

# STOCHASTIC DAMAGE MODELLING OF MIXED MODE FATIGUE DELAMINATION OF COMPOSITE WIND TURBINE BLADES

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## ABSTRACT

This paper deals with a stochastic approach to simulate mixed mode fatigue delamination development and obtain the failure probability of composite blades of wind turbines. Based on existing studies, various Paris law is used to predict the propagation of the fatigue delamination growth on mixed mode and reproduce the delamination length subjected to cyclically repeated loadings. Combined with a failure probability of the blades, the remaining service time is estimated by stochastic methods such as gamma process, and the evolution of fatigue delamination length is predicted. The failure time of wind turbine blades for mixed mode is estimated by the fatigue delamination crack growth of stochastic process with its predetermined critical delamination length. A numerical example is investigated to analyse the mixed mode fatigue delamination process by Paris law, and the failure probability of blades under different parameters is estimated by a stochastic gamma process for wind turbine composite blades. The results show that the Paris model with gamma process provides a good prediction of the failure time of the mixed mode fatigue delamination growth, and this stochastic method can be used for reliability analysis and determining optimum maintenance strategies.

## 1 INTRODUCTION

With the higher requirement of sustainable energy, wind farms have become one of most popular clean and renewable power in developed countries. As one typical offshore structures, wind turbines are suffered from the harsh marine environments and cyclic wind loadings during the design life, taking up nearly 23% of manufacturing costs. In order to improve the performance and structural resistance of wind turbines blades, layered fibre-reinforced polymer composite materials are usually manufactured for the large blades of wind turbines as they have better mechanical properties and lighter weight.

Delamination is one of the most important mechanical failures in composite blades during the designed lifetime, usually 25 years for offshore wind turbines according to design code. For glass fibre reinforced composite materials, there are three different delamination failure modes shown in Figure 1, namely, opening (mode I), sliding (mode II) and tearing (mode III) [1]. However, the mode III in fatigue growth is usually not considered in this structure, as the critical energy release rate values in this mode is high, and it is typical for small composite structures due to the constraints [2]. Nearly all fatigue delamination in the composite blade is mixed mode containing mode I and mode II, and it has a significant influence on the structure. Therefore, predicting the development of the delamination length for mixed mode is vital to plan proper maintenance and inspection for reducing lifecycle costs. The mixed mode delamination development of composite blades cannot be predicted deterministically, because of uncertainties in realistic situations like random wind loads. It is vital to select an effective stochastic damage model in order to evaluate the mixed mode delamination behaviour of the composite blades. Paris law is a useful tool for simulating deteriorating composite materials affected by cyclic fatigue damage during the service life [3]. In the review of Blanco et al. [2], various Paris law equations are discussed to simulate the mixed mode propagation of the fatigue delamination crack growth.

In previous studies, the gamma process has been considered as an appropriate stochastic approach to simulate the stochastic deterioration process, such as fatigue delamination damage of wind turbine blades [4, 5]. The evolution of fatigue delamination for wind turbine blades can be modelled as a

stochastic process because of uncertainties in offshore environments. Considering the nature of cumulative growth of fatigue delamination cracks, the gamma process model is an appropriate approach for performance deterioration since gamma process has been proved to be more versatile and increasingly used in optimal maintenance strategies. The gamma process can predict the fatigue crack growth accurately and estimate the time to reach the predefined crack size [6].

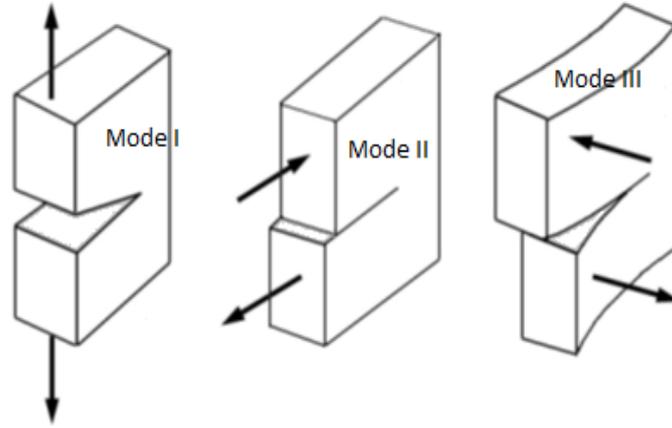


Figure 1: Three modes of fatigue: Model I opening, Mode II sliding, Mode III tearing

This paper adopts the Paris model with the stochastic deterioration modelling according to gamma process for predicting fatigue delamination development within the design service life of composite blades. A numerical example of composite materials is investigated to predict the mixed mode delamination crack growth by Paris law and to estimate the probability of fatigue failure by the gamma process. The probability of failure curves in different cases is then compared in this stochastic deterioration model. Finally, the gamma process can take the uncertainties into consideration for the delamination crack propagation of mixed mode, and the stochastic deterioration process shows a better agreement with the realistic delamination process for composite blades. From the results, the proposed stochastic fatigue delamination evolution modelling can provide a useful method for reliability analysis and optimum repairing plan for composite blades of wind turbines.

