SANDWICH STRUCTURES WITH 3D PRINTED FUNCTIONALLY GRADED LATTICE CORES

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ABSTRACT

Lattice structures are excellent candidates for sandwich cores since they are both light-weight and their cellular architecture can be advantageous to counteract buckling of the face sheets. Such lattices are ideally suited for additive manufacturing where complex designs can be easily produced with few associated manufacturing constraints. For the past few years, weight saving optimisation strategies have been available in many commercial software codes, utilising the finite element method for structural analysis (e.g. Dassault Systèmes Tosca, Autodesk Within and Altair OptiStruct). A new design feature in Altair's OptiStruct 14.0 enables the representation of light-weight lattice structures in the form of 1D truss elements. A range of unit cell configurations can be optimised through a combination of topology and size optimisation. Many unit cell designs have already been proposed along with characterised stiffness and strength behaviour. However, little attention has been given regarding the functional grading of these cells in order to further improve stiffness and strength.

1 INTRODUCTION

Cellular structures are useful in a variety of applications where a light-weight core is required to withstand external forces and moments. It has been demonstrated that cellular structures are particularly well suited to multifunctional applications where cell topology can be tailored to provide sufficient stiffness and yield strength while possessing other beneficial characteristics [1, 2]. Multifunctional applications include sandwich panels where the core material not only provides stiffness and strength but also buckling resistance [3]. Such cellular structures can be found all over the natural world including bamboo [4], sea sponges [5] and the internal cores of bones [6].

The most common cell arrangements are periodic, such as honeycomb cores with clear tessellation, or stochastic, such as open cell foams where the cellular structure is randomly distributed. Functionally graded versions of these topologies are also possible where there is a variation in density through the structure. Periodic truss-type lattice structures with a density variation through-the-thickness have been shown to have a progressive crushing behaviour with the sequential collapse of the graded layers [7]. Both periodic and stochastic functionally graded structures have also been investigated for orthopaedic hip implants [8, 9] where there is a requirement to match the stiffness and strength of bone and also tailor the surface texture to promote bone growth. Computational methods have been developed to address the difficult challenge of generating these functionally graded structures. Such approaches include a voxel-based method, well suited to periodic cellular structures [10]. A dithering approach combined with Voronoi tessellations has also been proposed to generate graded stochastic patterns [11].

Additive manufacturing technologies are particularly well suited to the creation of such optimised cellular structures. When compared with traditional manufacturing techniques, the available design space for additively manufactured parts is largely unconstrained by the manufacturing process or limitations on design complexity. The potential to generate highly complex parts also enables extensive use of topology optimisation as part of the design process. With topology optimisation an optimal density gradient can be converted into a cellular structure with a tailored density varying between cells. Topology optimisation approaches employing cellular structures include homogenisation approaches to simplify unit cell representation for modelling purposes [12]. Another approach is the multi-scale concurrent design method where cell level details are optimised concurrently with their macroscopic distribution [13]. One of the challenges encountered when optimising with lattices is the associated degradation in mechanical performance when compared with the parent solid material: a cellular geometry that can achieve the theoretical upper bounds for isotropic elasticity and strain energy storage is not achievable in most cases [14].

The optimisation of lattice structures presents a difficult challenge due to the potentially large number of design variables available. The ‘Michell Structure’ is widely considered to be the seminal work in this field [15, 16]. Michell devised a means of designing a load bearing truss structure with minimal mass. In the Michell Structure all members are subject to the same strain. This is achieved by aligning the truss members with respect to the principal stress/strain directions. The result of this process is a truss structure comprised of two sets of curves that are always perpendicular to each other at the points of intersection. One of the limitations of the method presented by Michell is that its application is limited to simple load cases and geometries.

More than a century has passed since the Michell Structure was first proposed and many computational optimisation techniques have since been developed, including size and topology optimisation [17]. Size optimisation defines the ideal structural parameters, such as the diameters of truss elements in a lattice structure, for known loads and boundary conditions. Size optimisation is a useful parametric optimisation tool but it does not address, for example, the optimal alignment of trusses within a lattice. Alternatively, topology optimisation is concerned with material layout in a given design space. An optimised design can be generated by assigning lower density properties to lower stressed areas and higher density material to more critical areas. In this way, by applying a threshold density, void and solid regions can be formed. Topology optimisation is useful for concept generation as it allows greater design freedom in comparison to size optimisation. Numerous topology optimisation techniques have been developed and have reached the point of becoming industry accepted design tools [18, 19, 20, 21].

