FREQUENCY EFFECT ON LIFETIME AND TEMPERATURE OF A TAILORED BLANK C/PPS THERMOPLASTIC COMPOSITE

Roman Ruzek[1], Martin Kadlec[2], and Lucie Nováková[3]

[1] VZLU - Aerospace Research and Test Establishment, Strength of Structures Department, Beranových 130, 199 05 Prague, Czech Republic, ruzek@vzlu.cz, www.vzlu.cz
[2] VZLU - Aerospace Research and Test Establishment, Strength of Structures Department, Beranových 130, 199 05 Prague, Czech Republic, kadlec@vzlu.cz, www.vzlu.cz
[3] VZLU - Aerospace Research and Test Establishment, Strength of Structures Department, Beranových 130, 199 05 Prague, Czech Republic, l.novakova@vzlu.cz, www.vzlu.cz

Keywords: Thermoplastic resin, Tailored blanks, Fatigue, Frequency, Temperature, Mechanical properties.

ABSTRACT

Applications of advanced thermoplastic matrix composites facilitate the use of modern manufacturing technologies such as hot forming and tailored blanks. This study investigates the influence of the loading frequency on the temperature, fatigue behaviour, and failure mechanisms of carbon fibre fabric–reinforced polyphenylenesulphide (PPS) laminates whose thickness is varied using ply drops. The effects of two ply drop configurations and fatigue loading frequencies from 0.5 to 15 Hz were considered. Fractographic examination revealed loading frequency based fracture surface features. The loading frequency had a significant influence on the fatigue lifetime of specimens with thickness variation. The local temperature increased significantly while the fatigue life decreased as the loading frequency was increased. In particular, the fatigue life decreased by more than one order of magnitude. An analytic relation between load rate and local temperature was defined. At low loading frequency, the matrix exhibited small plastic deformations and textured microflow was apparent. At high loading frequency, larger matrix features, such as rollers and debris, were found. Fibrils caused by large matrix deformation may be attributed to the higher temperature at higher frequencies.

1 INTRODUCTION

Thermoplastic matrix composites (TPMCs), which are used in airframes, have become increasingly popular in the aerospace industry owing to their superior impact damage resistance [1] or formability [2] compared to conventional epoxy-based laminates. Moreover, the possibility of recycling TPMCs [3] is a significant advantage in terms of environmental conservation. Previously, the large-scale use of TPMCs was hindered by their complex manufacturing process and high cost. These obstacles are less severe today, as larger batches of material are ordered and modern manufacturing technologies can be applied. TPMCs are used as constant-thickness materials in the construction of primary airframe components. A wide range of thermoplastic resins is available. Polyetheretherketone (PEEK) and polyphenylene sulphide (PPS) are among the most widely used high-performance TP resins [4]. This paper focuses on the mechanical behaviour of carbon/PPS (C/PPS) composites.

Fatigue loading is one of the main types of loading in structural elements, which can lead to catastrophic failure in certain situations. The fatigue response of C/PPS thermoplastic materials, especially those with varying thickness and ply drop, has not been investigated in sufficient detail thus far. Extensive experimental studies are required in this regard. Design against fatigue failure is complex because fatigue life can be influenced by a number of variables, such as maximum and minimum stress levels, temperature, loading frequency, component size and geometry, and environment. Continuous fibre–reinforced composites are characterized by the development and accumulation of several types of defects, including matrix cracking of the fibres, delamination between adjacent plies, debonding between matrix and fibres, and fracture of fibres. Such defects in composite systems are not isolated but interconnected. Thus, the identification of crack paths is a highly complex task. Moreover, most of these defects occur long before ultimate failure; hence, there can be many types of subcritical failure [5, 6-8].

In real applications, it is necessary to reduce the thickness of less loaded parts of the structure. Integral structures are loaded based on the outer loads, centre of mass, and neutral axis position. Thickness optimization is an ideal approach for decreasing the weight of constructions. The insertion of semipregs at suitable structure locations can facilitate a more uniform stress distribution as well as thickness variation. On the other hand, such a layout may result in an undesirable temperature increase that would affect the structure during fatigue loading. Thickness variation by ply drops can be classified as external, internal (longitudinal or transverse), and mid-plane ply drop constructions [9]. These constructions are either symmetric or asymmetric. Their applications include helicopter yokes, composite aircraft-wing skins, helicopter flexbeams, and flywheels. This technique is also useful for making airframe parts such as skins, ribs, stiffeners, and beams.

However, problems may arise from the use of ply drops (inserted ply ends), where the stress concentration can induce damage [9, 10]. To investigate these problems, numerical models based on global-local approaches have been developed using Timoschnecko beam elements [11] or cohesive zone elements with Hashin and LaRC failure theories [12]. Nevertheless, to gain a deeper understanding of the fatigue behaviour of composites with varying thickness, an extensive experimental program is required for the certification process of airframes structures, parts, and materials. Time and cost are the main considerations in material and structure certification processes.

This paper describes an experimental investigation of an asymmetrically tapered laminate with internal ply drops loaded under tension-tension fatigue conditions. The objective of this study is to obtain data for the specific design and fatigue strength of a rib demonstrator and to define the load rate limitation that must be considered during the certification process of experimental materials and structure parts.