## 2 MIXED MODE DELAMINATION CRACK GROWTH

Fatigue damage can weaken the resistance properties of the materials by cyclically loading on specific parts in a long time, and delamination is one of major failure for composite blades. The fatigue delamination crack propagation needs to be investigated to develop a proper model adopting the rate of fatigue crack growth under cyclic loadings. In order to find crack threshold and crack growth propagation, a most widely used method known as Paris law is often used to construct the relationship between the fatigue crack growth rate and strain energy release rate  $\Delta G_{Mix}$ . The crack growth propagation in Paris law can be divided into three stages, crack initiation stage, subcritical crack propagation stage, and critical crack propagation stage, as shown in Figure 2. With fatigue crack propagation under repeated loading, the structures easily reach the first stage of the failure process. Thus, the delamination crack propagation of a structure can be predicted by using the Paris law model.

The strain energy release rate  $\Delta G_{Mix}$  for mixed mode delamination propagation can be written as

$$\frac{da}{dN} = C(\Delta G_{Mix})^m \quad (1)$$

where  $a$  is the crack length;  $N$  is the number of load cycles;  $C$  and  $m$  are parameters related to material constants;  $\Delta G_{Mix}$  is strain energy release rate for mixed mode, defined as

$$\Delta G_{Mix} = G_{Mix,max} - G_{Mix,min} \quad (2)$$

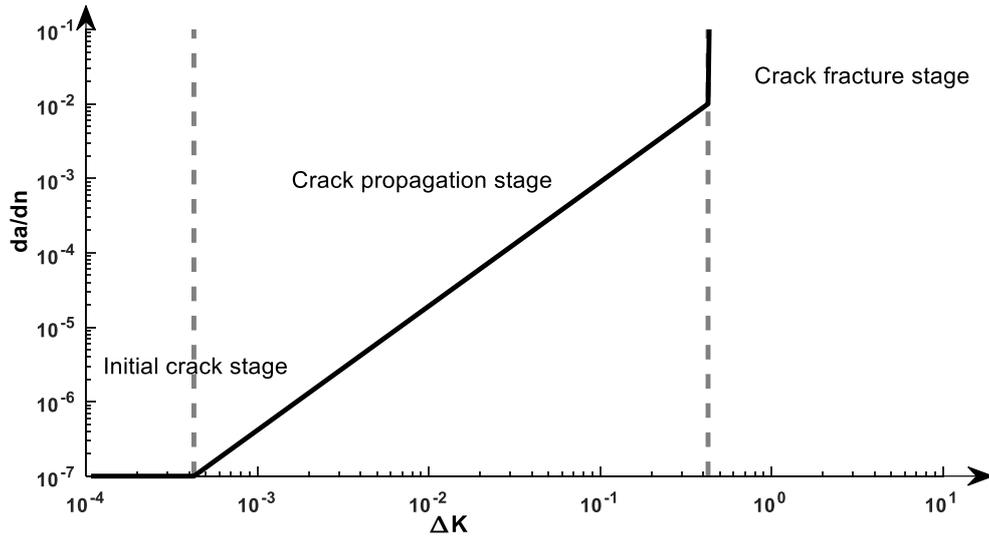


Figure 2: A schematic of the typical fatigue growth behaviour of cracks

According to Euler theory and fracture mechanics, the compliance  $C$  for SLB (Single Leg Bending) test defined as

$$C = \frac{\delta}{P} = \frac{2L^3 + 7a^3}{8EWh^3} + \frac{3(a + 2L)}{20GWh} \quad (3)$$

where  $E$  is Young's modulus and  $G$  is shear modulus;  $L$  is distance from the support beneath the cracked portion to the measurement point of longitudinal strain of SLB specimen;  $W$  is width of the surface of the SLB specimen, and  $h$  is distance between the neutral axis and bottom surface of the lower cracked portion of the SLB specimen, as shown in Figure 3 [7, 8].

