

# A CAPACITY STUDY OF COMPOSITE SANDWICH CYLINDRICAL SHELL UNDER HYDROSTATIC PRESSURE, PART II: CALCULATION STUDY

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## ABSTRACT

After the experimental study stated in paper Part I, this paper analyzed the composite sandwich cylindrical shell structure capacity under hydrostatic pressure. By using finite element calculation, comparing the calculation result of the two shell models with the experiment results. The finite element modeling software Abaqus has been introduced in this study, and the experiment working conditions have been calculated by it. As a result, the HW-S01 Shell has been confirmed that it is damaged by buckling effect, and the failure load is 1.40 MPa by experiment, which is indicated by maximum buckling load in calculation. And the HW-055 Shell is failed because of local composite matrix compressive stress overload, the structure failed at a load of 11.2 MPa in experiment, it can be modeled using Hashin criterion method and get the same conclusion. By comparing and analysis, the calculation method used in this research is capable to simulate the working condition of composite sandwich cylindrical shell under hydrostatic pressure, and it could be used to predict the buckling failure load or stress failure load.

## 1. INTRODUCTION

From recent studies, sandwich composite material is considered has many applications as structure material in different areas. Composite sandwich cylindrical shell is as a new type of material structure used in the field of underwater pneumatic shell. This structure provides many engineering advantages in designing and building, like designable core material, lay-up degree and lay-up thickness. These advantages is able to offer unique designing plans to solve critical engineering problems. Some experiments has been done to determine the capacity of the shells. Based on that, relative calculation studies has been carried out, toward the hydrostatic pressure capacity of the designed composite sandwich cylindrical shell, using finite element modeling to compare and analyze results between experiments and calculations and justify the calculation method by it. Beside this, the paper aims to explore the material behaviors and failure modes of sandwich composite pneumatic shell under deep-sea hydrostatic pressure.

In recent studies, the composite sandwich cylindrical shell is a brand new idea, and has not been studied comprehensively, but there still are few papers discussed similar structures, such as Hwu's [1] researches. In his work, the composite sandwich cylinder structure has been analyzed using theoretical calculation under free vibration condition. Most relative researches are focused on composite cylindrical shell without sandwich structure [2-9], or composite sandwich structure without been formed to be cylindrical shell [10-13]. In composite cylindrical shell studies, the main points are

dynamic studies [2]; static buckling studies [3-6] and static failure studies [7-9]. In which, Lopatin's work is under hydrostatic pressure [4]. In his research, mathematics analysis such as Fourier decomposition and the Galerkin method had been carried out and the result was justified by comparing with finite element analyses. It suggest that, for simple composite cylindrical shell without sandwich structure, finite element modeling method is valid. The resent researches about sandwich structures are mainly focused on response of impact [10, 11] and vibration problem [12, 13], which are all dynamic items. So the hydrostatic pressure capacity research for composite sandwich cylindrical shell is innovative in both structure and method, and is valuable to be proceed.

## 2. CALCULATION METHOD AND RESULT

### 2.1 Model Size Parameters

In order to ensure the accuracy of calculation, the finite element model is build exactly the same as the experiment models in detail, so the model size parameters should refer to the experiment model size parameters, which are listed in the table 1 below. Beside those, the solid cores' young's modules are 21.27 MPa for HW-S01 Shell, and 970 MPa for HW-055 Shell. The winding angle is also copied from the experiment models, the angle on the ending of the shell is 0/90 degree, and the main part winding angle is 55/-55 degree.

parameters	sizes(mm)	parameters	sizes(mm)
Shell length	560	Full-composite ends	65/65
Solid core length	430	Solid core thickness	10
Outer diameter	318	Outer layer thickness	1.9
Inner diameter	290.4	Inner layer thickness	1.9
Shell thickness	13.8		

Table 1. Size parameters of the models

### 2.2 Modeling Method

The experiment results described and analyzed above, can be verified by finite element calculation. The finite element software "Abaqus" is introduced in this case. A model of sandwich composite shell with flange system has been built in the software. The model is made using continuum shell layers and solid element, and the dimension properties and material properties are defined exactly the same as the experiment models. Because the main difference between HW-S01 Shell and HW-055 Shell is the solid core material, which can be defined by only change the material property of that part. And the other difference between two shells is the seal closure used in HW-055 Shell experiment, which has no significant effect on the shell during the loading and failure process. So the calculation for the two shells could use the same finite element model. The model is showed in the figure 7. below.