2 MATERIAL AND METHODS

2.1 Material configuration and parameters

The stacking and thickness variation correspond to a real structure design, i.e., a rib used in a large aerospace structure [13, 14]. The laminate thickness varied from 3.4 to 5 mm. The plates were manufactured from carbon fibre fabric prepreg according to AIMS 05-09-002 [15] with a real weight of 285±15 g/m^2. The fabric had a 5-harness satin weave, which is more flexible for a curved surface than a plain weave. The matrix was PPS with weight ratio of 43%. The nominal thickness of the lamina was 0.31 mm.

The two ply drop configurations were investigated by considering the thickness transition areas of the real structure part design. Alternative A had two transitions from 16 to 13 and from 13 to 11 layers (Fig. 1). The layup can be coded as follows for A: (0/90, ±45)∫ (0/90)| ±45| (0/90, ±45, 0/90)∫ ±45| (±45, 0/90, ±45, 0/90)∫ ±45| (0/90)| (±45, 0/90)∫, where ∫ denotes continuous plies and | denotes ply drops (cut lamina ends). The tapered angle for Alternative A was 3.5°. The actual thicknesses of the sections were 5.10 mm for 16 layers, 4.15 mm for 13 layers, and 3.55 mm for 11 layers. Alternative B had one transition from 16 to 11 layers (Fig. 2). The layup can be coded as follows for B: (0/90, ±45)∫ (0/90, ±45)| (0/90, ±45, 0/90)∫ ±45| (±45, 0/90, ±45, 0/90)∫ (±45, 0/90)| (±45, 0/90)∫. The tapered angle for Alternative B was 6°. The actual thicknesses of the sections were 5.05 mm for 16 layers and 3.53 mm for 11 layers.

The plates had these transitions on each side; thus, there were 16 layers on both the sides and 11 layers in the middle. The internal ply drops were not exactly positioned in columns as was intended, owing to movement of the layers during the layup and curing phase. Deviations of up to several millimetres were observed in the cross sections of the related ply drop positions in the longitudinal direction.

Four plates with dimensions of 400 × 400 mm were manufactured in two batches using hot forming. Two plates were configured as Alternative A (specimens 2-X and 3-X) and two plates were configured as Alternative B (specimens 1-X and 4-X). Twelve specimens were extracted from each plate using a water jet cutter. Further, specimens 3-X and 4-X underwent additional mechanical machining (milling) of the edges for better surface quality and were thus expected to have better fatigue characteristics. The specimens were 370 mm long and 25 mm wide.

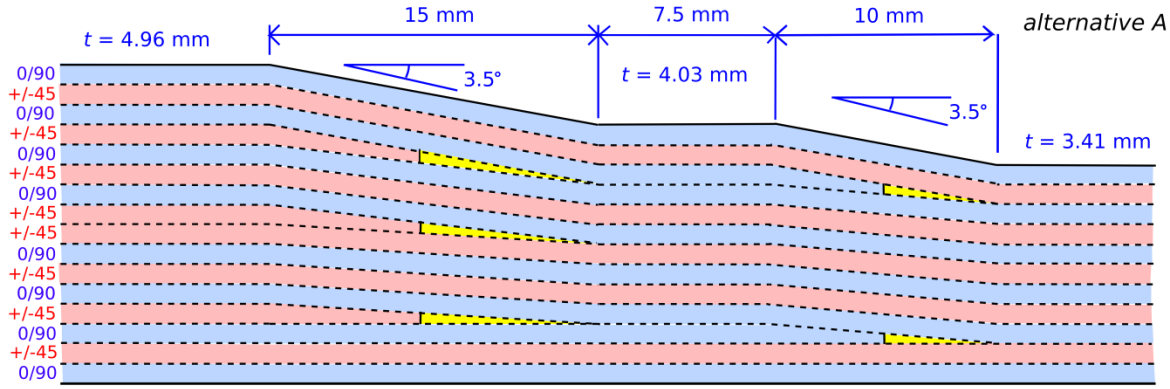

Figure 1: Layup schematic of the tapered section of Alternative A with nominal dimensions.

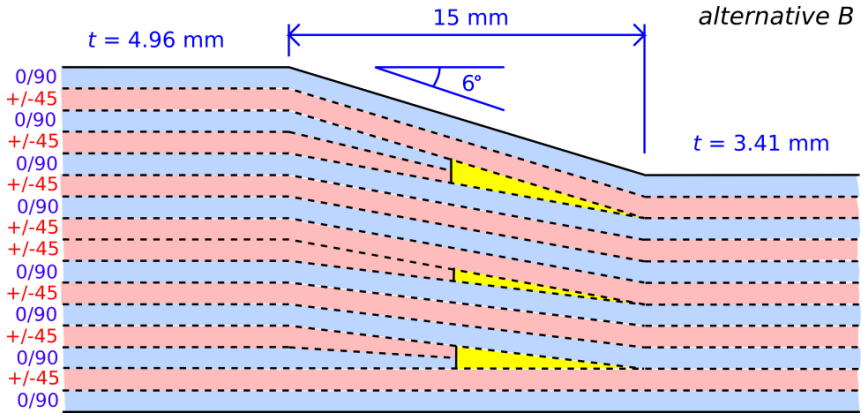

Figure 2: Layup schematic of the tapered section of Alternative B with nominal dimensions.