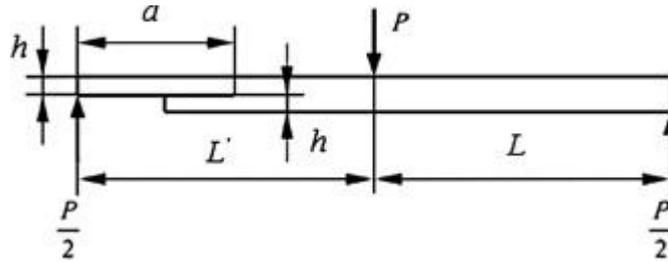


Figure 3: Mixed mode fatigue testing of SLB specimen

Thus, the initial compliance  $C_0$  for an initial crack size  $a_0$  can be written as

$$C_0 = \frac{2L^3 + 3a_0^3}{8EWh^3} + \frac{3(a_0 + 2L)}{20GWh} \quad (4)$$

and, the strain energy release rate of SLB beam test is defined as

$$G_{Mix} = \frac{P^2}{2W} \frac{\partial C}{\partial a} = \frac{21P^2 a^2}{16EW^2 h^3} + \frac{3P^2}{20GWh} \quad (5)$$

Based on the theory for energy release rate in fracture mechanics, the mixed mode strain energy release rate  $\Delta G_{Mix}$  is defined as

$$\Delta G_{Mix} = G_{Mix,max} - G_{Mix,min} = \frac{21(1-R)^2 P_{max}^2 a^2}{16EW^2 h^3} + \frac{3(1-R)^2 P_{max}^2}{20GWh} \quad (6)$$

in which

$$P_{max} = \frac{2P_a}{1-R} \quad (7)$$

where  $P_a$  is constant amplitude at a stress ratio of  $R$  and  $P_{max}$  is maximum constant amplitude.

Substituting Equation 6 into Equation 1 by integrating, the Paris law equation for mixed mode is

$$N = \frac{a(xa^2 + y)^m \times {}_2F_1\left(\frac{1}{2}, -m; \frac{3}{2}; -\frac{a^2x}{y}\right)}{C\left(\frac{a^2x}{y} + 1\right)^m} - \frac{a_0(xa_0^2 + y)^m \times {}_2F_1\left(\frac{1}{2}, -m; \frac{3}{2}; -\frac{a_0^2x}{y}\right)}{C\left(\frac{a_0^2x}{y} + 1\right)^m} \quad (8)$$

where  $x = \frac{21(1-R)^2 P_{max}^2}{16EW^2h^3}$ ,  $y = \frac{3(1-R)^2 P_{max}^2}{20GWh}$ , and  ${}_2F_1(a, b; c; z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n z^n}{(c)_n n!}$  is the hypergeometric series function with  $(a)_n = a(a+1)(a+2)\cdots(a+n-1)$ .

Therefore, the critical cycle number of fatigue leading to composite blades failure  $N_f$  can be represented by

$$N_f = \lim_{a \rightarrow \infty} \frac{a(xa^2 + y)^m \times {}_2F_1\left(\frac{1}{2}, -m; \frac{3}{2}; -\frac{a^2x}{y}\right)}{C\left(\frac{a^2x}{y} + 1\right)^m} - \frac{a_0(xa_0^2 + y)^m \times {}_2F_1\left(\frac{1}{2}, -m; \frac{3}{2}; -\frac{a_0^2x}{y}\right)}{C\left(\frac{a_0^2x}{y} + 1\right)^m} \quad (9)$$

Equation 8 is adapted to the fatigue model to describe the mixed mode fatigue delamination crack propagation between the laminates in composite blade versus cycle number of fatigue loads. It is clear that if  $N \rightarrow 0$ , then  $a \rightarrow a_0$ , and if  $N \rightarrow N_f$ , then  $a \rightarrow \infty$ .

### 3 FAILURE PROBABILITY

Gamma process is a stochastic process with an independent non-negative gamma distribution increment with identical scale parameter monotonically accumulating over time in one direction, which is suitable to model gradual damage such as wear, fatigue, corrosion, erosion [9]. The advantage of this stochastic process is that the required mathematical calculations are relatively straightforward and the results are trustful. Gamma process with uncertainties is a stochastic process and should be an effective approach for simulating the deterioration process.

The relationship between the mixed mode delamination crack length and the cycles of stress given in Equation 8 can be used for fatigue reliability analysis. Because of cyclic fatigue loading, a failure can occur even under the stress resistance of the materials. The evaluation of fatigue delamination crack growth is an uncertain process because of various conditions such as wind speed, wave loads and humidity. Thus, it can be considered as a time-dependent stochastic process  $\{X(t), t \geq 0\}$  where  $X(t)$  is a random quantity for all  $t \geq 0$ .