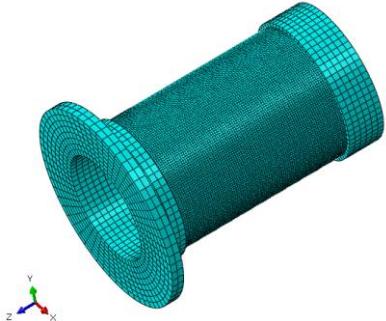


Figure 1. Finite element model with element mesh

In order to verify the failure process of the experiments, two calculation method has been used. The first one is buckling calculation, which aims to determine the maximum buckling load. And the second method is statics calculation, which able to indicate material damage by using different criterions. To determine the failure of composite material, Hashin criterion is introduced in this calculation. There four indicators in Hashin criterion statics calculation, which are Hashion`s fiber tensile damage initiation criterion (Hashin FT); Hashin`s fiber compressive damage initiation criterion (Hashin FC); Hashin`s matrix tensile damage initiation criterion (Hashin MT) and Hashin`s matrix compressive damage initiation criterion (Hashin MC). These four criterions respectively indicate four types of failure mode for composite material. When the one of the calculated indicator maximum numbers is larger than 1, it indicate a material failure of certain kind.

$$\begin{array}{lll}
 \frac{\sigma_x}{x^t} = 1 & \sigma_x \text{ is tensile} & (1) \\
 \frac{\sigma_x}{x^c} = 1 & \sigma_x \text{ is compressive} & (2) \\
 \frac{\sigma_y^2}{(Y^t)^2} + \frac{\tau_{xy}^2}{S^2} = 1 & \sigma_y \text{ is tensile} & (3) \\
 \frac{\sigma_y^2}{(Y^c)^2} + \frac{\tau_{xy}^2}{S^2} = 1 & \sigma_y \text{ is compressive} & (4)
 \end{array}
 \left. \begin{array}{l}
 \left. \begin{array}{l}
 \sigma_x \text{ is tensile} \\
 \sigma_x \text{ is compressive}
 \end{array} \right\} (1), (2) \\
 \left. \begin{array}{l}
 \sigma_y \text{ is tensile} \\
 \sigma_y \text{ is compressive}
 \end{array} \right\} (3), (4)
 \end{array} \right\} \text{fiber failure}$$

Hashin criterion

There for, the finite element calculation is not perfectly accrete, there might be some difference between calculation results and experiment results, so it is defined that, if in the same working condition the calculation result is within 10% larger or smaller than the experiment result, it can be considered that the calculation result is similar to the experiment result. Through the two calculation method mentioned, the shell failure process can be confirmed by the fallowing process. First, calculate the maximum buckling loads of the shells, and then calculate the Hashin criterions of the shells at the experiment failure load. If the calculated maximum buckling load is similar to the experiment failure load, and the maximum Hashin criterions at the experiment failure load are all smaller than 1, it can be confirmed that, the shell is damaged because of buckling effect. If the calculated maximum buckling load is significantly larger than the experiment failure load, and the largest Hashin criterion number is around 1 at the experiment failure load, it means that, the shell is failed start with stress damage.