2.2 Test Procedures

Fatigue testing was performed according to ASTM D3479M [16] on IST Hydropuls Sinus 100 kN and Schenck 250 kN at frequencies in the range of 0.5 and 15 Hz. The tension-tension fatigue test with a stress ratio of 0.05 was selected because it is considered to be the most appropriate test for the investigated structure part. Moreover, the tension-tension fatigue test represents the most uniform and severe form of fatigue loading. The modulus of elasticity was measured regularly for preselected numbers of elapsed cycles.

During fatigue loading, the surface temperature of specimens was measured using an FLIR E60 infrared camera (Fig. 3) with a sensitivity of <0.07°C and precision of 2°C. The temperature was either measured in predefined intervals or continuously recorded into the memory card. The measurements were adjusted such that the warmest point on the specimen surface was recorded. Selection of the measured specimens was performed to obtain data under different loading conditions in the entire range of used frequencies (f = 0.5–15 Hz).

Figure 3: Fatigue test set-up with focused infrared camera.

2.3 Methods of morphological analysis

For the edgewise ply drop and delamination description, the selected specimens were cut using a precise linear metallographic saw (IsoMet 4000) in the longitudinal direction in order to observe the centre plane part of a specimen along its length. A diamond wafering blade designed for composite materials was used; it was cooled by cutting fluid. A common method for preparing (manual grinding and polishing) metallographic samples was applied using the KOMPAKT 1031 grinder-polisher. The samples were observed using a metallographic microscope (Olympus GX-51) and photographs were subsequently taken using a digital camera (ARTRAY ARTCAM-300MI) with QuickPHOTO Industrial software.

3 RESULTS

3.1 Strength and fatigue life

The average static strength of Alternative A was 533 MPa, whereas that of Alternative B was 551 MPa (3 % higher than that of Alternative A). However, the difference was not statistically significant with a p-value of 0.22.

The fatigue test results are graphically presented as maximum stress vs. fatigue life graphs. Fig. 5 shows the effect of cycling frequency for non-milled specimens. The increase in frequency from 0.5 to 4–7 Hz decreased the fatigue life by 10 orders of magnitude when jet cutting was used. The temperature was measured as 35°C when a frequency of 4–7 Hz was applied. This is in line with the observations made in [16], according to which, for some material systems, a change of 10°C is accompanied by measurable degradation of the material properties. The typical fracture was out of ply drop at the thinnest part.

Figure 6 shows the effect of the cycling frequency for the milled specimens. No degradation was observed in the fatigue life when the frequency increased from 0.5 to 5 Hz. A significant decrease in the fatigue life was observed when the frequency increased to 10 and 15 Hz. For these frequencies, the stable temperature was already over 50°C. Typical fracture occurred at ply drop of 0/90 near the thinnest part. For higher frequencies, fracture was observed more often out of ply drop. A smoother thickness transition (from 11 to 13 and consequently to 16 plies) does not causes local concentration, i.e., all the fatigue failures are initiated in a larger area.

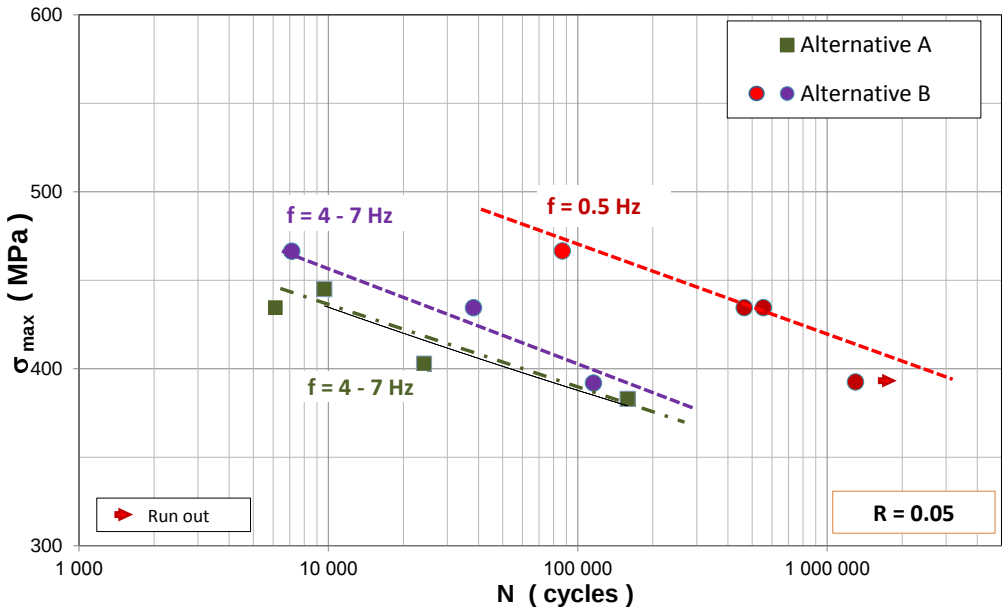

Figure 5: Effect of edge cycling frequency: jet cutting. The frequency was increased from 0.5 to 4–7 Hz, which caused the fatigue life to decrease by 10 orders of magnitude.

