

# FATIGUE LIFE PREDICTION OF COMPOSITE LAMINATES BASED ON MASTER FATIGUE CURVE

Song Zhou<sup>1\*</sup>, Yi Sun<sup>1</sup>, Xiaojie Chen<sup>1</sup>

<sup>1</sup> Department of Astronautic Science and Mechanics, Harbin Institute of Technology, Harbin 150001, China

\*corresponding author: dagaier@163.com; Phone: +86 451 86418100

**Keywords:** Composite laminates, Fatigue life, Multiaxial stress

## ABSTRACT

Fatigue life of fibre reinforced composite is an important aspect in composite failure investigation. In this paper, master fatigue curve method is adopted to predict fatigue life of unidirectional and braid composite laminate. Different fatigue loading frequency and stress ratios could be used in the master fatigue curve to predict the fatigue life of composite laminate. Equivalent stress is introduced to predict the fatigue life when laminate is under multiaxial stress state. Fatigue life of unidirectional and braid composite laminate predicted by the master fatigue curve show good agreements with experiment.

## 1 INTRODUCTION

Fatigue of composite laminates is very important during the whole service period of composites, which has obvious influence on the cost control and reliability design of composite structure. Traditional unidirectional and braid composite laminates are the most common used in the composite structure. Fatigue properties of composites have drew many researchers attention in the past few decades.

Fatigue life models of composite laminate under constant amplitude fatigue loading are established on the S-N curves or Goodman-type diagrams and a fatigue failure criterion. S-N curves and Goodman diagrams are quite familiar to the researcher. So the failure criterion is quite important for the composite fatigue life prediction. The first fatigue failure criterion was proposed by Hashin [1], in his model fibre and matrix failure were taken into consideration. The strain energy density was introduced in the failure criterion by Ellyin [2]. Reifsnider [3] established a fatigue failure criterion, based upon an average stress formulation of composite derived from the Mori-Tanaka method. Fawaz [4] proposed a semi-log linear relationship between applied cyclic stress and the number of cycles to failure. Harris [5, 6] proposed a normalized constant-life model which is the combinations of mean and peak stress amplitudes. Modified the Tsai-Hill failure criterion was proposed by Jen [7] for plane stress multiaxial fatigue loading. Philippidis[8] adopted a modified Tsai-Wu quadratic failure criterion for multiaxial fatigue life prediction. They implied their fatigue life model was fit for any kinds of laminate, including the braid laminates. Plumtree [9] modified the metal multiaxial fatigue failure criterion to predict the composite laminate fatigue life. Bond [10] developed a semi-empirical fatigue life prediction methodology for variable-amplitude loading of glass fibre-reinforced composites. Castillo [11] used more complex fatigue models than the traditionally used linear model. Tay [12] adopted master fatigue curve method to predict the multiaxial fatigue life of composite laminate. In his model, the fatigue master curve is established by the Time-Temperature Superposition Principle (TTSP) which is first established by Miyano [13], and the failure criterion is modified Puck [14, 15] failure criterion.

In this paper, master fatigue curve is adopted to predict fatigue life of unidirectional and braid composite laminates. The fatigue life prediction of unidirectional composite laminate by master fatigue curve is compared with the experimental result in literature [16]. Braid composite fatigue experiments are carried out by ourselves using stress ratios  $R=-1$ , 1 and 10, which the CLD (constant life diagram) is obtained from. The numerical results show good agreements with the experimental results.

## 2 MASTER FATIGUE CURVE METHOD

### 2.1 MASTER FATIGUE CURVE

The fatigue master curve is established by the Time Temperature Superposition Principle (TTSP) under uniaxial loadings. Fatigue life of composites can be described in wide ranges of loading and environmental conditions. The fatigue master curve can be generally formulated as:

$$\log \sigma_f = \log \sigma_{s,0} - \log \left[ 1 + \frac{(2ft)^{n_c}}{(2ft_1)^{n_r}} \right] - \log (2ft)^{n_t} \quad (1)$$

Where  $\sigma_f$ ,  $f$ ,  $t$  are the uniaxial fatigue failure stress, frequency, and failure time, respectively.  $\sigma_{s,0}$  is the constant strain rate (CSR) strength at the initial reduced time.  $t_1$  is the transient reduced time.  $n_c$ ,  $n_f$ ,  $n_r$  are exponent parameters determined by fitting test data. By introducing the  $g_f$ , Eq. (1) can be expressed as follow:

$$\log \sigma_f = \log \sigma_{s,0} - \log \left[ 1 + g_f \left( \frac{2ft}{2ft_1} \right)^{n_r} \right] - g_f \log (2ft)^{n_t} \quad (2)$$