### 2.3. HW-S01 Shell Calculation Results

Based on the experiment result, it can be assumed that, the failure mode for this shell is overall bulking failure, then carry out the calculations mentioned above, to confirm it. The calculated

maximum buckling load is 1.4015 MPa 4.59% larger than the experiment failure load 1.34 MPa. The buckling mode is showed in the figure 8 below. Comparing the calculation result and experiment result, the two results are similar to each other, so the calculation method is trustable. Then get the calculation result of Hashin criterions at the load of 1.34 MPa, as the numbers shows in the table 2. below. The indicators are all smaller than 1, and consider the buckling calculation result, so it is sure that the HW-S01 Shell is damaged because of buckling effect. And the buckling mode shows in figure 8 present a overall buckling mode, so the failure process should be as: When the hydrostatic pressure reached 1.34 MPa, the overall buckling start to effecting on the shell and create local defect, which lead to fiber break and water leak and finally to the failure phenomenon observed during the experiment.

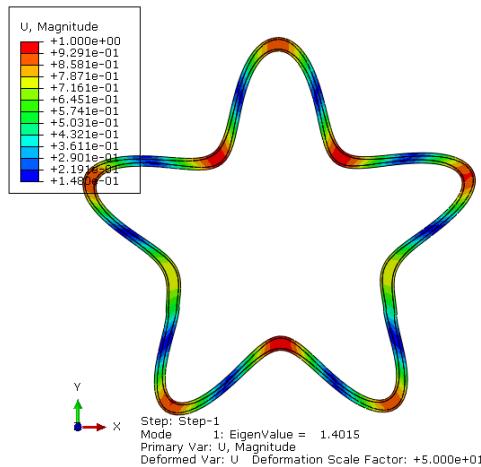


Figure 2. Buckling calculation result and buckling mode of HW-S01 Shell

Indicator	Result	Indicator	Result
Hashin FT	$1.230 \times 10^{-4}$	Hashin MT	$7.148 \times 10^{-3}$
Hashin FC	$7.197 \times 10^{-3}$	Hashin MC	$5.076 \times 10^{-3}$

Table 2. Hashin criterions for HW-S01 Shell at 1.34 MPa

## 2.4 HW-055 Shell Calculation Results

The HW-055 Shell experiment result indicate a stress failure process, so proceed the calculations, then compare and analyze the results. The maximum buckling load is 17.349 MPa by calculation, which is significantly larger than the experiment failure load, 11.2 MPa, the buckling mode is showed in figure 9 below. So it can be confirmed that, the failure appeared in the experiment is not caused by buckling effect. Then calculate Hashin criterions at 11.2 MPa to check if the stress calculation method is suitable, and if the experiment failure is stress failure. The indicate numbers are listed in the table 3. It appears that in the calculation mode, when load reached 11.2 MPa, the model is already damaged. So check the loading pressure when the largest Hashin criterion number, , is just equal to 1. As a result, when the load reached 10.6 MPa, Hashin MC number is 1. So 10.6 MPa is the calculated failure load, which is 5.4% smaller than the experiment failure load, 11.2 MPa. Therefore, the stress calculation method is suitable, which can correctly simulate the stress failure in the experiment.

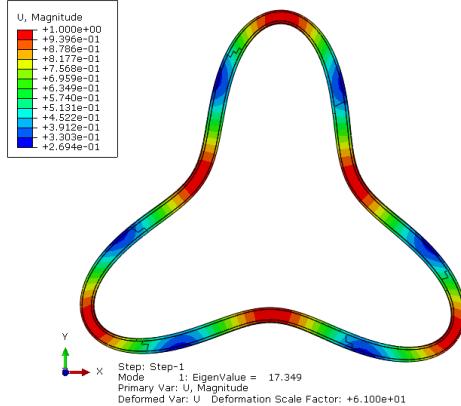


Figure 3. Buckling calculation result and buckling mode of HW-055 Shell

Indicator	Result	Indicator	Result
Hashin FT	0.07104	Hashin MT	0.2792
Hashin FC	0.3241	Hashin MC	1.191