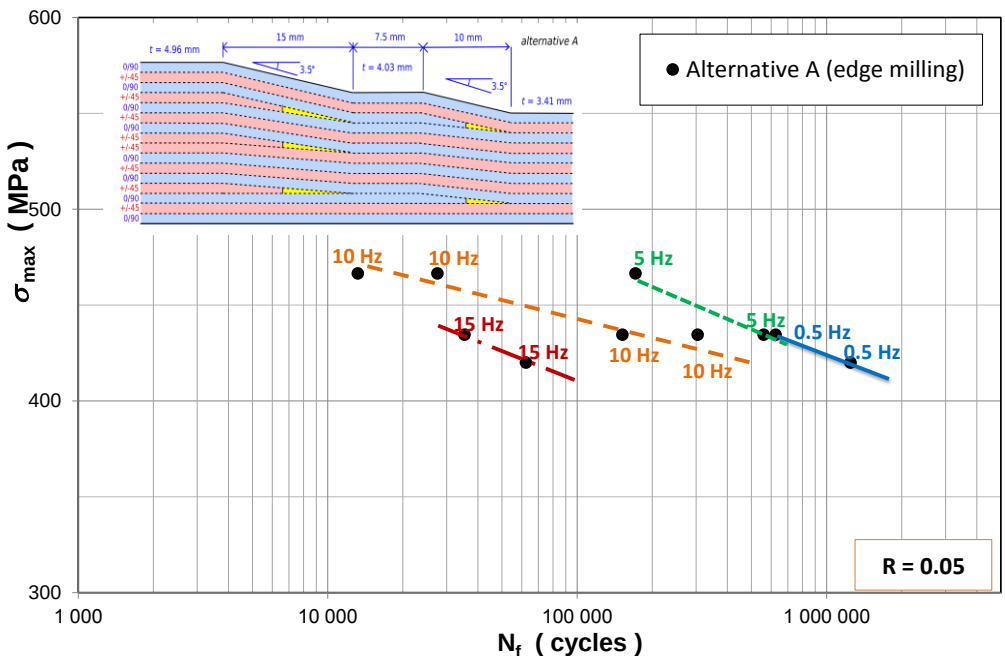

Figure 6: Effect of edge cycling frequency: edge milling (0.5 – 15 Hz). An increase in frequency from 5 to 15 Hz caused the fatigue life to decrease by more than 10 orders of magnitude.

3.2 Temperature

Initially, the highest temperature area was always formed in the middle part of a specimen, where the thickness was constant (11 plies). This area corresponds to the area with the highest stress level. During the following fatigue cycling, the warmest point was subsequently shifted to an area located much closer to the thickness transition zone where the final failure occurred. The global temperature increase was caused by rapid cyclic deformation; no global microcracking was observed in the cross section. The highest temperature increase corresponds to the area with fatigue crack (delamination) initiation and propagation. Examples of the highest temperature area gradually moving from the middle part of the specimens with constant thickness to the failed area are shown in Fig. 7 (test frequency of 10

Hz). In the first stage of the fatigue test, the stabilized warmest point had a temperature of 32.2°C in the middle part of the specimen with constant thickness (Fig. 7a). During continuous loading, the heat produced by rapid deformation accumulated and increased the temperature of the warmest point to 64.4°C (Fig.7b). In addition, this point moved towards the thickness transition area (ply drop) where the initial delamination shearing led to an even higher temperature. Fig. 7c shows the area with the highest temperature of 85.6°C just before failure. Fig. 7d shows the specimen just after failure, when the individual failed ends disconnected (subsequently, the temperature rapidly decreased).

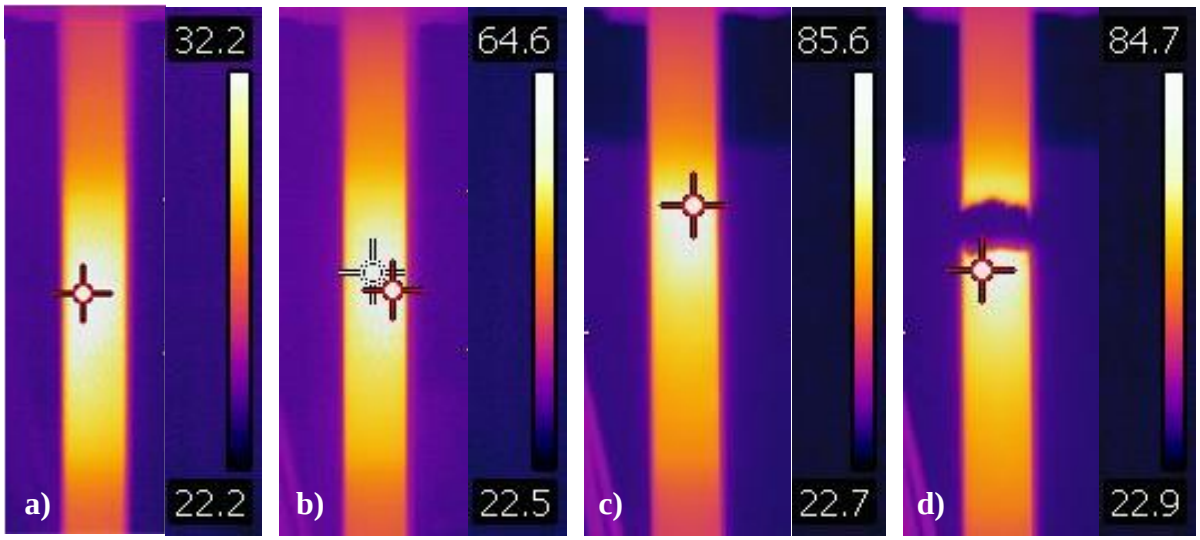

Figure 7: Thermo-images of fatigued specimen. The upper numbers denote the temperature in degrees Celsius at the red crosses, and the bottom numbers denote the temperature in degrees Celsius of the laboratory environment.