This paper begins with an outline of a state-of-the-art commercially available lattice optimisation method. This method, available in Altair’s OptiStruct 14.0, is capable of generating functionally graded lattice structures where the diameter of each lattice member is generated and then uniquely sized. This commercially available approach is then compared with two in-house developed methods that can tailor cell size, shape and orientation based on either optimal strain or optimal density distributions. A sandwich panel is presented as a case study. Optimised lattice designs are presented and compared with purely solid and hybrid solid-lattice designs. Polymer demonstrators are manufactured, tested and then compared with their predicted performance.

2 METHODOLOGY

2.1 State-of-the art methodology for optimised lattice generation

The two-stage topology plus size optimisation procedure to generate lattice structures in Altair’s OptiStruct 14.0 is summarised, in the green box, on the right hand side of Figure 1. Herein such structures are referred to as ‘diameter graded’ lattices since the diameters of the constituent trusses form the final design variables. The first step in the procedure is to run a topology optimisation on a 3D solid element structure to determine the optimal density distribution. With conventional solid topology optimisation, at this point, a threshold density is applied to the optimised density and any elements falling below this limit are converted to voids and any elements above this density limit are converted to solids, resulting in a binary void/solid optimised solution. Lattice optimisation in OptiStruct 14.0 takes this one step further by allowing these intermediate densities to be represented by lattice trusses, of varying diameter, in the form of beam elements. This solution can be considered as a grayscale interpretation of the idealised optimal density distribution.

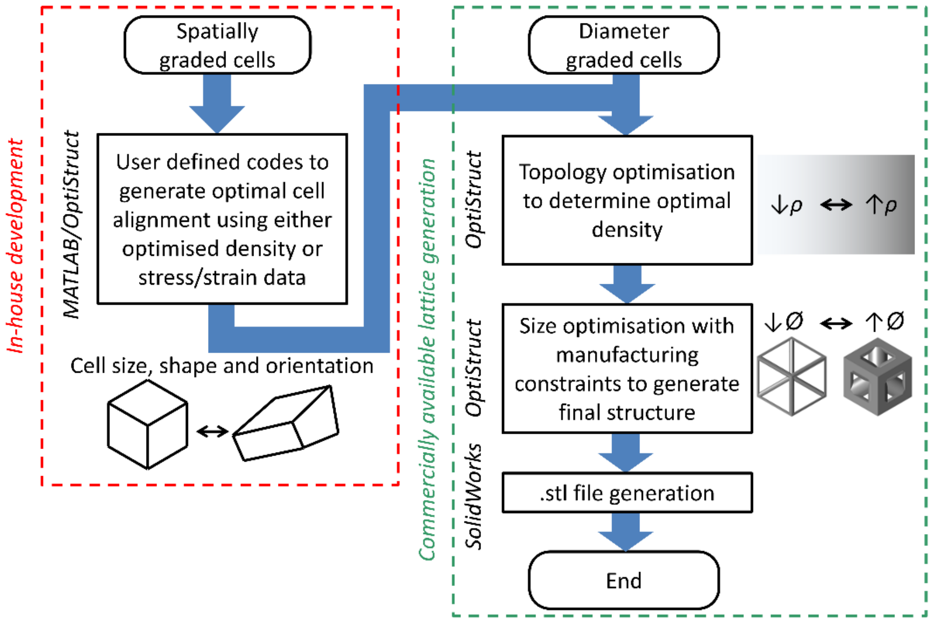

Figure 1: Optimisation procedures to generate either spatially graded or diameter graded cellular structures.

