EXTENDED MULTISCALE FINITE ELEMENT METHOD FOR GEOMETRICALLY NONLINEAR ANALYSIS OF THIN COMPOSITE PLATES ON BENDING PROBLEMS

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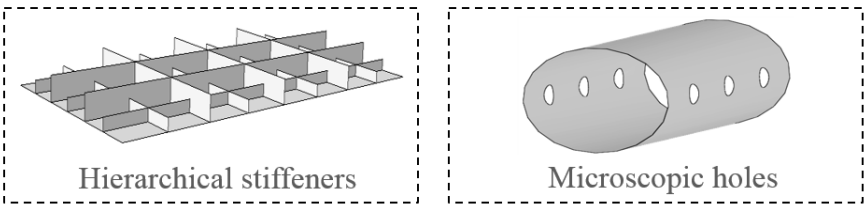
ABSTRACT

An efficient multiscale finite element method is developed for geometrically nonlinear analysis of thin composite plates on large deflection bending problem. On the basis of our existing work about small deflection problems, the coupling effects of composite laminates are also considered by introducing the additional coupling terms among translations and rotations into the multiscale base functions. Then, the decoupling displacement boundary conditions for deflections and rotations are employed to construct the multiscale base functions numerically at the micro scale. After that, the equivalent tangent stiffness matrix and equivalent secant stiffness matrix secant stiffness of macroscopic elements can be obtained based on the Total-Lagrange incremental approaches. Furthermore, the equilibrium macro scale finite element iterations for each load step can be performed directly and the macroscopic response fields will be obtained efficiently. Finally, the microscopic response fields can be obtained through downscale computation easily. Numerical examples demonstrate that the developed method possesses high computing accuracy and efficiency compared with the conventional finite element method.

1 INTRODUCTION

Composite structures have been widely used in the field of aerospace due to its high mechanical properties and outstanding designability characteristic. For thin composite structures subjected to bending load, it is usually a large-deflection problem due to small bending stiffness. In order to obtain higher stiffness-weight properties, these structures often have complicated microstructures such as hierarchical stiffeners, microscopic holes [1,2] as shown in Fig. 1. Due to these microstructures, analysis of the thin composite laminates conducted by the conventional finite element method (FEM) is often time consuming. Therefore, it is necessary to seek for a numerical computation method with high computing efficiency and precision.



(a) (b)

Fig. 1 Thin composite structures with different microstructures.

The multiscale finite element method (MsFEM) is named officially by Hou et al [3, 4]. The basic idea of MsFEM is to capture the properties of microstructure effectively by constructing the multiscale base functions numerically. On the basis of the multiscale base functions, the problems can be solved at the macro scale directly with high computing efficiency and accuracy. It is worth mentioning that,

the MsFEM is not restricted by the periodicity assumption and the scale separation assumption. Thus, The MsFEM has been widely used [5, 6]. The extended multiscale finite element method (EMsFEM) is developed by Zhang Hongwu et al [7] based on the MsFEM. In order to consider the coupling effects among different directions for the multi-dimensional vector field problems, the additional coupling terms of the multiscale base functions are introduced. It has been successfully applied in the research of heterogeneous porous media and composite materials [8-10].

In this paper, an efficient multiscale finite element method is developed for geometrically nonlinear analysis of thin composite laminates with microscopic holes on large deflection bending problems. First, on the basis of our existing work for small-deflection analysis of thin composite plates, the coupling effects of composite laminates are considered by introducing the additional coupling terms among translations and rotations into the multiscale base functions and the decoupling displacement boundary conditions for deflections and rotations are reconstructed to construct the multiscale base functions according to their coupled displacement modes of thin plates. Then, the equivalent tangent stiffness matrix and equivalent secant stiffness matrix secant stiffness of macroscopic elements can be obtained based on the Total-Lagrange incremental approaches. Finally, the equilibrium macro scale finite element iterations for each load step can be performed directly and the macroscopic and microscopic response fields will be obtained efficiently through the macro scale and downscale computation respectively on the basis of multiscale base functions Several numerical examples demonstrate that the developed method possesses high computing accuracy and efficiency compared with the conventional finite element method.