The evolution of the temperature measured during fatigue testing for individual specimens under different load conditions is shown in Figs. 8–13. The initial phase in which the temperature rapidly increased can be recognized for higher frequency values. After several elapsed cycles, the temperature stabilized at a certain value. This value depends on the maximum stress value, test frequency, and layup configuration. The stabilized temperature was achieved between 5,000 and 20,000 cycles. A maximum stabilized temperature level of 75°C was measured. This value is relatively close to the glass transition temperature of C/PPS composites (i.e., *T*g = 98°C). The fatigue life was significantly reduced in this case. At a frequency of 0.5 Hz, no surface temperature changes were observed (see Figs. 8 and 9). On the other hand, the stabilized temperature was not reached in certain conditions (highest frequency and load levels; see Fig. 13).

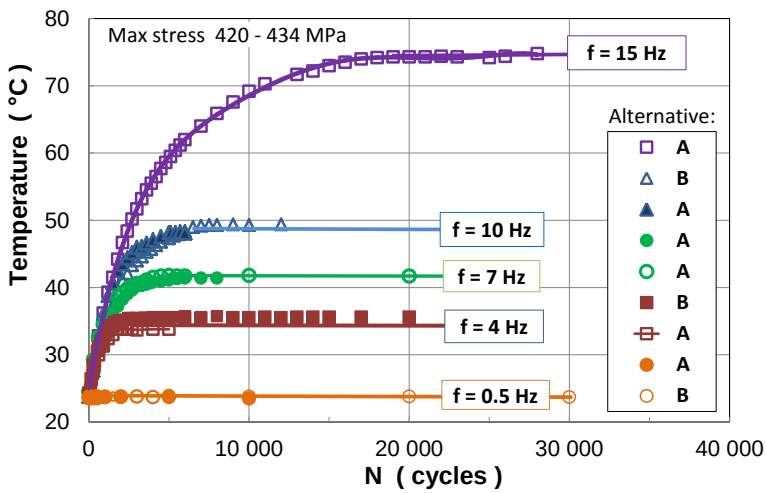

Figure 8: Temperature evolution up to stabilized value during cyclic loading for various frequencies (maximum stress in the range of 420–434 MPa).

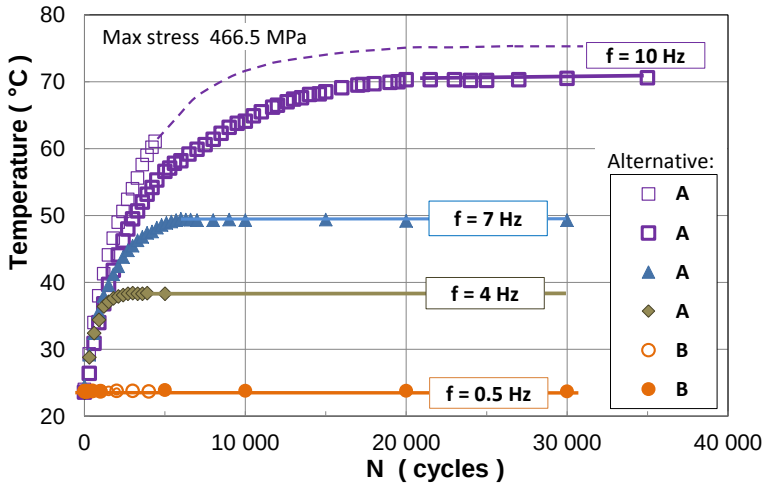

Figure 9: Temperature evolution up to stabilized value during cyclic loading for various frequencies (maximum stress level of 466.5 MPa).

When the stabilized surface temperature was lower than 38°C, no relationship was observed between temperature and fatigue life for the milled specimens. On the other hand, when the stabilized surface temperature exceeded 75°C, a significant reduction in fatigue was observed in all the cases.

During continuous fatigue loading, the temperature stabilizes at a certain time as the exposed heat is sufficiently conducted away. After this phase, cracks initiate near the transition area and the temperature begins to increase gradually. The warmest point moves towards this area, in which failure occurs afterwards. Temperature evolution example until failure is shown in Figs. 10, compared to the steady-state temperature at different frequencies. The temperature increase is significant before the failure of the specimens. The difference between the temperature in the steady-state phase of the fatigue process and the surface specimen temperature just before failure may be greater than 30°C. This difference depends on the parameters of fatigue loading. The maximum temperature value just before the specimen failure was measured to be higher than 110°C. This value is higher than the glass transition temperature of C/PPS composites.