The density based topology optimisation problem considered in this work, using linear static finite element analysis, is formalised in Equation (1).

$$\begin{aligned} \text{minimise:} \quad & f(\boldsymbol{\rho}, \mathbf{U}) = \mathbf{U}^T\mathbf{K}\mathbf{U} \\ \text{subject to:} \quad & \mathbf{K}(\boldsymbol{\rho})\mathbf{U} = \mathbf{F}(\boldsymbol{\rho}) \\ & g(\boldsymbol{\rho}, \mathbf{U}) = V - V_f \leq 0 \\ & 0 \leq \boldsymbol{\rho} \leq 1. \end{aligned} \tag{1}$$

The objective function is given by f and the density design variables are contained within the vector $\boldsymbol{\rho}$. The global stiffness matrix is given by $\mathbf{K}$ and $\mathbf{U}$ is the displacement vector. The objective is to minimise structural compliance; $\mathbf{U}^T\mathbf{K}\mathbf{U}$. The stress-strain constitutive equation, relating stiffness and displacements to the force vector $\mathbf{F}$, is required to satisfy static equilibrium. The function g, relates the design space material volume fraction V and the allowable volume fraction upper bound, V_f.

Several lattice unit cell topologies are available in OptiStruct, such as the body-centred orthorhombic cell shown on the right hand side of Figure 1, as used in this present work. At the lattice generation step, where solid elements with idealised densities are converted to beams, upper and lower density thresholds can be applied. This enables an optimised design space to contain either void, lattice or solid. A penalty option is applied to the formation of lattice regions since the parent solid material will typically have superior stiffness and yield strength per unit mass [14]. In this work we assume the relationship between solid and lattice stiffness performance is approximated by the relationship given in Equation (1) [22, 23, 24, 25]. The choice of the 1.8 exponent is semi-empirical and depends upon details of a cell's geometry and loading. An exponent of 2 is characteristic of bending-dominated behaviour whereas an exponent of 1 is typical for stretch dominated behaviour [24].

$$\frac{E_{lattice}}{E_{solid}} = \left(\frac{\rho_{lattice}}{\rho_{solid}}\right)^{1.8} \tag{2}$$

Once lattice members have been generated, size (parametric) optimisation can then be carried out on each individual lattice member to fine tune the design, which can introduce additional anisotropy. At this point, stress and buckling constraints can be considered at the beam element level. OptiStruct 14.0 also enables upper and lower bounds to be placed on potential lattice diameters. These bounds are particularly important as manufacturing considerations for a particular additive manufacturing technique. Lattice stress constraints are dealt with in OptiStruct using the 'σ_{norm}' method [26]. Two stress constraints are generated using this method, one for the most highly stressed 10% of lattice members and one constraint for the remaining elements. This stress criterion is defined as

$$\sigma_{norm} = \left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\sigma_i}{\sigma_{bound}}\right)^6\right]^{1/6} \tag{3}$$

where σ_i denotes the stress in beam element i, σ_{bound} is the material strength upper bound and n is the number of beam elements in the stress response.

The final step in the process shown in Figure 1 is to export the optimised lattice structure for printing. The structure is exported from OptiStruct 14.0 as solid geometry (typically STEP *.step or Parasolid *.x_t work best) and imported into SolidWorks. When imported into SolidWorks, each lattice member is an independent solid cylinder that needs to be merged together to form a unified structure. Once combined the structure is exported to stereolithography *.stl format for 3D printing.

Although the thickness of each lattice member can be controlled at the size optimisation stage in OptiStruct 14.0, it is not clear whether this is the most desirable solution from a static strength point of view since any large changes in thickness at the lattice nodes (the joints) can lead to large stress concentrations. Furthermore, the ability to change lattice cross-section thicknesses alone will only be effective within a certain range of values as, for example, each 3D additive manufacturing technology has a minimum and maximum printable member size. The current state-of-the art modelling approaches do not consider variation of the size, shape and orientation of each individual lattice cell as design variables. The design space can be further expand if lattice cells are functionally graded in such a way [27, 28]. These structures are referred to herein as 'spatially graded' lattices. The left hand side of Figure 1 shows that this can be achieved by re-meshing the 3D finite element model after an initial topology optimisation step. In this way, the solid mesh geometry can be tailored with response to either the optimal density distribution or the optimised stress/strain field. Details of these in-house developed methods are given in Sections 2.2 and 2.3.

