisely and chordwisely varying lamina scheme. VABS, powered by PreVABS, can easily provide an efficient high-fidelity analysis for real girders with a model setup effort similar to that of PreVABS.

3.2 CrossX

CrossX is the result of an activity that started with two student project/diploma works at the department of structural engineering at the former Norwegian Institute of Technology (NTH), which is now part of the Norwegian University of Science and Technology (NTNU), in 1993.

CrossX is a windows-based, interactive program for calculation of sectional parameters and stiffness of, and stress distributions due to user supplied forces and moments on arbitrary beam cross sections composed of one or more isotropic and linearly elastic materials. Thin-walled sections and massive sections developed independent. The focused on the computational aspects of the problem, and the resulting programs had rather crude user interfaces. The torsional stiffness is computed based on St. Venant's theory in massive sections, neglecting the effects of the warping functions altogether; while in the thin-walled sections, the warping effect is considered.

In massive part if CrossX, it invokes the following assumptions

- The cross section of the beam is uniform over its entire length.
- The x-axis is straight and coincides with the centroid of the cross section.
- The x-axis is straight and coincides with the centroid of the cross section.
- The body forces are small compared to he stresses and can be neglected.

While in thin-walled part of CrossX, all computations are based on the following basic assumptions:

- The cross section belongs to a straight beam with constant cross section.
- Local deformations of the cross section are neglected, i.e., the section maintains its geometrical shape with no distortions in its own plane.
- Local (transverse) moments and shear forces are neglected.
- Shear strain due to torsion is neglected for the open parts of the section.
- Bending deformations obey Navire's hypothesis (plane sections remain plane).

CrossX allows arbitrary cross-sectional geometry. It can predict the complete set of stiffness coefficients needed for the Euler-Bernoulli beam model in Eq. (5), and the inertial properties including mass per unit length, mass moments of inertia, and the principal inertial axis. CrossX can also calculate the shear center, tension center and the shear deformation factors. The advantage of CrossX lies in its efficiency because it is based on the finite element method. It is also general enough to deal most of beams with with very few restrictions. However, because of its adoption of oversimplified assumptions, there are some concerns about its accuracy in addition to its admitted approximation in shear center calculations.

3.3 Sigma-X

Sigma-X section is developed by sigma-X Ltd, Ireland. Sigma-X Section is a structural engineering software that accurately calculates the geometric and material properties of standard, arbitrary and built up sections. it can calculate area, section centroid, first and second moments of area, radii of gyration, principle properties, elastic and plastic neutral axes, shear properties, torsion and warping constants.

The approach used in the software is based on three-dimensional descriptions using boundary element method (BEM). The advantage of the Sigma-X is that it is very efficient because all the calculation is based on analytical formulas. However, Sigma-X only produces a subset of the inertia and structural properties of the section. Particularly it misses the increasingly important coupling terms between twist and other deformation modes and shear center location, which is indispensable for one to use the Euler-Bernoulli beam model. Second, since it depends on a FEA using either 3D brick elements or 2D shell elements, this approach will be more labor intensive and computational expensive in comparison to the other approaches.

3.4 Overall assessment

Although it is difficult to provide conclusive statement about each tool purely based its publications, we can at least provide a qualitatively assessment regarding their theoretical foundations and their functionalities. Among the three cross-sectional analysis tools, it seems the theory of CrossX is the most rudimentary since it invokes a lot of assumptions. Although it is hard to assess the theory of Sigma-X because of its very limited description, it does show a level of sophistication with its ability to calculate the shear center of an arbitrary composite cross section. VABS has a unique mathematical foundation which is far more sophisticated than the other tools. As far as efficiency is concerned, CrossX, and Sigma-X should be similar because their calculations are based on analytical formulas, and should be more efficient than VABS which is based on a 2D finite element analysis of the cross-section. As far as functionalities are concerned, usually it is not that difficult for any tool to calculate the inertial properties including mass per unit length, mass moments of inertia, and principal inertial axis. For structural properties, VABS can provide the most information for a given cross-section including Euler-Bernoulli model, Timoshenko model and Vlasov model, and characteristic centers including mass center, shear center and tension center. The CrossX method can only provide a Timoshenko model for the beam along with shear center and tension center. Sigma-X can provide the structural properties for the Euler-Bernoulli model with the shear center location, and the principal bending and torsional stiffness values along with mass per unit length and mass center, but it cannot provide a Timoshenko model or a Vlasov model. Among all the cross-sectional analysis tools, only VABS can accurately recover the 3D displacement, stress, and strain field, comparable to a 3D FEA using brick elements.