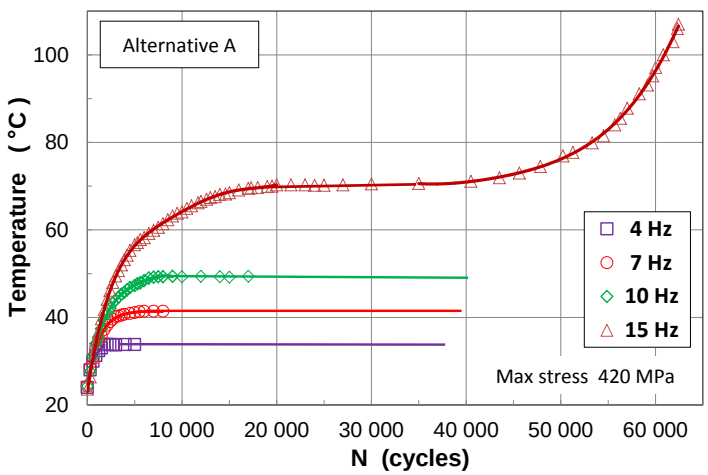

Figure 10: Temperature evolution during cyclic loading under different frequencies until failure (maximum stress level of 420 MPa, stress ratio 0.05).

3.3 Fractography

First, edge appearance was evaluated using macro-photography. According to the location, there were two types of failure: failure at ply drop and failure out of ply drop. Figure 14 shows macro-photos of both types of failures taken for Alternative B. The ply drops were not positioned exactly in a column but with a scatter of up to ±5 mm. The actual positions of the drops are visible in the following images of the cross sections.

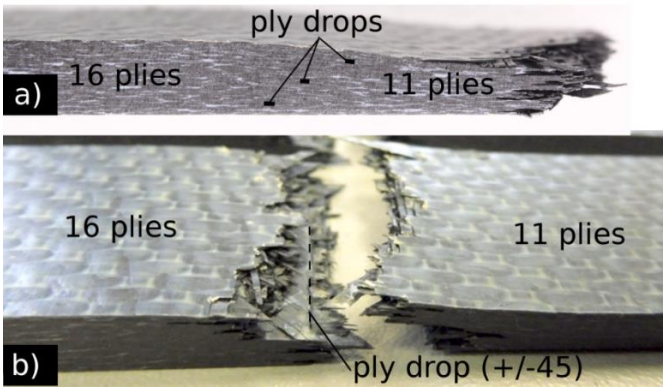

Fig. 14: Typical failure macro-photography of (a) out of ply drop (spec. 1.11) and (b) ply drop failure (spec. 4.4).

Longitudinal cross sections of the selected specimens were observed. The cross sections showed actual ply drops as well as cracks caused by the loading. Fatigue cracks initiated at 0/90 ply drops for both alternatives. No fatigue initiation was observed on +/-45 ply drops. For some specimens, compression failure was observed near the upper ply drop at the end of the fatigue crack. This was caused by impact waves in the part of the specimen opposite to the tensile failure.

With regard to Alternative A with two tapers, the following fractographic features were investigated: fatigue crack initiation was observed at both 0/90 ply drops positioned near the gauge section with growth towards the thick section (both jet cutter and milled specimens). This can be explained by the higher stress in the thinner tapering and also by the 0/90 ply drop that is the most critical because of its high stiffness. The cracks grew between the carbon tows preferentially. Figure 15 shows an example of specimen 3.6.

With regard to Alternative B with one taper, the similar features were found. Ply drops that were planned to be in one column were highly scattered. The fatigue crack initiated from the 0/90 ply drop.

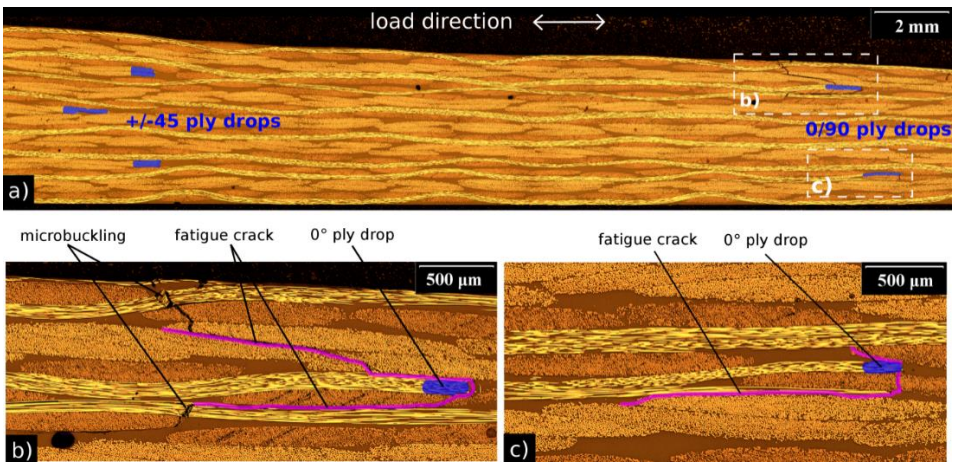

Figure 15: Cross section of spec. 3.6 (Alt. A) after 62,000 cycles. (a) ply drop overview, (b) fatigue crack growth from the upper 0/90 ply drop towards the thick section and compression microbuckling failure from released energy after tensile failure on the other side of the specimen, and (c) fatigue crack initiated from the bottom 0/90 ply drop.