Table 3. Hashin criterions for HW-055 Shell at 11.2 MPa

Indicator	Result	Indicator	Result
Hashin FT	0.06132	Hashin MT	0.241
Hashin FC	0.2809	Hashin MC	1

Table 4. Hashin criterions for HW-S01 Shell at 10.6 MPa

After verify the correctness of calculation method, it is able to use the stress calculation result to analyze the failure process happened in the experiment. The figure 10 shows a local cloud picture of Hashin MC number, is the one indicator reached 1, which present the failure start area. The cloud picture content, half of the shell, which shows a profile surface of the shell, and one side of the flange which connect to the shell. The large white structure on the left of the figure present part of the flange, and the white area on the right shows the solid core of sandwich composite shell, and the part between flange and sandwich shell with core is the part of full composite shell. To the shell, the Hashin MC number reaches 1 means the shell is damaged due to the matrix of composite material is overloaded by the compressive stress. Observe the colour regions on the cloud picture in Figure 10, the region which content the indicator over 1 is just at the interface between sandwich composite shell area and full-composite area. So it can be considered that, the failure is because of the local strain at the interface between sandwich composite shell area and full-composite area. The local strain number lead to composite matrix failure and then caused fiber break and after that the shell lose the capacity to hydrostatic pressure.

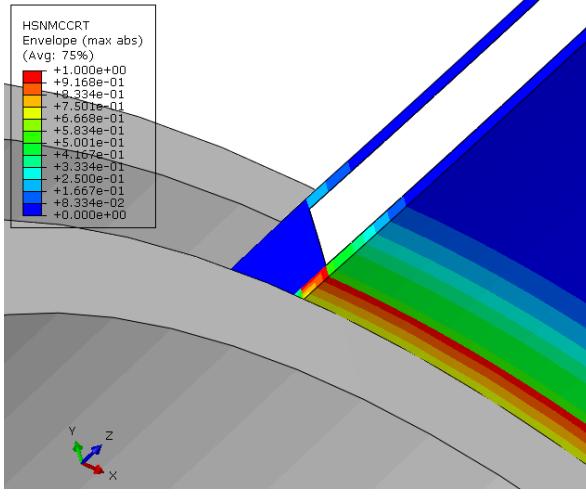


Figure 4. The failure caused part on HW-055 Shell based on Hashin MC number.

### 3. STRAIN NUMERICAL ANALYSIS IN EXPERIMENT AND CALCULATION

During the two hydrostatic experiment, strain data from certain measure points on the composite material surface is collected and processed. And through Abaqus software, the strain data from these measure points can be calculated at the same load. So by comparing the strain data can be used to check the correctness of calculation method. Because the limit of the software, the strain data output is logarithmic strain but not strain. The relationship between strain and strain is: Strain is  $E = \frac{\Delta l}{l_0}$  ; Logarithmic strain is  $LE = \int_{l_0}^{l_1} \frac{dl}{l} = \ln \frac{l_1}{l_0} = \ln \left(1 + \frac{\Delta l}{l_0}\right) = \ln(1 + E)$ . Based on the Taylor expansion:

$$f(x) = \ln(x + 1); f'(x) = \frac{1}{x+1}; f''(x) = -\frac{1}{(x+1)^2}; \dots \quad (5)$$