4 ASSESSMENT EXAMPLES

This section presents a detailed and systematical assessment of several commonly used and easily accessible beam design and analysis tools, including VABS, Sigma-X and CrossX (massive sections or thin-walled sections). Various examples of isotropic sections with different geometry schemas, including a homogeneous or highly heterogeneous open section, such as π beams, and a closed section, such as box-beams are analyzed and the resulting sectional properties such as the mass and stiffness coefficients, the locations of the mass center and the shear center are compared to assess the accuracies and limitations of these tools. Although VABS can provide various common engineering beam models such as the Euler-Bernoulli model, the Timoshenko model, and the Vlasov model, only the sectional properties for the Euler-Bernoulli model are provided in this assessment, to facilitate our comparison with other two tools, which can only provide the Euler-bernoulli model. As point out in [5], for reliable prediction of behavior of beam using the Euler-Bernoulli model, the analyst must choose the reference line along the generalized shear centers. Hence the shear center location is also provided for cases where it is not at the origin of the coordinate system. The inertia properties provided for the following examples are referred to the principal inertia axes at the mass center. The structural properties are referred to the shear center with axes parallel to the user defined axes x2 and x3. In this work analytical results are obtained for isotropic sections with simple geometry and more complex sections. The analytical method used in this article is either based on the elasticity theory or based on the thin-walled theory readily available in textbooks on isotropic beam theories [10].

Items	Cross-X	VABS	Sigma-X	Analytical	Diff. (Cross-X)	Diff. (VABS)	Diff. (Sigma-X)
$EA(N)$	3.8220E+09	3.8220E+09	3.8220E+09	3.8200E+09	0.05%	0.05%	0.05%
$EI_{33}(Nmm^2)$	2.1334E+11	2.1334E+11	2.1300E+11	2.1334E+11	0.00%	0.00%	-0.16%
$EI_{22}(Nmm^2)$	1.9800E+09	1.9800E+09	1.9800E+09	1.9800E+09	0.00%	0.00%	0.00%
$GI_t(Nmm^2)$	5.1195E+08	5.1265E+08	5.1680E+08	5.1265E+08	-0.14%	0.00%	0.81%
$\mu(kg/m)$	1.4287E+01	1.4287E+01	1.4300E+01	1.4287E+01	0.00%	0.00%	0.09%
$x_{s3}(m)$	1.1100E+00	1.1300E+00	1.1300E+00	1.1300E+00	-1.77%	0.00%	0.00%
$x_{m3}(m)$	-7.4450E-01	-7.4450E-01	-7.4450E-01	-7.4450E-01	0.00%	0.00%	0.00%

Table 1: Sectional properties of a isotopic section.

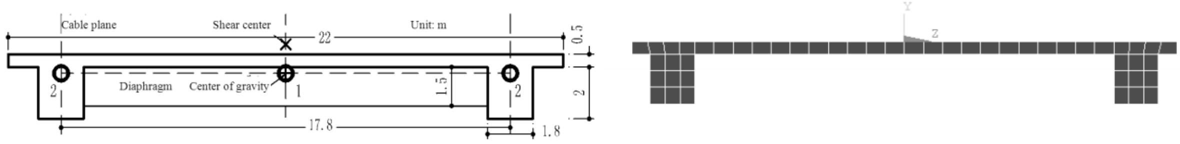

Figure 2: Schematic and mesh result of a isotopic open section by VABS.