2.2 Spatially graded lattice structures - cells graded by strain

In this section we present a method for tailoring the solid mesh geometry based on the idealised optimal solid element strain distribution. The stress/strain graded approach is a patented method developed by the authors [27, 28]. After the initial topology optimisation run, data is then output from OptiStruct for either the in-plane stress or strain components for each element (σ_x, σ_y, σ_{xy} or ε_x, ε_y, ε_{xy} respectively). This data enables the two orthogonal principal stress/strain magnitudes to be determined in addition to their orientation (σ_1, σ_2 or ε_1, ε_2 respectively and θ) using either the equations of statics or by finding the Eigen vectors from the initial stress or strain tensors [29, 30, 31]. During this process the sign of the principal stress directions cannot be determined. A MATLAB script has been developed by the authors for this purpose. The script is able to recognise patterns between neighbouring principal stress or strain values and can sort them into the most consistent order possible. Once this principal stress/strain data has been sorted it is a simple process of tracing the resulting isostatic lines. The spacing between the lines can be based on either a maximum 'force line' constraint [32] or an average line spacing. In this current work, the latter approach is used for simplicity of implementation.

2.3 Spatially graded lattice structures - cells graded by density

A second in-house spatially graded approach has been developed that uses the optimised 3D solid element density distribution to functionally grade cell size, shape and orientation. Topology optimised density data, along with the respective element locations, are exported from OptiStruct after an initial topology optimisation phase. A MATLAB script then takes this density data and constructs a new OptiStruct input file where the effective thermal strain in each element is now a function of this optimised density:

$$\alpha\Delta T = h(\rho_{lattice}/\rho_{solid})\Delta T = C(\rho_{lattice}/\rho_{solid})\Delta T. \tag{4}$$

The exact form of the function h is a user input but for the example in this paper a simple linear relationship is used, where C is a scaling factor with units of 1/temperature. The result of this process is that higher density elements tend to 'shrink' whereas lower density elements 'expand' into a new state of static equilibrium when a temperature difference is applied in OptiStruct.

3 SPECIMEN DESIGN, MANUFACTURING AND TESTING

Sandwich structures are commonly found in light-weight designs, for applications such as aircraft floor panels and skins [33, 34]. In the following sections a case study of a sandwich panel is presented that combine macro scale topology optimisation and parametric cell size optimisation along with in-house developed methods for generating spatially graded cellular structures. Comparison is also made with a conventional 'void-solid' topology optimised design that contains no lattice regions.

The available design space is shown in Figure 2a. 2 mm thick upper and lower skins serve to provide stiffness and strength whereas the main purpose of the core material is to prevent these skins from buckling. The inner core region forms the design space for this optimisation study. The skins form the non-design space and are shown in red in Figure 2a. The load case considered is 4-point bending, which results in a near-constant bending moment applied to the central section of the structure between the two applied loads, see Figure 2b. Features known as 'ramp downs' [33] are included toward the two simply supported boundary conditions. Ramp downs are commonly used when point loads are applied to a sandwich panel, such as a joint. Another common design feature, a recess, is also included on the lower surface of the panel. The objective function for all designs is to minimise compliance subject to a design space volume fraction constraint of 0.25 and a yield stress constraint of 39 MPa. The lattice designs also have an additional buckling constraint applied at the size optimisation stage. Each of the two applied loads are 900 N and the length, height and thickness of the panel section is 250 mm × 30 mm × 15 mm. Due to symmetry only half the length of the panel is modelled with a symmetry line placed between the two applied loads.

The material used for this study is known as VisiJet CR-WT 'ABS like' and has a Young's modulus, E_{solid}, of 1700 MPa. This material was selected since it has high rigidity and can be printed using a ProJet MJP 5500X [35] to a typical resolution of 600 × 600 × 1600 DPI. Such high resolution is ideal for printing intricate lattice structures for validation purposes. Another benefit of this printing technology for printing lattices, which have large amounts of overhanging material, is that the printer uses a wax based supporting material that can be easily melted away after printing is completed. 4-point bending tests were carried out on an Instron 5982 universal testing machine fitted with a 10 kN load cell. Compressive load was applied at a rate of 1 mm/min.