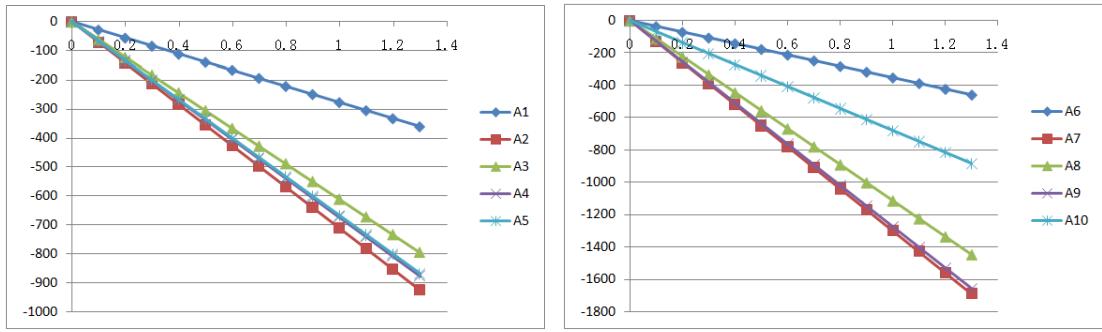
$$LE = f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots + \frac{f^{(n)}(0)}{n!}x^n + \dots = x - \frac{1}{2}x^2 + \dots + (-1)^{n-1} \frac{x^n}{n} + \dots \quad (6)$$

In the formula above,  $x=E$ , so when  $|E| \ll 1$ , then  $LE \approx x$ . In this study, the strain number is at an order of magnitude of  $10^{-2}$  to  $10^{-3}$ , so it can be defined that,  $LE \approx E$  in this study.

In the analysis of this section, minus number in strain data means compressive strain, and positive number present tensile strain. In this case, because the shell is under a hydrostatic compressive pressure, so for most part of the shell, the strain number are minus. In order to simplify the math, call the numbers closer to 0 is smaller than the numbers further to 0.

#### 3.1. Numerical analysis for HW-S01 Shell

For the HW-S01 Shell experiment, the strain data is only collected from the inner composite surface. The experiment strain data of HW-S01 Shell is plotted in the figure 11 below, it is clear that the data are all perfectly linear distributed. All the data are collected from the inner composite layer surface. It presents the elastic deformation part of the shell during the loading procedure. Therefore the strain data are all collected validly, which can be used to compare with calculation result to justify the calculation method and analyze the deformation pattern of the shell on the radial direction.



(a) radial strain;

(b) circumferential strain

Figure 5. HW-S01 Shell experiment strain data

Collect the strain data of calculation result at the corresponding positions on the finite element model as the experiment model. Compare the strain data at a load of 1.3 MPa between experiment data and calculation data as the line chart shows below. Due to the error in production procedure and deviation of material characteristics, the experiment data and calculation data showed in figure 12 clearly has differences. However, by observing the trend lines obviously has the same pattern. For both experiment data and calculation data in radial strain and circumferential strain, the strain on the ends of the shell (A1; A5; A6; A10) the compressive strain numbers are smaller than the data collected on the middle part of the shell, which is because of boundary effect caused by the flanges. And for the middle part of the shell, the data on the quarter positions (A2; A4; A7; A9) are larger than the ones in the middle (A3 A8), it might because of in the middle of the shell, the strain data content both minus number compressive strain and positive number buckling strain, as a superposition the total minus strain numbers are smaller than the ones without buckling strain on the quarter positions. Based on the analysis above, it can be said that, the calculation method is correct, which can simulate the failure mode and strain data trend. Compare the calculated radial strain and circumferential strain at the 3 measure points in the middle of the shell. The strain between A2 and A7; A3 and A8; A4 and A9, the numbers are similar to each other, which correctly response to the equal strength designing and 55/-55 degree winding angle, so it also proved the calculation method is correct.

	A1	A2	A3	A4	A5
EXP	-361.925	-925.874	-796.178	-877.246	-868.472
CAL	-346.4	-848.9	-537.4	-810.8	-438.6
	A6	A7	A8	A9	A10
EXP	-460.383	-1688.23	-1448.48	-1656.95	-884.783
CAL	-194.3	-682.9	-677.2	-744	-341