4.1 Isotropic cases

4.1.1 Open section

The first example is an isotropic and homogeneous open section with width b = 21m and thickness *h* = 2.5m (as shown in Figure 2). The material has Young's modulus E = 2.1 × 105N/mm2, Poison's ratio ν = 0.3, and density ρ = 7850kg/m3. With the origin at the center, only diagonal terms of the cross-sectional stiffness and mass matrix are not zero. Table 1 lists the results obtained by using CrossX (massive sections), VABS, Sigma-X and the ANSYS.

The section is discretized for VABS cross-sectional model used 8-noded quadrilateral elements (Figure 2), while triangle elements are used in CrossX and Sigma-X model (as shown in Figure 3). The unit listed for each quantity is according to the International Standard and will remain the same for all the other examples. The relative error in Table 1 is defined as $\left|\left(X - X_E\right) / X_E\right| \times 100\%$, where X is a special cross-sectional property evaluated using one of aforementioned tools and XE is the corresponding exact solution obtained using the ANSYS software (Figure 3). It can be observed that both mass and stiffness coefficients obtained by VABS are almost exactly the same as those calculated by the ANSYS, with maximum error less than 0.05%. This observation confirms the proof in Ref. [10] that VABS can reproduce the Elasticity theory results for isotropic prismatic beams. The relative errors of these coefficients predicted by CrossX becomes larger, especially the torsion stiffness GIt and shear center xs3. Sigma-X also has an excellent prediction for the bending stiffness and mass per unit length. It is also strange that error of the torsional stiffness predicted by Sigma-X reaches more than 0.8%.

For reference, the 6×6 Timoshenko stiffness matrix calculated by VABS for the open section is listed as Eq. 11:

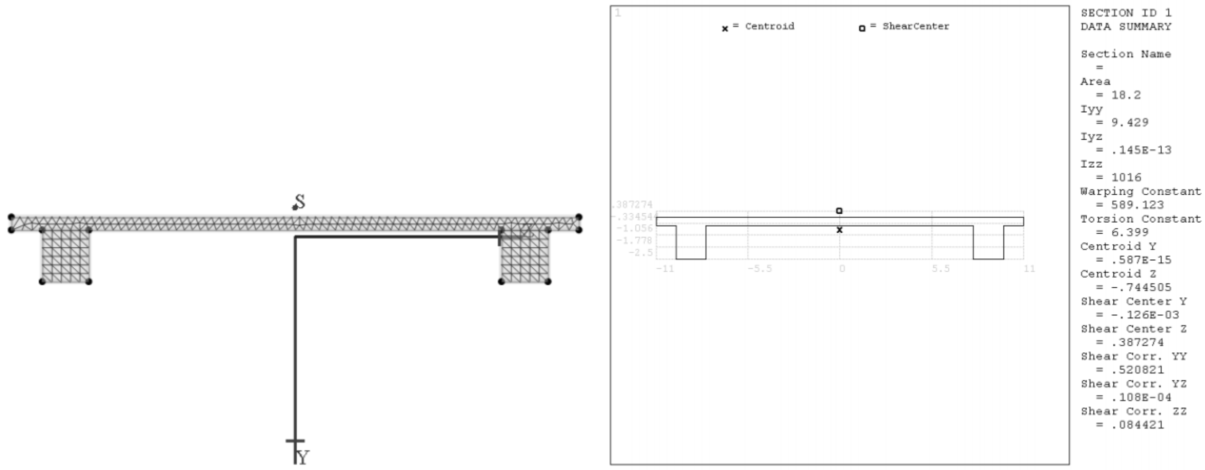

Figure 3: Mesh and numerical result of open section by CrossX and ANSYS, respectively.