The gamma process is a continuous stochastic process  $\{X(t), t \geq 0\}$  with the following three properties: (1)  $X(t) = 0$  with probability one; (2)  $X(t)$  has independent increments; (3)  $X(t) - X(s) \sim Ga(v(t-s), u)$  for all  $t > s \geq 0$ . [4, 9]

The probability density function  $Ga(x|v, u)$  is given by

$$Ga(x|v, u) = \frac{u^v}{\Gamma(v)} x^{v-1} e^{-ux} I_{(0, \infty)}(x) \quad (10)$$

where  $v$  is shape parameter;  $u$  is scale parameter, and  $I_{(0, \infty)}(x)$  is defined as

$$I_{(0, \infty)}(x) = \begin{cases} 1 & \text{if } x \in (0, \infty) \\ 0 & \text{if } x \notin (0, \infty) \end{cases} \quad (11)$$

and the complete gamma function  $\Gamma(v)$  and incomplete gamma function  $\Gamma(v, x)$  are defined as

$$\Gamma(v) = \int_0^{\infty} x^{v-1} e^{-x} dx \text{ and } \Gamma(v, x) = \int_x^{\infty} x^{v-1} e^{-x} dx \quad (12)$$

where  $v \geq 0$  and  $x > 0$ .

According to the delamination crack length, the probability density function can be written as:

$$f(a) = Ga(a|v, u) = \frac{u^v}{\Gamma(v)} x^{v-1} e^{-ua} I_{(0, \infty)}(a) \quad (13)$$

According to the study by [10], the development of the fatigue crack length can be calculated by the Increment Sampling of Gamma (ISG) process. When the fatigue crack evolution of the blades is modelled as gamma process by ISG process, the increment of fatigue crack length at each jumping point is required. A path of the gamma process is sampled at these time points:  $t_0, t_1, t_2, t_3, \dots, t_{n-1}, t_n$ , which means  $n$  times per unit are the same and the increment by crack models in each time interval can be represented by

$$\Delta a_i = a(t_i) - a(t_{i-1}) \quad (14)$$

Considering the gamma process, each fatigue crack increment by ISG process  $\Delta a_{ii}$  at  $t_i$  can be represented as

$$\Delta a_{ii} \sim Ga(a|[v(t_i) - v(t_{i-1})]\Delta a_i^2/u, u/\Delta a_i) \quad (15)$$

Thus, the probability density function of  $\Delta a_{ii}$  is

$$f(\Delta a_{ii}) = \frac{(u/\Delta a_i)^{[v(t_i) - v(t_{i-1})]\Delta a_i^2/u}}{\Gamma([v(t_i) - v(t_{i-1})]\Delta a_i^2/u)} \Delta a_{ii}^{[v(t_i) - v(t_{i-1})]\Delta a_i^2/u - 1} e^{-(u/\Delta a_i)\Delta a_{ii}} \quad (16)$$

The fatigue crack length by ISG process can be accumulated by summing the sampled length in each time point, namely

$$a_{ii} = \sum_0^i \Delta a_{ii} \quad (17)$$

where  $a_{ii}$  is the crack length by ISG process at time  $i$ .

For the composite wind turbine blades, the fatigue failure is defined as experiencing  $N$  times loading at  $t_N$  time, where fatigue delamination length reaches the critical crack propagation stage. The critical delamination length depends on the construction design requirement, environmental conditions and the safety factor of the structures. The service life of the wind turbine blades can be predicted by accumulating the increased delamination in each time before reaching critical delamination length. From S-N curves, the bearing capacity of structures decreases when the number of loading increases. Also, the fatigue failure probability of the structure increases when the resistance for blades reduces. The maintenance for repairing, therefore, should be undertaken in time to prevent structural failure.

The equation for the failure probability can be calculated from [4, 5]

$$F(t) = Pr\{t \geq t_N\} = Pr\{a \geq a_{cr}\} = \int_{a_{cr}}^{\infty} f(a) da = \frac{\Gamma(v(t), u(a_{cr} - a_0))}{\Gamma(v(t))} \quad (18)$$

where  $v(t)$  is shape function and can be obtained from the expected delamination growth discussed in the previous section as

$$v(t) = u\Delta a(t) = u(a(t) - a_0) \quad (19)$$

where  $a_0$  is initial delamination length,  $a(t)$  is the delamination length at time  $t$ ;  $a_{cr}$  is critical delamination length, and  $u$  ( $u > 2$ ) is scale parameter for composite blades. The value  $u$  could be determined from statistical estimation methods such as a maximum likelihood method and method of moments [5].