Table 5. HW-S01 Shell experiment and calculation strain data at 1.3 MPa

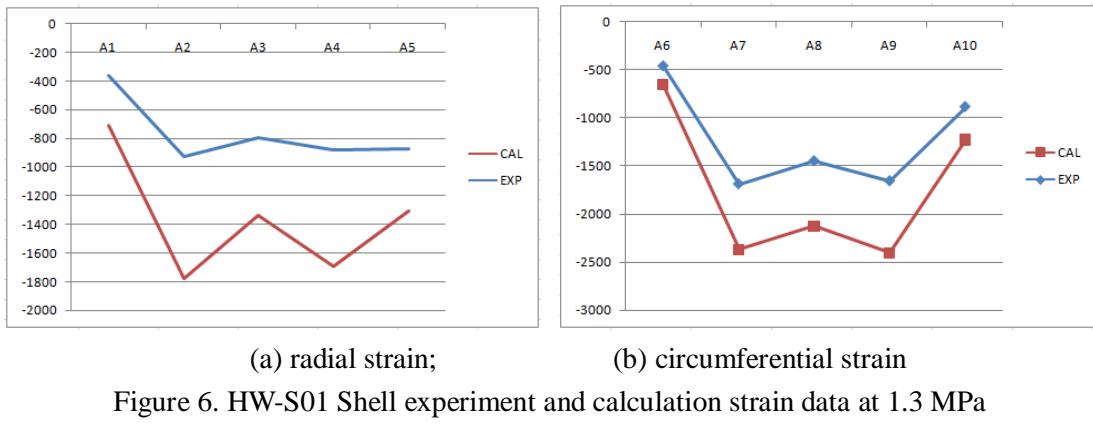


Figure 6. HW-S01 Shell experiment and calculation strain data at 1.3 MPa

### 3.2. Numerical analysis for HW-055 Shell

Same as the analysis above, the strain data comparing can also be done to test the correctness of calculation method for HW-055 Shell. During the HW-055 Shell experiment, the strain data is collected from both inner and outer composite surface. The data collected form outer surface are numbered as A1 to A10; and the data from inner surface are numbered as A'1 to A'10. However, because the outer composite surface is sink in the water during the experiment at a high water pressure, water invaded into the waterproof layer which covered on the strain gauges and effected the validation of strain gauges at some of the measure points on the outer composite surface. So part of the data from outer surface are not useable. The valid strain data are plot in the figure 13 below. Apparently, the data linearity are all good, which can present the mechanical property of the shell during the elastic deformation part.