2 METHODOLOGY

2.1 Incremental macro and micro scale finite element formulation

On the basis of incremental theory and Newton-Raphson iteration method, the multiscale finite element computation can be performed for large deflection problems of thin composite laminates. The incremental macro and micro scale finite element formulations from t to $t+\Delta t$ are derived here according to Total-Lagrange formulation.

At the micro scale, the weak form of equilibrium equations and boundary conditions at $t+\Delta t$ can be derived as Eq. (1) based on the principle of virtual displacement.

$$\sum_e \left\{ G_E^T \int_{\Omega_e} {}_0^t B_e'^T \, {}_0^{t+\Delta t} F_e dxdy \right\} = \sum_e \left\{ G_E^T \int_{\Omega_e} N_e^T \, {}_0^{t+\Delta t} \bar{f}_e dxdy \right\} + \sum_e \left\{ G_E^T \int_{\Gamma_\sigma^e} N_e^T \, {}_0^{t+\Delta t} \bar{T}_e ds \right\} \tag{1}$$

The simplified form can be expressed as Eq. (2):

$$\left(K_{L0}^E + {}_0^t K_{L1}^E + {}_0^t K_\sigma^E \right) \Delta u_E = {}_0^t K_T^E \Delta u_E = {}_0^{t+\Delta t} \bar{P}_E - {}_0^t K_C^E \, {}_0^t u_E \tag{2}$$

where ${}_0^t K_T^E$ and ${}_0^t K_C^E$ are the tangent and secant stiffness matrices of a macroscopic element, ${}_0^{t+\Delta t} \bar{P}_E$ is the equivalent microscopic nodal force within a macroscopic element. They can be expressed as:

$$K_{L0}^E + {}_0^t K_{L1}^E = \sum_e \left\{ G_E^T \int_{\Omega_e} \left(B_{L0}' + {}_0^t B_{L1}' \right)^T \begin{bmatrix} A & B \\ B & D \end{bmatrix} \left(B_{L0}^* + {}_0^t B_{L1}^* \right) dxdy G_E \right\}$$

$${}_0^t K_\sigma^E = \sum_e \left\{ G_E^T \int_{\Omega_e} \left(HN_e \right)^T \, {}_0^t F_e \left(HN_e \right) dxdy G_E \right\}$$

$${}_0^t K_C^E = \sum_e \left\{ G_E^T \int_{\Omega_e} \left(B_{L0}' + {}_0^t B_{L1}' \right)^T \begin{bmatrix} A & B \\ B & D \end{bmatrix} \left(B_{L0} + {}_0^t B_{L1} \right) dxdy G_E \right\}$$

$${}_0^{t+\Delta t} \bar{P}_E = \sum_e \left\{ G_E^T \int_{\Omega_e} N_e^T \, {}_0^{t+\Delta t} \bar{f}_e dxdy \right\} + \sum_e \left\{ G_E^T \int_{\Gamma_\sigma^e} N_e^T \, {}_0^{t+\Delta t} \bar{T}_e ds \right\}$$

Eq. (2) is the incremental micro scale finite element formulation. It is worth noting that the number of

increment step and iteration step should be set to one when Eq. (2) is applied to construct the multiscale base functions.

At the macro scale, a composite laminate is discretized into the macroscopic finite elements. It is assumed that the incremental macroscopic nodal displacement ΔU_E within a macroscopic element satisfy

$$\Delta u_e = N_E \Delta U_E \tag{3}$$

where N_E is the multiscale base function of a macroscopic element.