$$\begin{bmatrix} 3.822E+9 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ & 7.643E+8 & 0.0 & 0.0 & 0.0 & 0.0 \\ & & 1.114E+8 & 0.0 & 0.0 & 0.0 \\ & & & 1.493E+9 & 0.0 & 0.0 \\ & sym. & & & 1.980E+9 & 0.0 \\ & & & & & 2.133E+11 \end{bmatrix} \tag{11}$$

4.1.2 Closed section

The previous open cross-section has very simple geometry. It will be interesting to point out how different methods perform for more complex geometry, particularly, box-like geometry, which are the target applications of all the tools we are currently assessing. A box combines the advantages of lightness and stiffness in both bending and torsion and, as a result, has largely superseded the ”π”-section, whether of steel or concrete, for longer span beam-type bridges. To this end, we suggest an isotopic box-like section as shown in Figure 4 with width b = 16.5m, depth h = 13m and thickness t = 0.6m. Material properties of this section are the same as the case 1. To complete our assessment, the closed box section is analyzed and compared using VABS, CrossX (thin-walled sections), and Sigma-X. All the stiffness properties are given in a coordinate system with the origin located at the shear center and x2 and x3 axes parallel to the axes sketched in Figure 1. The mesh result by VABS is shown in Figure 4 and numerical result by ANSYS is shown in Figure 5.

The differences between various approaches are listed in Table 2, where the relative differences are calculated with respect to the analytical results. It can be observed that for this section, results predicted by VABS match well with those of the analytical results based on the thin-walled theory, with the maximum percentage difference (0.18%) occurring for the bending stiffness EI33. Since this section is indeed a thin-walled section, it is not a surprise that analytical results based on the thin-walled theory match VABS results very well.

However, Sigma-X results exhibit very large relative difference on literally all sectional properties except extensional stiffness and mass per unit length. While CrossX provides an accurate prediction of mass moments of inertia as well as the tensional stiffness, torsional stiffness, and bending stiffness EI22, it demonstrates relative larger errors on predicting the torsional stiffness GIt. This is attributed to the fact that the analytical approach based on the thin-walled theory cannot accurately locate the shear center. The results predicted by Sigma-X are even worse. It is a thin-walled structure with a wall thickness to chord length ratio of 0.0046, for which one might expect the thin-walled theory, CrossX, and Sigma-X to have a relative good performance. However, this example shows that this is not the case.

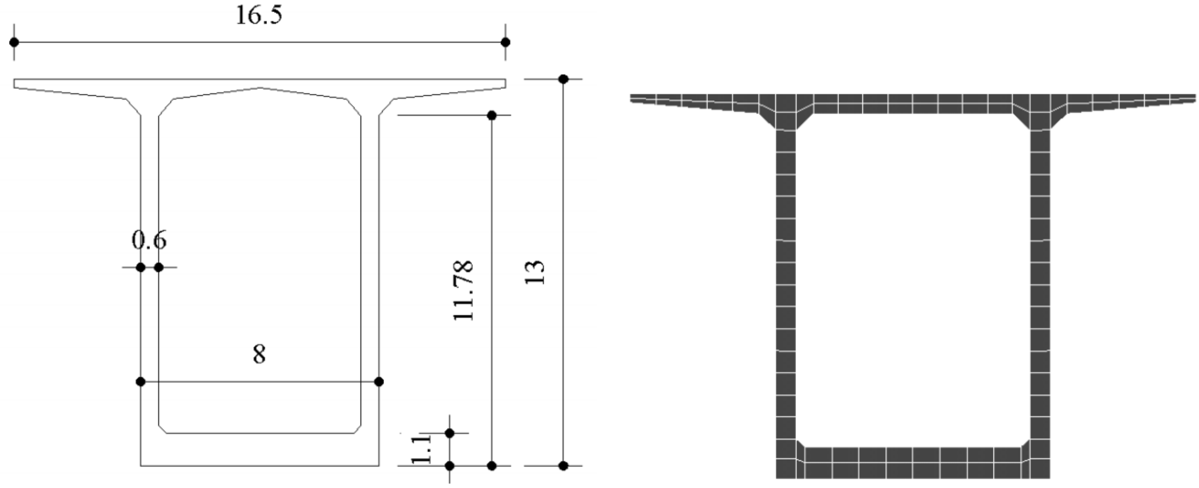

Figure 4: Schematic of a isotopic closed section and the mesh result by VABS; unit (m)