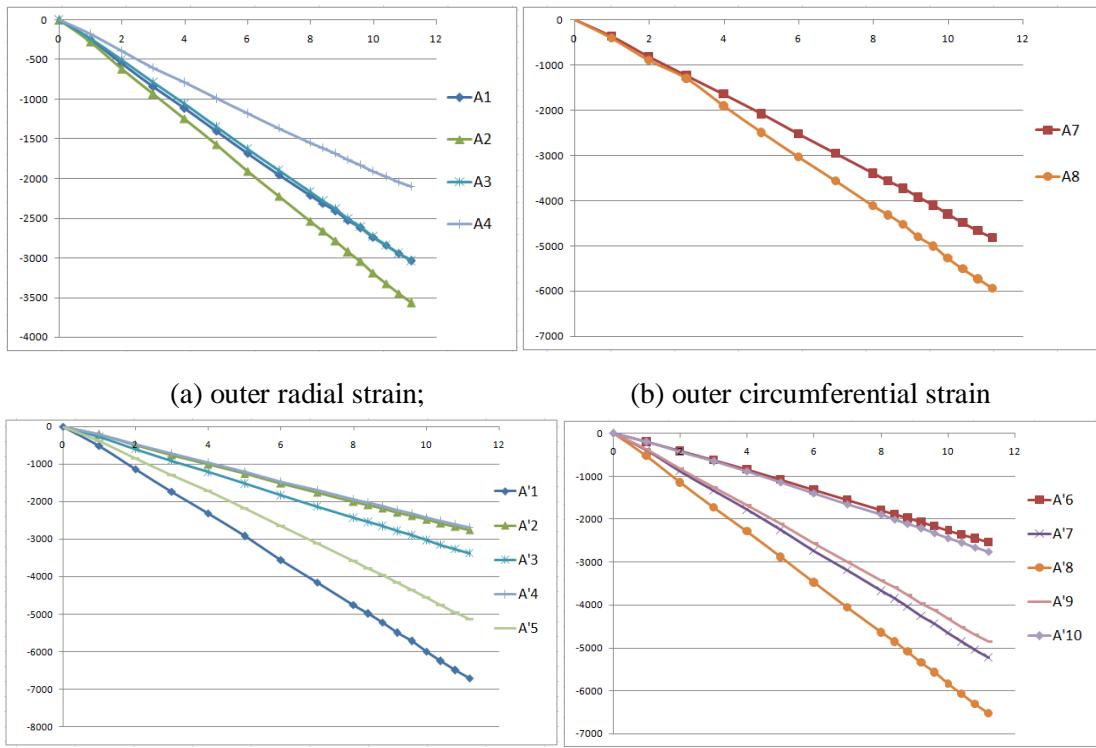


Figure 7. HW-055 Shell experiment strain data

Compare the strain data at the failure load from both experiment and calculation result. Because there is an offset between experiment and calculation results, the experiment failure load is 11.2 MPa and the calculation failure load is 10.6 MPa. On the other hand, the failure load indicate the same mechanical condition for the shell. So in order to compare the strain data of the shell at the same mechanical condition, should not compare the experiment and calculation strain data at the same load, but should compare them both at the failure loads. The strain data from experiment and calculation has been listed in the table 6 below. And the data has been processed and plotted in figure 14. For the outer surface strain data, a large part of the experiment data are not valid, so it cannot be used to analysis the strain pattern on the shell. However, in data A7, A8, apparently the number of calculation strain and experiment strain are similar to each other. And the data at 5 measure points on the outer surface has the same distribute pattern as the 5 measure points on HW-S01 Shell. Then, for the inner surface, the experiment data quality is good, and all the data group can be used in the analysis. Numerically, the radial strain data from A'1 to A'5, the calculation data and experiment data are similar to each other, which can indicate the correctness of calculation method. And for the circumferential strain from A'6 to A'10, the two data groups has the same trend, but the number difference is large due to the error in production procedure and deviation of material characteristics. In addition, the number of the radial strain on the inner surface (shows in figure 14. (c)) has a different trend to other data group. For the other data groups, the strain on the ends of the shell (A1; A5; A6; A10; A'6; A'10) are significantly smaller than other the data from other measure points, it is because of the boundary effect caused by the flanges. However, the data from A'1 and A'5 are significantly larger than other data in the inner surface radial strain data group. One possible reason might be the matrix compressive failure mode. Based on the Hashin failure theory, the failure mode indicate high local compressive stress and high local compressive strain at the place where Hashin criterion reach 1 first, which are the places where A'1 and A'5 located. So the high compressive strain number at A'1 and A'5 justified the Hashin failure theory used in this calculation. Same as the result in HW-S01 Shell numerical analysis, the strain number in the table 6 below also, indicate the equal strength designing and 55/-55 degree winding method is effective. So the calculation method is correct and valid to be used in sandwich composite cylindrical shell finite element calculation.