The weak form of equilibrium equations and boundary conditions at $t+\Delta t$ can be derived as follows

$$\sum_E \left\{ G^T \sum_e \left\{ N_E^T \int_{\Omega_e} {}_0^t B_e'^T \, {}_0^{t+\Delta t} F_e dxdy \right\} \right\} = \sum_E \left\{ G^T \sum_e \left\{ N_E^T \int_{\Omega_e} N_e^T \, {}_0^{t+\Delta t} \bar{f}_e dxdy \right\} \right\} + \sum_E \left\{ G^T \sum_e \left\{ N_E^T \int_{\Gamma_\sigma^e} N_e^T \, {}_0^{t+\Delta t} \bar{T}_e ds \right\} \right\} \tag{4}$$

Eq. (4) can be further simplified as follows

$$\left(K_{L0} + {}_0^t K_{L1} + {}_0^t K_\sigma \right) \Delta U = {}_0^t K_T \Delta U = {}_0^{t+\Delta t} \bar{P} - {}_0^t K_C \, {}_0^t U \tag{5}$$

where ${}_0^t K_T$ and ${}_0^t K_C$ are the tangent and secant stiffness matrices of whole structure, ${}_0^{t+\Delta t}\bar{P}$ is the equivalent macroscopic nodal force of whole structure. They can be expressed as:

$$K_{L0} + {}_0^t K_{L1} = \sum_E \left\{ G^T \sum_e \left\{ N_E^T \int_{\Omega_e} \left(B_{L0}' + {}_0^t B_{L1}' \right)^T \begin{bmatrix} A & B \\ B & D \end{bmatrix} \left(B_{L0}^* + {}_0^t B_{L1}^* \right) dxdy N_E \right\} G \right\}$$

$${}_0^t K_\sigma = \sum_E \left\{ G^T \sum_e \left\{ N_E^T \int_{\Omega_e} \left(HN_e \right)^T \, {}_0^t F_e \left(HN_e \right) dxdy N_E \right\} G \right\}$$

$${}_0^t K_C = \sum_E \left\{ G^T \sum_e \left\{ N_E^T \int_{\Omega_e} \left(B_{L0}' + {}_0^t B_{L1}' \right)^T \begin{bmatrix} A & B \\ B & D \end{bmatrix} \left(B_{L0} + {}_0^t B_{L1} \right) dxdy N_E \right\} G \right\}$$

$${}_0^{t+\Delta t}\bar{P} = \sum_E \left\{ G^T \sum_e \left\{ N_E^T \int_{\Omega_e} N_e^T \, {}_0^{t+\Delta t} \bar{f}_e dxdy \right\} \right\} + \sum_E \left\{ G^T \sum_e \left\{ N_E^T \int_{\Gamma_\sigma^e} N_e^T \, {}_0^{t+\Delta t} \bar{T}_e ds \right\} \right\}$$

Eq. (5) is the incremental macro scale finite element formulation which is applied to calculate the incremental macroscopic nodal displacement.

Then, the macroscopic nodal displacement can be obtained by Eq. (6).

$${}_0^{t+\Delta t} U = {}_0^t U + \Delta U \tag{6}$$

2.2 Multiscale base functions

The multiscale base functions are the displacement functions of the macroscopic elements which are employed to establish the relationship between the macro scale and the micro scale stiffness matrices. Besides, the multiscale base functions are also applied to obtain the microscopic strain and stress fields in the downscale computation. Thus, it can be seen that the construction of multiscale base functions is a key step in the EMsFEM.

Consider that the coupling effects are obvious as a result of layup configurations and coupled displacement modes between deflections and rotations for the thin composite plates. The displacements of a microscopic node within a macroscopic element can be expressed as