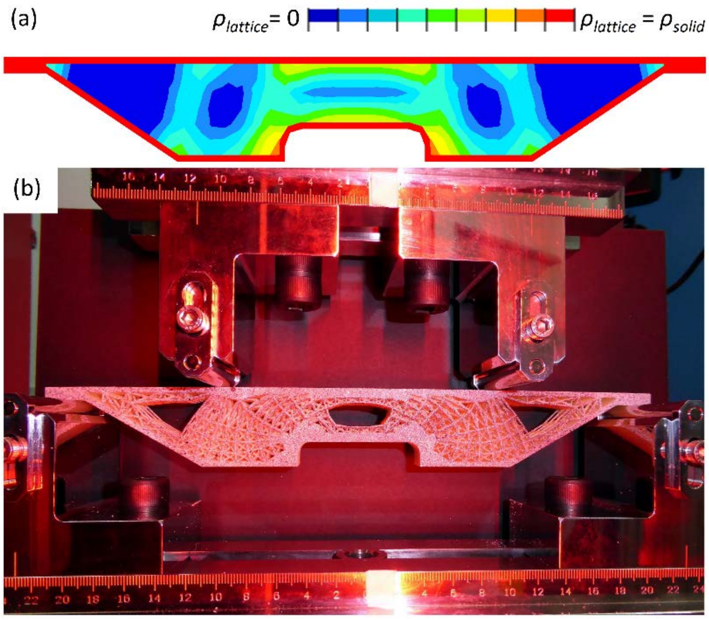

Figure 2: Comparison between (a) normalised density distribution after topology optimisation and (b) strain graded cell distribution of printed sandwich panel.

4 RESULTS AND DISCUSSION

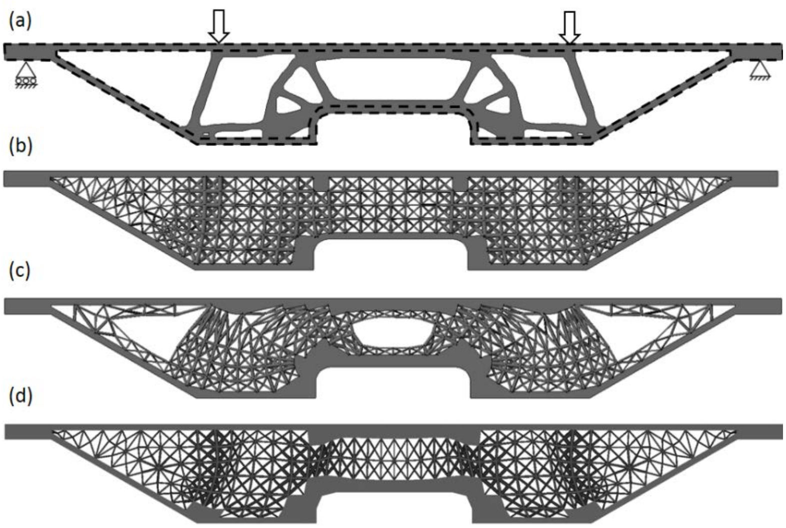

Figure 3: Sandwich panel optimised for a 4-point bend load case (a) solid design, (b) diameter graded cells, (c) strain graded cells and (d) density graded cells.

Four sandwich panel designs were investigated, see Figure 3. The first two designs use commercially available OptiStruct functionality without any further modifications. The design in Figure 3a consists of only solid material with the void-solid density threshold set at 0.5. The design in Figure 3b includes the orthorhombic lattices unit cell with an average cell size of 5 mm and a minimum diameter bound of 0.5 mm. The lower bound density cut-off for void-lattice formation is 0.1 and the lattice-solid transition density is varied between 0.5 and 0.9 in the results that follow. The final two designs use our spatially graded lattice generation approaches. Figure 3c shows a strain graded design with an average isostatic line spacing of 5 mm. The density graded design shown in Figure 3d also has an initial mesh seed of 5 mm. The thermal strain scaling factor C in Equation (4) is set equal to -1 K^{-1} and the temperature difference ΔT is set to 1 K.

A correlation can be seen in Figures 2 and 3 between the arrangement of the isostatic lines and the target optimal density. Aligning the cells with these principal strain directions cause voids to form when the lower 0.1 density cut-off was applied. Some amount of user input is required to convert the isostatic mesh generated by the MATLAB routine into a usable finite element mesh. This was a fairly labour intensive process that prevented a full parametric investigation at multiple lattice-solid density cut-offs, as was conducted for the diameter graded and density graded designs.

An extensive series of parametric studies have been carried out for these four sandwich panel designs and they are summarised in Figure 4, along with experimental results. The results show that there is a trade-off between stiffness and strength. The solid design is the stiffest of all designs investigated but also the weakest because its slender features are prone to buckling. The diameter graded and density graded designs have relatively similar performance with the density graded approach generally creating panels that are slightly stiffer but weaker. Both approaches show that reducing the percentage of lattice in the design space is beneficial for stiffness but not for strength. Apart from the solid design all structures failed by yielding. Finite element analysis shows that the strain graded approach is beneficial for recovering both stiffness and strength over the other two lattice generation methods for the same lattice percentage.