	A1	A2	A3	A4	A5
CAL	-4191	-5102	-4974	-5118	-3607
EXP	-3038	-3568	-3041	-2099	--
	A6	A7	A8	A9	A10
CAL	-2102	-5028	-4894	-5023	-1716
EXP	--	-4825	-5946	--	--
	A'1	A'2	A'3	A'4	A'5
CAL	-9545	-4965	-5164	-4960	-10200
EXP	-6714	-2754	-3373	-2689	-5130
	A'6	A'7	A'8	A'9	A'10
CAL	-3026	-5233	-5250	-5226	-3145
EXP	-2533	-5223	-6528	-4851	-2751

(the "--" in the table means data is not valid)

Table 6. HW-055 Shell experiment and calculation strain data at failure loads

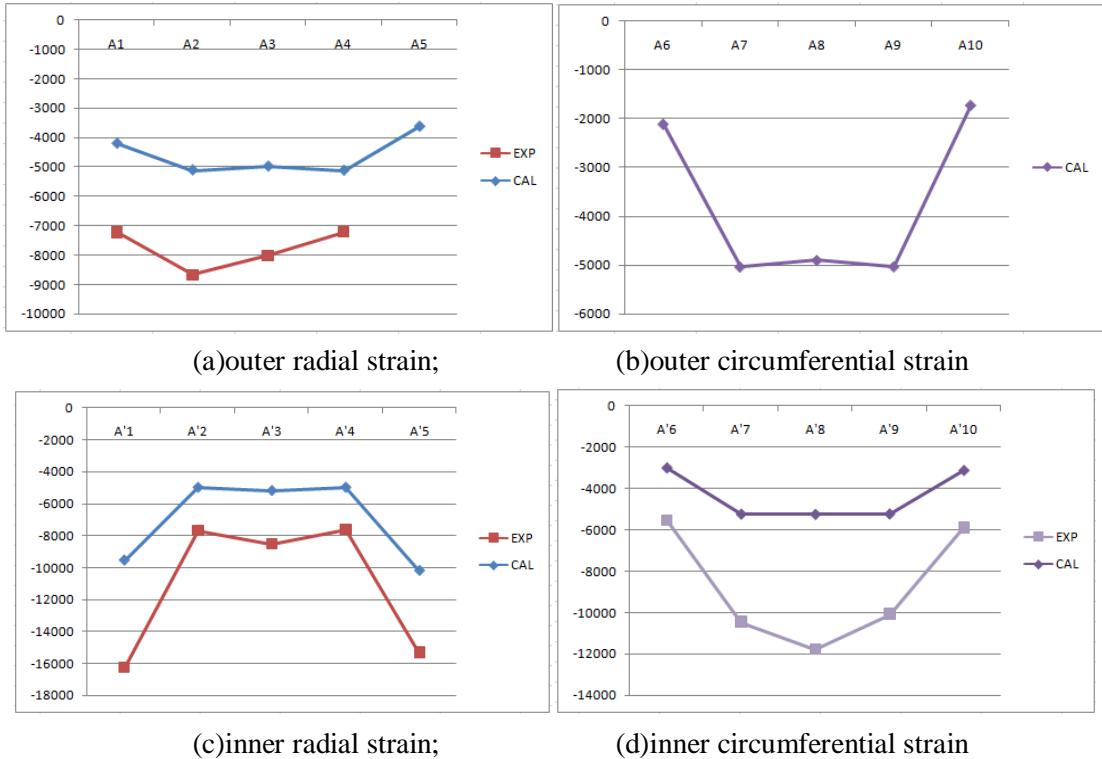


Figure 8. HW-055 Shell experiment and calculation strain data at failure load

#### 4. CONCLUSION

From the experiment and calculation studies analyzed above, it can be conclude that, the finite element modeling method used in this research is capable to simulate the working condition of composite sandwich cylindrical shell under hydrostatic pressure, and it could be used to predict the buckling failure load or stress failure load. Same to the conclusion got from experiment study, compare the maximum buckling load of HW-S01 Shell and HW-055 Shell, by the calculation, it can be confirmed that the young's modules of the solid core has significant influence to the structure stability of composite sandwich cylindrical shells. If other size and material parameters are the same, when the young's modules of the core is 21.27 MPa, the maximum buckling load of the shell 1.40 MPa by calculation; and when the core's young's modules increase to 970MPa, the maximum buckling load reaches 17.35 MPa.

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