The probability of failure per unit time at  $t_i$  is computed from

$$p_i = F(t_i) - F(t_{i-1}), \quad \text{for } i = 1, 2 \dots T \quad (20)$$

When the fatigue delamination crack length reaches the critical level, the probability of failure reaches unity, and the structure fails. The requirement for maintenance becomes critical to reduce the risk of structural failure and to prevent the possible unacceptable loss before reaching this stage. Thus, the service time of composite blades can be extended under proper maintenance.

## 4 NUMERICAL EXAMPLE

### 4.1 Fatigue delamination crack growth

A fibre reinforced composite blade is used to examine the effectiveness of proposed approach. The parameters for simulating fatigue crack growth and reliability analysis related to Paris law and gamma process are obtained from existing studies. According to the experiment study by [8], the length  $L$  is 180mm, the width  $W$  is 25mm, and the height  $h$  is 2.7mm. The initial crack length  $a_0$  is 45mm at the beginning of SLB test for mixed mode.  $C$  and  $m$  for mixed mode delamination at subcritical crack propagation stage in Paris law are 1 and 3, respectively [8]. Consistent with design code of offshore wind turbines, the typical service life is 25 years. The properties of the composite material are measured that  $E=150\text{GPa}$  and  $G_{13} = 4315\text{MPa}$  [7].

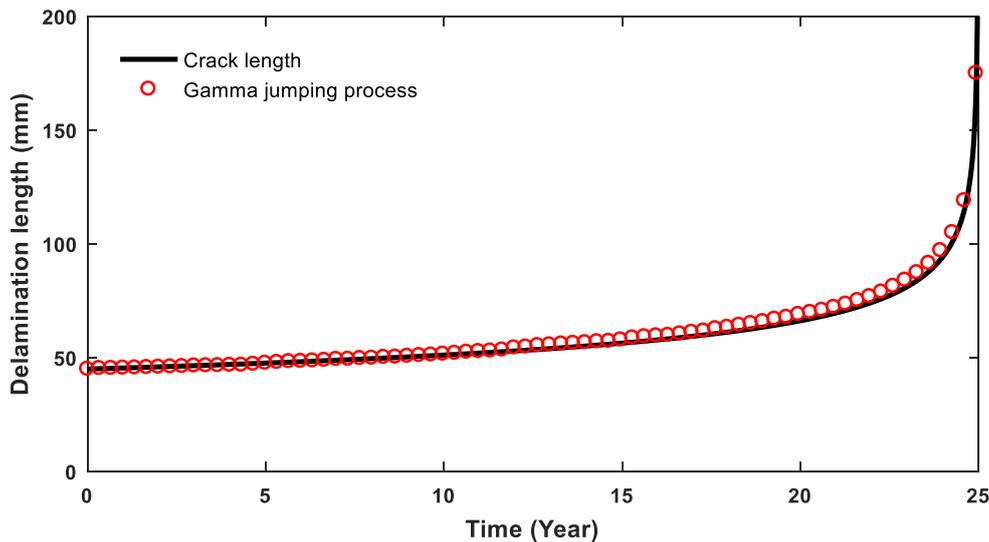


Figure 4: Fatigue crack growth of three types of composite specimens

The trend of delamination curves for this glass fibre reinforced materials is significant. Firstly, the crack lengths grow slowly and gradually at beginning. Secondly, as the service time increases, delamination length becomes unstable and develops quickly. When the delamination length reaches the threshold length, the crack becomes uncontrolled and the material failures in the final stage.

The propagation of fatigue delamination of composite blades simulated by gamma process is also plotted in Figure 4. The results show that the delamination cracks growth for mixed mode predicted by Paris law match well the ISG gamma process. Although slightly different values from gamma process exist at some points, the trend of the fatigue delamination development is almost the same compared with Paris curve. It is obvious that the delamination length development is accumulated by jump values for these minor periods as time increases. These random values between two close time points can reflect the uncertainties in the loading and environments affecting the crack process development.

### 4.2 Failure Probability for Composite Blades

It is assumed the delamination threshold is  $a_{cr}=100\text{mm}$  when the structure failures and the scale factor is taken as 6 in this example, which are used to analyse failure probability by the gamma process. Combined the propagation of delamination length with gamma process, the deterioration of the performance for wind turbine blades during service life can be modelled, and the results of the probability of failure curves are shown in Figure 5.