$$\begin{cases} u=\sum_{i=1}^{4}N_{uui}u_i^E+\sum_{i=1}^{4}N_{uvi}v_i^E+\sum_{i=1}^{4}N_{uwi}w_i^E+\sum_{i=1}^{4}N_{uxi}\theta_{xi}^E+\sum_{i=1}^{4}N_{uyi}\theta_{yi}^E \\ v=\sum_{i=1}^{4}N_{vui}u_i^E+\sum_{i=1}^{4}N_{vvi}v_i^E+\sum_{i=1}^{4}N_{vwi}w_i^E+\sum_{i=1}^{4}N_{vxi}\theta_{xi}^E+\sum_{i=1}^{4}N_{vyi}\theta_{yi}^E \\ w=\sum_{i=1}^{4}N_{wui}u_i^E+\sum_{i=1}^{4}N_{wvi}v_i^E+\sum_{i=1}^{4}N_{wwi}w_i^E+\sum_{i=1}^{4}N_{wxi}\theta_{xi}^E+\sum_{i=1}^{4}N_{wyi}\theta_{yi}^E \\ \theta_x=\sum_{i=1}^{4}N_{xui}u_i^E+\sum_{i=1}^{4}N_{xvi}v_i^E+\sum_{i=1}^{4}N_{xwi}w_i^E+\sum_{i=1}^{4}N_{xxi}\theta_{xi}^E+\sum_{i=1}^{4}N_{xyi}\theta_{yi}^E \\ \theta_y=\sum_{i=1}^{4}N_{yui}u_i^E+\sum_{i=1}^{4}N_{yvi}v_i^E+\sum_{i=1}^{4}N_{ywi}w_i^E+\sum_{i=1}^{4}N_{yxi}\theta_{xi}^E+\sum_{i=1}^{4}N_{yyi}\theta_{yi}^E \end{cases} \tag{7}$$

where N is the multiscale base function of the macroscopic element. Take an additional coupling terms N_{uvi} as an example. It means the displacement field u when unit displacement v is applied on the macroscopic node i.

2.3 Boundary condition

The decoupling displacement boundary conditions of macroscopic nodes on the macroscopic element boundary can be expressed as follows:

$$\begin{cases} \bar{N}_{uw1}\big|_i=0 \\ \bar{N}_{vw1}\big|_i=0 \\ \bar{N}_{ww1}\big|_i=\delta_{1m} \quad (i=1,2,3,4) \\ \bar{N}_{xw1}\big|_i=0 \\ \bar{N}_{yw1}\big|_i=0 \end{cases} \tag{8}$$

The decoupling displacement boundary conditions of microscopic nodes on the macroscopic element boundary can be expressed as follows:

$$\begin{cases} \bar{N}_{uw1}=\sum_{i=1}^{4}\left(N_{ui}^e\,\bar{N}_{uw1}\big|_i\right) \\ \bar{N}_{vw1}=\sum_{i=1}^{4}\left(N_{vi}^e\,\bar{N}_{vw1}\big|_i\right) \\ \bar{N}_{ww1}=\sum_{i=1}^{4}\left(N_{wwi}^e\,\bar{N}_{ww1}\big|_i\right) \\ \bar{N}_{xw1}=\sum_{i=1}^{4}\left(N_{xwi}^e\,\bar{N}_{xw1}\big|_i\right) \\ \bar{N}_{yw1}=\sum_{i=1}^{4}\left(N_{ywi}^e\,\bar{N}_{yw1}\big|_i\right) \end{cases} \tag{9}$$

Eqs. (8) and (9) are the decoupling displacement boundary conditions corresponding to the multiscale base functions N_1^w. Under these decoupling displacement boundary conditions, the multiscale base functions N_1^w can be constructed. The other multiscale base functions can be

constructed in a similar way.

3 NUMERICAL EXAMPLES

A thin square composite plate with layup [0/90/90/0] is examined. The plate is composed of 8×8 macroscopic elements and it is subjected to the uniform distributed load with all edges clamped. There are two kinds of microscopic finite element models, where the numbers of microscopic elements with a macroscopic element are 1×1 and 3×3 (see Fig. 2). The two kinds of numerical multiscale models are designated as EMsFEM_1×1 and EMsFEM_3×3. The length of plate is 12 inch. The thickness is 0.096 inch. The lamina constants are given: E_1=1.8282×10^6 psi, E_2=1.8315×10^6 psi, G_{12}= G_{13}= G_{23}=3.125×10^5 psi, μ_{12}=0.2395. Note that 1 inch is 2.54 cm and 1 psi is 6.9 KN/m^2.