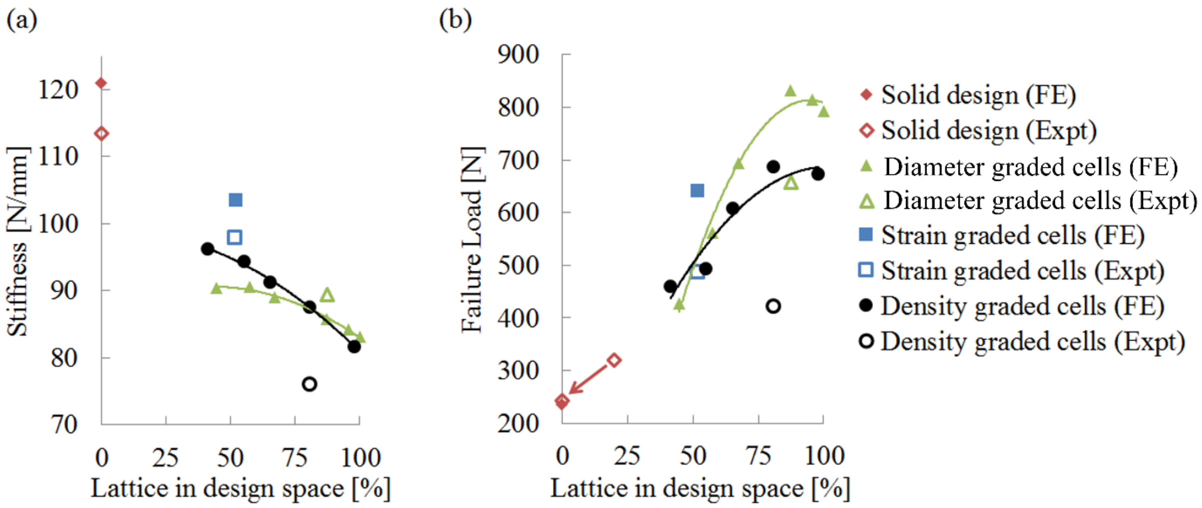

Figure 4: Influence of solid-lattice density threshold and lattice design on (a) stiffness and (b) failure load of floor beams.

Comparisons of stiffness values obtained from finite element analysis and experimental testing are generally better than the correlation for strengths. The strengths of the lattice structures manufactured on the Projet MJP 5500X were all below their predicted values. A similar observation was previously made for additively manufactured metallic lattice structures [36]. These discrepancies might in part be down to the 'σ_{norm}' method used in this work. This strength model is beneficial for promoting convergence during size optimisation as it effectively smears the stress state of multiple lattice members. However, this strength criterion might be too simplistic to represent the actual failure load. In addition, the beam elements used to construct the models cannot accurately model the detailed stress state around each joint. Another limitation of this strength method is that is does not account for statistical influence of any manufacturing flaws [37].

5 CONCLUSIONS

We have investigated and compared four optimisation strategies; namely solid topology designs and diameter graded, strain graded and density graded lattice designs. These optimisation strategies have been applied to a sandwich panel case study. It can be concluded that the selection of the most appropriate functionally grading optimisation methodology is application specific and is determined by loads, geometry and material properties. Determination of an optimal design is also dependent on whether structural strength or stiffness is of primary importance, along with a wider range of potential multifunctional design considerations.

For the lattice designs investigated a trade-off between stiffness and strength has emerged. The solid optimised designs created the stiffest structures and introducing more lattice into the design space is generally detrimental to stiffness. This observation is consistent with previous studies [14]. However, strength can be improved when the failure mode of an equivalent solid design is dominated by buckling. Buckling is not easily accounted for during macro scale topology optimisation but can be considered during size optimisation at the truss level.

Important design variables identified include the lower and upper bounds of lattice formation and the minimum thickness constraint. Changing these parameters has a significant influence on the optimised designs obtained. The two spatially graded approached developed in-house are shown to be effective means of improving the stiffness of the lattice designs in the majority of cases. These approaches can be used to improve lattice designs over commercially available topology and size optimisation approaches.

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