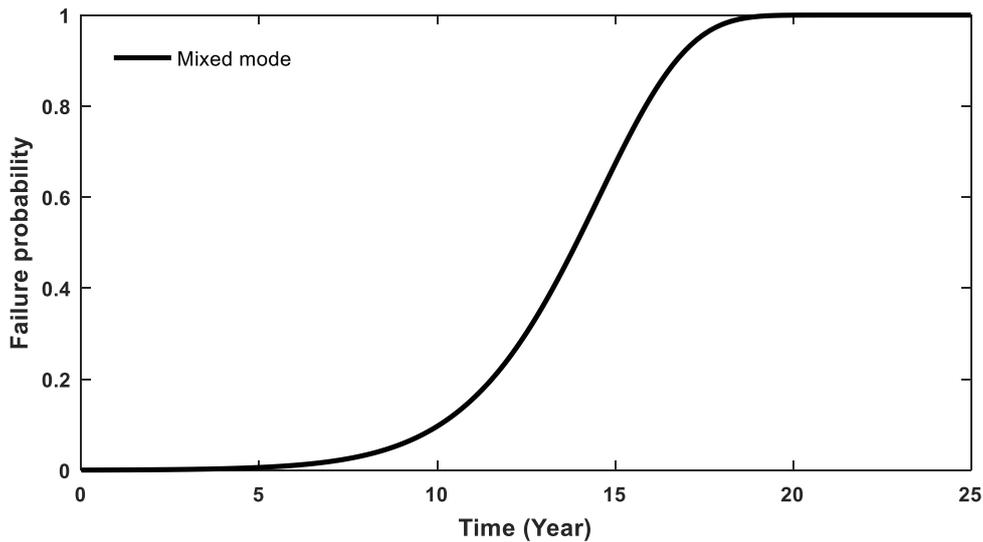


Figure 5: Failure probability for different of three types of composite specimens

At first, as the delamination length grows slowly, the probability of fatigue failure also increases slowly, which indicates structures are under normal use. With the increase in service time, the failure probability increases gradually until reaching certain time points between 14 and 21 years and then the curve has a rapid rise as the crack is reaching the critical length. The probability of failure increases dramatically over time and reaches approximately 50% at the time when the expected fatigue crack length is equal to the given acceptable limit. Finally, the failure probabilities reach to a value of very close to unity where the structure completely fails.

#### 4.3 Parameter Sensitivity Analysis

The fatigue delamination growth is modelled as the gamma process to analyse the reliability of composite wind turbine blades measured by the growth of the fatigue delamination length. The failure probability of the structure by gamma process is influenced by several parameters, i.e. initial delamination length, critical delamination length. The sensitivity of these parameters is discussed here to check the efficiency of proposed stochastic method.

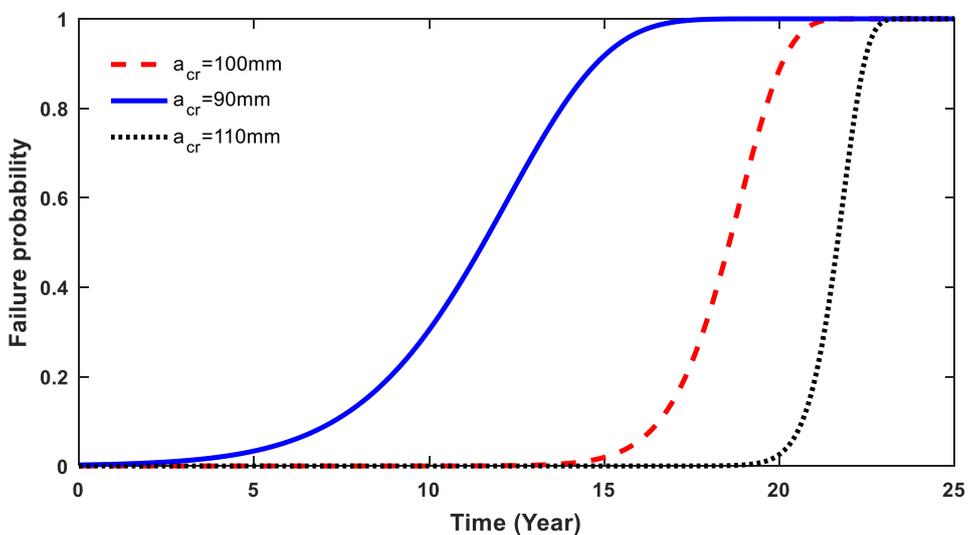


Figure 6: Failure probability for different critical crack lengths

The results of the lifetime failure probability are shown in Figure 6 for three different critical

delamination length limits, i.e.  $a_{cr}=100\text{mm}$ ,  $90\text{mm}$ ,  $110\text{mm}$ , respectively. The shapes of failure probability curves for different critical delamination length are similar. The probability of failure associated with the critical delamination length depends on the given acceptable limit, with a higher probability of failure for a lower acceptable level at any given time and vice versa. The time of failure probability reaching unity is close for these three predefined critical delamination lengths where the blade needs to be repaired or replaced.