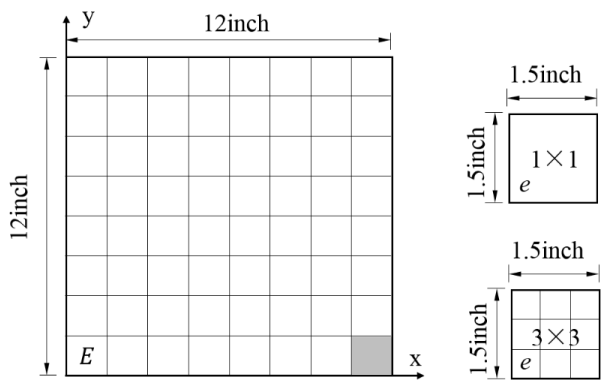


Fig. 2 Multiscale finite element model of a thin square composite plate: a macroscopic finite element model; b microscopic finite element model

The central deflections obtained from the developed method, together with those from reference [11] are presented in Table 1.

P (psi)	EMsFEM_1×1	EMsFEM_3×3	Reference solutions[11]
0.2	0.038416	0.038534	0.03773
0.4	0.065400	0.065812	0.06504
0.6	0.084756	0.08544	0.08489
0.8	0.099807	0.100712	0.10039
1.0	0.112191	0.113276	0.11316
1.2	0.122773	0.124009	0.12406
1.4	0.132057	0.133418	0.13362
1.6	0.140360	0.141831	0.14216
1.8	0.147893	0.14946	0.14991
2.0	0.154807	0.156459	0.15701
2.2	0.161209	0.162938	0.16359
2.4	0.167182	0.168984	0.16972

Table 1 Center deflections of a simply supported thin square plate

The Load-deflection curves are plotted in Fig. 3.

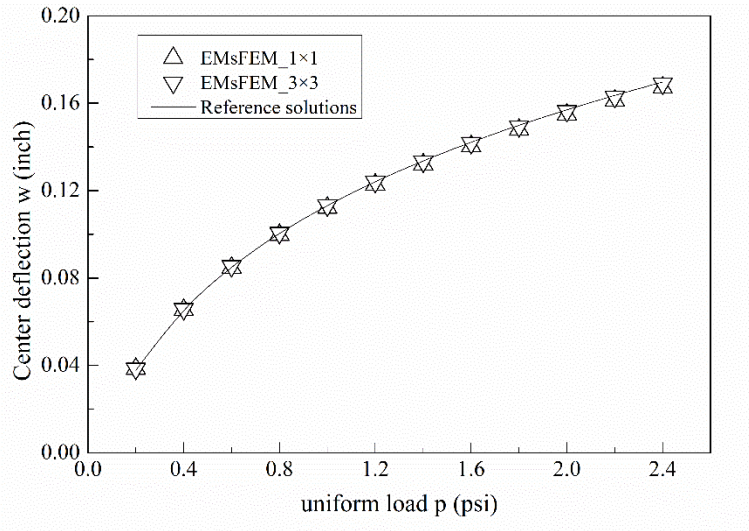


Fig. 3 Load-deflection curves of a simply supported thin square plate subjected to the uniformly distributed load

It is obvious that the nonlinear results obtained by the developed method are in good agreement with those from reference for thin composite plates. It means that the developed method has high computing accuracy. Besides, the developed method has a good convergence.

4 CONCLUSIONS

In this paper, a novel multiscale finite element method suitable for small-deflection analysis of thin composite plates with aperiodic macrostructure features is developed on the basis of the theoretical framework of the extended multiscale finite element method.

The numerical examples illustrate that the developed method has the characteristic of high computing accuracy for thin composite plates in consideration of layup configurations and aperiodic microstructure characteristics, especially under the relaxed decoupling displacement boundary conditions. Meanwhile, the computer memory is reduced and the computing efficiency is improved significantly compared with the conventional finite element method.

The developed method has great potential for strength prediction for composite structures with complicated microstructures. That will be carried out in our future work.

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