6 DISCUSSION

The data presented above indicate a strong correlation between the loading parameters, temperature, and its influence on fatigue life. The local ply drop shift leads to friction between plies and results in a temperature increase above acceptable levels. Vieille at al. [17] and Franco at al. [18] stated that the mechanical properties of notched and un-notched laminates subjected to temperatures higher than the glass transition temperature of C/PPS composites (i.e., T_g = 98°C) can severely degrade the mechanical properties of angle-ply laminates. The results presented herein show that in the case of ply drop composition, the critical temperature considering the degradation of fatigue properties is significantly lower. The critical temperature seems to be around 40°C. The temperature is significantly dependent on the loading parameters.

Figure 20 shows the dependence of the temperature on the load rate. A linear relation between load rate and temperature can be observed. The load rate in MPa.sec^{-1} is defined as the ratio of stress amplitude to loading frequency. Using linear regression, the following equation was defined for the stabilized temperature evaluation:

$$T = 0.0091v + 21.6, \quad (1)$$

where T is the temperature and v is the load rate [MPa.sec^{-1}]. The value of 21.6°C represents the ambient temperature. This equation can serve as an important tool for design and control of fatigue experiments for composites with ply drops. From the results, it is evident that temperatures higher than 40°C are attained for loading frequencies in the range of 4 to 7 Hz, or for load rates higher than 2,000 MPa.sec^{-1}. These values imply a significant influence on the fatigue life, and they can be adopted as criteria for fatigue test design and execution.

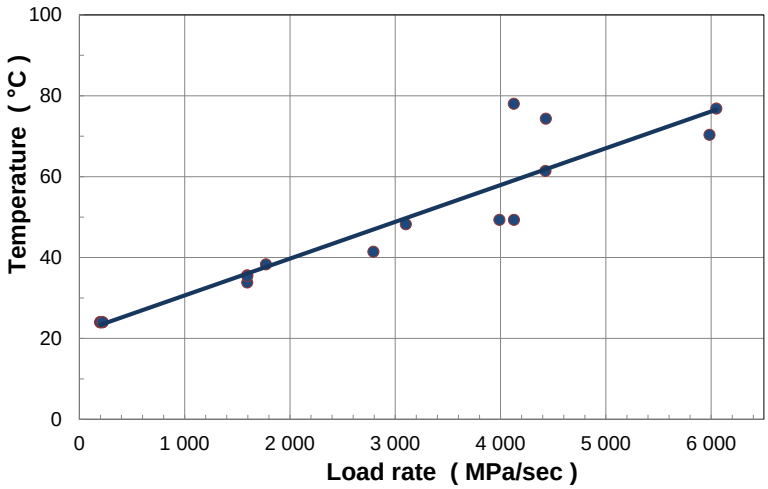

Figure 20: Temperature dependence (steady-state phase) on the load rate.

Significant scatter was observed in the horizontal positioning of the ply drops. This may be due to manual positioning or movement during moulding. The positions of the ply drops can affect the fatigue initiation, as the ply drops near the gauge section are more susceptible to damage, as will be discussed later. The fatigue crack initiation occurred only for the 0/90 ply drops. For the +/-45 ply drops in the second taper, no delamination was observed. For Alternative A (two tapers), the crack initiated only from the tapering near the gauge section. Crack initiation near the thin region is in line with the observations made in [19], which investigated fatigue at R = -1, and the first two ply drops close to the thin section were observed to be the most critical for damage initiation. In [20], for laminates with ply drops in multiple steps, the onset of delamination was observed to occur at the terminated ply group closest to the thinnest (gauge) region. An optimal approach would be to end the 0° plies in the thicker section and the 45° plies in the thinner section with higher stress. This would also be in line with the

design guidelines presented in [21], which state, 'The plies should be dropped in decreasing order of their stiffness. 0° plies should be dropped first.'

According to [22], the failure mechanisms of fatigue are very similar to those under static loading. It is believed that the main driving force is the straightening out of the kinked cover plies. However, for the present specimens, the layers are pressed into each other with no significant kink. Therefore, the straightening of the cover plies is not the issue here.

6 CONCLUSION

This paper discussed the influence of loading frequency on the temperature, fatigue behaviour, and failure mechanisms of carbon-fibre-fabric-reinforced PPS laminates whose thickness was varied using of ply drops. The main objective was to evaluate the material characteristics and parameters that significantly influence the fatigue characteristics, especially the influence of frequency. This requirement arises from the need to reduce costs and save time in the experimental verification of structures. The effects of two ply drop configurations and fatigue loading frequencies from 0.5 to 15 Hz were investigated. The fatigue crack initiation occurred only for the 0/90 ply drops. For the +/-45 ply drops in the second taper, no delamination was observed.

The loading frequency was shown to have a significant influence on the fatigue lifetime of specimens with varying thickness. The local temperature increased significantly whereas the fatigue life decreased as the loading frequency was increased. In particular, the fatigue life decreased by more than one order of magnitude. Frequencies higher than 4–7 Hz were shown to significantly influence the fatigue life of the investigated tailored blank structures in relation to the applied load level. Load rates higher than 2,000 MPa.sec^{-1} are critical from the viewpoint of their influence on fatigue life.

ACKNOWLEDGEMENTS

This research was funded by institutional support from the Ministry of Industry and Trade of the Czech Republic.

REFERENCES

[1] G. Reyes, U. Sharma, Modeling and damage repair of woven thermoplastic composites subjected to low velocity impact, Composite Structures, Volume 92, Issue 2, January 2010, pp. 523-531.