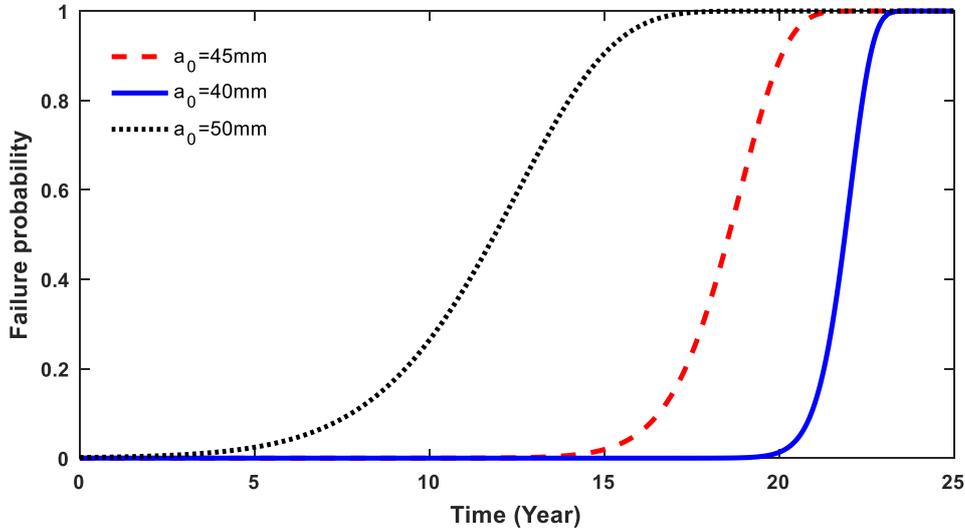


Figure 7: Failure probability for different initial crack lengths

The lifetime failure probability for various initial delamination lengths ( $a_0=45\text{mm}$ ,  $40\text{mm}$ ,  $50\text{mm}$ ) is shown in Figure 7. It is obvious that the probability of failure changes with different initial fatigue crack lengths, with a higher probability of failure for a higher initial crack length. For smaller initial crack length, the results show that the period before the sudden rise in the probability of failure becomes much longer.

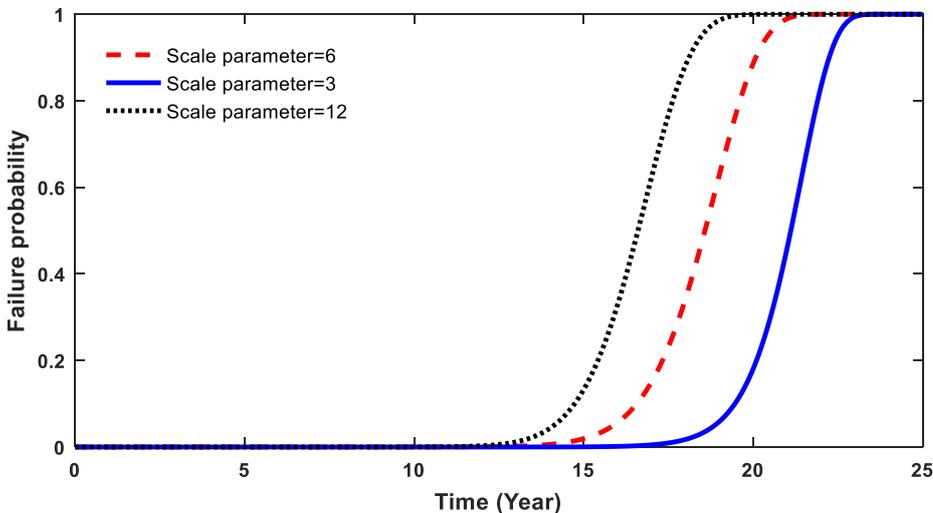


Figure 8: Failure probability for different scale parameters in gamma process

In gamma process, the scale parameter  $u$  has an influence on the fatigue property curve. The results are shown in Figure 8, and it is obvious that the smaller scale parameter makes the curve reaching the fatigue failure longer. The failure probability for larger scale parameters increases faster. However, compared with other factors such as initial crack length  $a_0$ , and critical crack length  $a_{cr}$ , the scale

parameters  $u$  has less influence on the failure probability and the failure time.

## 5 CONCLUSION AND FUTURE WORK

This paper uses a stochastic method to analyse the evolution of mixed mode fatigue delamination development for composite blades of wind turbines. A numerical case study is presented to investigate the effectiveness of this stochastic method. The failure probabilities of different fatigue cases are predicted by gamma process based on Paris law. The results show that gamma process gives reliable simulations and can be used for simulating on the mixed mode fatigue delamination propagation of composite blades. All of the parameters are analysed, and the following conclusions can be drawn, as listed below.