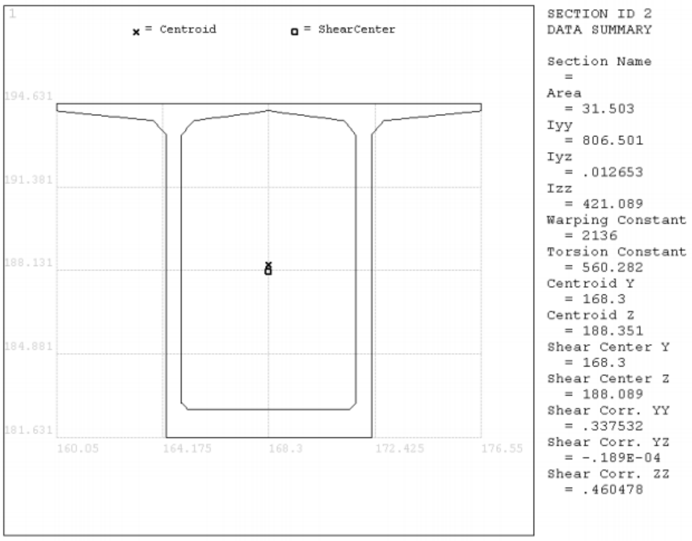

Figure 5: The numerical result of the closed section by ANSYS

None of these two tools (CrossX and Sigma-X) can predict the significant twist-bending coupling, S24, for this section, although this coupling is almost 5 times of the torsional stiffness. This is because the accuracy of the sectional properties are strongly dependent on an accurate calculation of the warping functions. Methods based on apriori assumptions for the section to warp in a certain fashion or completely neglecting the warping effect will have a hard time to provide an accurate prediction for the sectional properties.

4.2 Heterogeneous case

The third example is highly heterogeneous opening section with two different materials having E = 3000N/mm2, μ = 0.225,ρ = 2700kg/m3 (concrete) and E = 210000N/mm2, μ = 0.3,ρ = 7850kg/m3 (steel), respectively. (see the sketch in Figure 6):

This provides a challenging test case for cross-sectional tools to handle highly heterogeneous sections. This section is modelled by both CrossX, Sigma-X and VABS, where the relative differences is calculated with respect to analytical results.

Due to the symmetry, centroid and generalized shear center are coincides with the origin of the coordinate system. It can be observed from Table 3 that the VABS results show an excellent agreement with theory results (with the maximum relative difference less than 0.04%). The results calculated by CrossX and Sigma-X demonstrate large deviations, with the prediction of torsional stiffness GIt and shear center xs3 being the worst, indicating that CrossX and Sigma-X are not suitable for analyzing highly heterogeneous cross-sections if the material properties between different segments of the beam section are drastically different.