[2] B. Maron, Ch. Garthaus, A. Hornig, F. Lenz, M. Hübner, M. Gude, Forming of carbon fiber reinforced thermoplastic composite tubes – Experimental and numerical approaches, Journal of Manufacturing Science and Technology, 7 October 2016, ISSN 1755-5817, http://dx.doi.org/10.1016/j.cirpj.2016.09.004.

[3] S. Boria, A. Scattina, G. Belingardi, Experimental evaluation of a fully recyclable thermoplastic composite, Composite Structures, Volume 140, 15 April 2016, pp. 21-35.

[4] R. Vodicka, Defence Science and Technology Organization, Australian Government of Defence, Report n◦ DSTOTR-0424, 2006.

[5] V. Bellenger, J. Decelle, N. Huet, Ageing of a carbon epoxy composite for aeronautic applications, Composites Part B, **36**, 2005, pp. 189–194.

[6] R. Talreja, Damage mechanics of composite materials, Vol. 9, 1st Edition, Elsevier, ISBN: 978044488852, 1994.

[7] E. Dan-Jumbo, S.G. Zhou, C.T. Sun, Load-frequency effect on fatigue life of IMP6/APC-2 thermoplastic composite laminates, Advances in thermoplastic matrix composite materials, ASTM STP 1044, 1989, pp. 113- 132.

[8] L.A.L. Franco, Fractographic Analysis of Advanced Polymeric Composites, MSc Thesis, ITA, Sao Jos´e dos Campos, Brazil, 2003.

[9] K. He, S.. Hoa, R. Ganesan, The study of tapered laminated composite structures: a review, Compos. Sci. Technol. **60**, 2000, pp. 2643–2657 (doi:10.1016/S0266-3538(00)00138-X).

[10] S. V. Hoa, J. Daoust, B.L. Du, T. Vu-Khanh, Interlaminar stresses in tapered laminates, Polym. Compos., **9**, 1988, pp. 337–344 (doi:10.1002/pc.750090506).

[11] K.W. Gan, G. Allegri, S.R. Hallett, A simplified layered beam approach for predicting ply drop delamination in thick composite laminates, Mater. Des., **108**, 2016, pp. 570–580 (doi: 10.1016/j.matdes.2016.06.105).

[12] J. Šedek, R. Hron, M. Kadlec, Bond Joint Analysis of Thermoplastic Composite Made from Stacked Tailored Blanks. Applied Mechanics and Materials, **827**, 2015. pp. 161-168, ISSN 1662-7482 (doi: 10.4028/www.scientific.net/AMM.827.161).

[13] J. Šedek, Optimization of composite airframe rib using tailoring blank technology, In Plánička F., Krystek J. (eds.), EAN 2016, 54th International Conference on Experimental Stress Analysis, Hotel Srní, Srní, Czech Republic, May 30 – June 2, 2016, Department of Mechanics, Faculty of Applied Sciences, University of West Bohemia, pp. 401-404, ISBN: 978-1-5108-2670-0.

[14] R. Růžek, J. Šedek, M. Kadlec, P. Kucharský, Mechanical behavior of thermoplastic rib under loading representing real structure conditions. 54th International Conference on Experimental Stress Analysis, EAN 2016, Hotel Srni, Srni; Czech Republic; 30 May 2016 through 2 June 2016; Code 123122, Department of Mechanics, Faculty of Applied Sciences, University of West Bohemia, pp. 356-359, ISBN: 978-1-5108-2670-0.

[15] AIMS05-09-002. Airbus Material Specification / Carbon Fiber Reinforced Thermoplastic Materials. Airbus Industrie Engineering Directorate 31707 Blachnac Cedex France: Airbus Industrie, 1998.

[16] ASTM D 3479/D3479M - 12. Standard Test Method for Tension-Tension Fatigue of Polymer Matrix Composite Materials, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA 19428-2959, United States: ASTM International, 2007.

[17] B. Vieille , J. Aucher, L. Taleb, Influence of temperature on the behavior of carbon, Materials Science and Engineering A, **517**, 2009, pp. 51–60 (doi: 10.1016/j.msea.2009.03.038).

[18] L.A.L. Franco, M.L.A. Graca, F.S. Silva, Fractography analysis and fatigue of thermoplastic composite laminates at different environmental conditions. Materials Science and Engineering A, **488**, 2008, pp. 505–513 (doi: 10.1016/j.msea.2007.11.053).

[19] A. Weiss, W. Trabelsi, L. Michel, J.J. Barrau, S. Mahdi, Influence of ply-drop location on the fatigue behaviour of tapered composites laminates. Procedia Engineering, **2**, 2010, pp. 1105–1114.

[20] Shim, D. J., Role of delamination and interlaminar fatigue in the failure of laminates with ply drop offs. Massachusetts Institute of Technology, Dept. of Aeronautics and Astronautics, Thesis (Ph. D.), Cambridge, MA, 2002 (URL: http://hdl.handle.net/1721.1/29245).

[21] A. Mukherjee, B. Varughese, Design guidelines for ply drop-off in laminated composite structures. Composites: Part B, **32**, 2001, pp. 153–164.

[22] M. R. Wisnom, R. Dixon, G. Hill, Delamination in asymmetrically tapered composites loaded in tension, Composite Structures, Vol. 35, Iss. 3, 1996, pp. 309-322.