(1) The gamma process gives good simulations on the mixed mode fatigue delamination crack propagation of composite blades. The numerical results by the stochastic method are in good agreement with the available data because the gamma process considers uncertainties in the modelling of composite wind turbine blades.

(2) The proposed stochastic deterioration model based on the gamma process can be used for time-dependent reliability analysis. This method evaluates the lifetime probability of failure for the deteriorating blades and can be used to assist in the inspection and maintenance of the blades in advance. The optimum maintenance strategies according to the results from the predictive stochastic model can save costs for maintenance.

(3) The results from proposed gamma model show how the parameters affect the failure probability. It is helpful in evaluating the failure time of composite blades of wind turbines for inspection and maintenance in various situations.

Further work is needed to determine the optimum repair strategy for composite wind turbine blades.

## REFERENCES

- [1] J.F. Mandell, D.S. Cairns, D.D. Samborsky, R.B. Morehead and D.J. Haugen, Prediction of Delamination in Wind Turbine Blade Structural Details, *Journal of Solar Energy Engineering*, **125**, 2003, pp. 522-530 (doi: [10.1115/1.1624613](https://doi.org/10.1115/1.1624613)).
- [2] N. Blanco, E.K. Gamstedt, L.E. Asp and J. Costa, Mixed-mode delamination growth in carbon–fibre composite laminates under cyclic loading, *International journal of solids and structures*, **41**, 2004, pp. 4219-4235 (doi: [10.1016/j.ijsolstr.2004.02.040](https://doi.org/10.1016/j.ijsolstr.2004.02.040)).
- [3] N. Pugno, M. Ciavarella, P. Cornetti and A. Carpinteri, A generalized Paris' law for fatigue crack growth, *Journal of the Mechanics and Physics of Solids*, **54**, 2006, pp. 1333-1349 (doi: [10.1016/j.jmps.2006.01.007](https://doi.org/10.1016/j.jmps.2006.01.007)).
- [4] H.P. Chen and A.M. Alani, Optimized maintenance strategy for concrete structures affected by cracking due to reinforcement corrosion, *ACI Structural Journal*, **110**, 2013, pp. 229-238 ([www.concrete.org/publications/internationalconcreteabstractsportal.aspx?m=details&ID=51684403](http://www.concrete.org/publications/internationalconcreteabstractsportal.aspx?m=details&ID=51684403)).
- [5] J.M. van Noortwijk, A survey of the application of gamma processes in maintenance, *Reliability Engineering & System Safety*, **94**, 2009, pp. 2-21 (doi: [10.1016/j.ress.2007.03.019](https://doi.org/10.1016/j.ress.2007.03.019)).
- [6] M. Guida and F. Penta, A gamma process model for the analysis of fatigue crack growth data, *Engineering Fracture Mechanics*, **142**, 2015, pp. 21-49 (doi: [10.1016/j.engfracmech.2015.05.027](https://doi.org/10.1016/j.engfracmech.2015.05.027)).
- [7] M.V. Fernández, M.F.S.F. de Moura, L.F.M. da Silva and A.T. Marques, Mixed-mode I+ II fatigue/fracture characterization of composite bonded joints using the Single-Leg Bending test, *Composites Part A: Applied Science and Manufacturing*, **44**, 2015, pp. 63-69 (doi: [10.1016/j.compositesa.2012.08.009](https://doi.org/10.1016/j.compositesa.2012.08.009)).
- [8] M.F.S.F. de Moura and J.P.M. Gonçalves, Cohesive zone model for high-cycle fatigue of composite bonded joints under mixed-mode I+ II loading, *Engineering Fracture Mechanics*, **140**, 2015, pp. 31-42 (doi: [10.1016/j.engfracmech.2015.03.044](https://doi.org/10.1016/j.engfracmech.2015.03.044)).

- [9] H.P. Chen and A.M. Alani, Reliability and optimised maintenance for sea defences, *Proceedings of the ICE-Maritime Engineering*, **165**, 2012, pp. 51-64 (doi: [10.1680/maen.2010.37](https://doi.org/10.1680/maen.2010.37)).
- [10] A.N. Avramidis and P. L'Ecuyer, Efficient Monte Carlo and quasi-Monte Carlo option pricing under the variance gamma model, *Management Science*, **52**, 2006, pp. 1930-1944 (doi: [10.1287/mnsc.1060.0575](https://doi.org/10.1287/mnsc.1060.0575)).