Items	Cross-X	VABS	Sigma-X	Analytical	Diff. (Cross-X)	Diff. (VABS)	Diff. (Sigma-X)
$EA(N)$	1.8421E+09	1.8421E+09	1.8421E+09	1.8421E+09	0.00%	0.00%	0.00%
$EI_{33}(Nmm^2)$	1.3340E+11	1.3340E+11	1.3341E+11	1.3340E+11	0.00%	0.00%	0.01%
$EI_{22}(Nmm^2)$	9.3429E+08	9.3400E+08	9.3457E+08	9.3404E+08	0.03%	0.00%	0.06%
$GI_t(Nmm^2)$	3.3597E+08	3.4036E+08	3.3674E+08	3.4034E+08	-1.28%	0.01%	-1.06%
$\mu(kg/m)$	8.6220E+04	8.6220E+04	8.6380E+04	8.6220E+04	0.00%	0.00%	0.19%
$x_{s3}(m)$	7.6380E-01	7.6200E-01	7.6410E-01	7.6230E-01	0.20%	-0.04%	0.24%
$x_{m3}(m)$	-7.4450E-01	-7.4450E-01	-7.4400E-01	-7.4450E-01	0.00%	0.00%	-0.07%

Table 3: Sectional properties of a highly heterogeneous open section.

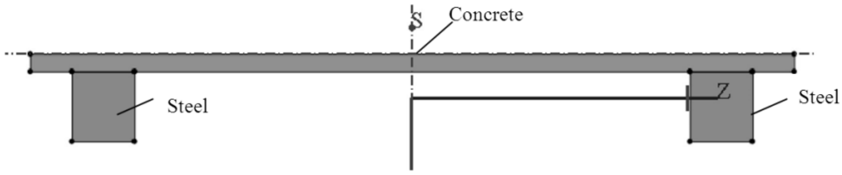

Figure 6: Schematic of a highly heterogeneous open section.

5 DISCUSSIONS

From all these examples, VABS has demonstrated a consistent and reliable prediction for all the material properties in comparing to the analytical results based on thin-walled theory, and other well-accepted tools including ANSYS. However such consistency is not found for CrossX and Sigma-X.

When the cross-section is an isotropic, homogeneous open section with simple geometry, such as the ”π” section, and when the height is small compared to the dimension of the cross section, CrossX and Sigma-X can provide a reasonable prediction for the inertial and structural properties. When the isotropic homogenous, closed-section becomes more complex in geometry, such as the closed box section, the prediction of CrossX becomes worse, particularly for the shear center prediction and torsional stiffness coefficients. CrossX cannot provide reliable predictions for most of the properties except the extensional stiffness and mass per unit length, and some of the coupling terms cannot be predicted by these two tools. It is also worthy to note that predictions made by CrossX and Sigma-X vary with big differences when compared to each other although they are all implementations of analytical formulas based on a similar theoretical foundation and are common tools currently used by bridge engineers. As the girders get more and more sophisticated, real girders will become highly heterogeneous and highly anisotropic. A cross-sectional tool with solid mathematical foundation and demonstrated performance, such as VABS, should be used to accurately predict the sectional properties which are crucial for dynamic and aeroelastic simulations of the complete bridge system so that high-performance systems can be designed and build more cost effectively.

6 CONCLUSIONS

In this paper, we have critically assessed several computer tools commonly used for calculating beam properties including CrossX, VABS, and Sigma-X. The meaning of sectional properties including both inertial and structural properties is precisely described and transformation of the sectional properties to different coordinate system is clearly specified. The theoretical foundation of each tool is brief summarized and the advantages and disadvantages of each tool are pointed out. Several bench-mark examples are used to evaluate the performance of different tools and huge differences have been found among these beam tools. We have also observed that only VABS consistently provides reliable predictions for all the cross-sections we have tested. Such a systematic and critical assessment should provide some guidance for engineers to choose the right tool to effectively design and analyze beams with confidence. Because of the poor and inconsistent performance of CrossX and Sigma for simple cross-sections, their applicability to real complex, composite beams is questionable. On the other hand, through this assessment, VABS, a proven technology in Helicopter Industry, demonstrates its clear advantage and performance over other tools. Particularly, empowered with PreVABS, one should be able to use VABS to perform an efficient yet accurate modeling of the beams with nominal human interaction efforts not more than CrossX or Sigma-X. This assessment paper also points out that we need to provide more extensive validation of the computational tools currently using in the bridge engineering.

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