And the normalized fatigue master curve:

$$M_f = \left[ (2ft)^{g_f n_r} + g_f \left( \frac{t}{t_1} \right)^{n_r} (2ft)^{n_a} \right]^{-1} \quad (3)$$

where

$$g_f = \begin{cases} (1 - c^2)/(1 - c_0^2), & \text{if } c > 0 \\ [1 - c(|c| + c_0)/2]/(1 - c_0^2), & \text{if } c < 0 \end{cases} \quad (4)$$

$$\begin{cases} M_f = \sigma_f / \sigma_{s,0} \\ n_a = g_f n_r + n_c - n_r \end{cases} \quad (5)$$

$$c = \operatorname{sgn}(\sigma_{\min} \sigma_{\max}) \frac{|\sigma_{\min}|}{|\sigma_{\max}|} = \begin{cases} R, & \text{if } -1 \leq R < 1 \\ 1/R, & \text{if } |R| > 1 \end{cases} \quad (6)$$

Empirical coefficient  $g_f$  is the effect of mean and alternating stresses relative to the reference state.  $-1 \leq c < 1$  is a load amplitude ratio.  $R$  and  $R_0$  are the stress ratios in the actual and reference states, respectively. If  $0 \leq R_0 < 1$   $c_0 = R_0$ ; If  $R_0 > 1$   $c_0 = 1/R_0$ . From the Eq. (1),  $\sigma_{s,0}$  is quite important for the fatigue life prediction, a relationship between  $\sigma_{s,0}$  and  $\sigma_0$  (static strength) is as follow:

$$\sigma_{s,0} = \sigma_0(1 + a) \quad (7)$$

Where:

$$a = g_f / (2ft_1)^{n_r} \quad (8)$$

For a constant load frequency and using the relationship  $N_f = ft$ , the traditional uniaxial S-N curve can be obtained from the fatigue master curve formulation of Eq. (2):

$$\sigma_f = \frac{\sigma_0(1+a)}{(2N_f)^{n_1} + a(2N_f)^{n_2}} \quad (9)$$

So when master fatigue curve is established by the uniaxial S-N curves, then uniaxial S-N curve with any stress ratio and stress amplitude could be obtained from Eq. (9).

## 2.2 Multiaxial equivalent stress

Due to the anisotropy of composite and contact between different structure components, composite is under multiaxial stress state even when the composite under uniaxial fatigue loading. However, the S-N curves are obtained using uniaxial fatigue loading. So the uniaxial S-N curves cannot be used in the multiaxial fatigue life prediction. In this paper, multiaxial equivalent stress is introduced into the multiaxial fatigue life calculation in Eq. (10):

$$\sigma_{eq} = \sqrt{\sigma^T \sigma} = |\sigma_N| \sqrt{\sum_i \lambda_i^2} \quad (10)$$

Equivalent stress  $\sigma_{eq}$  can be obtained with the above mentioned fatigue model and given multi-axiality ratios  $\lambda_i = \sigma_i / |\sigma_N|$ .  $\sigma_N$  is the first non-zero stress component of  $\sigma$ . Modified Puck's fatigue criterion is adopted in the fatigue failure determination.

## 3 FATIGUE EXPERIMENTS OF BRAID COMPOSITE LAMINATES

The braid composite laminate for the fatigue experiments are made by T700/Epoxy 2D woven prepreg according to GJB 3065-1997. As shown in Fig. 1, geometric parameters of tensile-tensile fatigue specimens can be seen, and the thickness of laminate is 3mm with each ply thickness 0.2mm. Fatigue mode of composite laminate under stress ratio R=0.1 also can be seen in the Fig. 1. The fatigue experiments are carried out by Instron-880 fatigue equipment with fatigue loading frequency 10Hz.

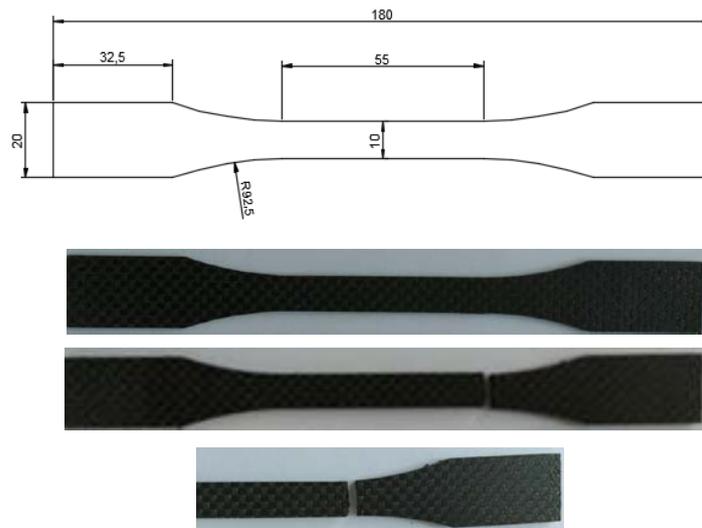


Figure 1: Tensile-tensile fatigue specimens and failure mode.

Geometric parameters of compressive-compressive and tensile-compressive fatigue specimens are shown in Fig. 2. The thickness of specimen is same with the tensile-tensile fatigue specimen. Fatigue modes of these two kinds of specimens are more complicated than the one of tensile-tensile specimen. Because the compressive strength of matrix is lower than the tensile strength of fibre, so when laminate is under compressive loading, matrix fatigue failure firstly occurs, then the fibre damage appears. The local buckling can be found in these two kinds of failure modes.

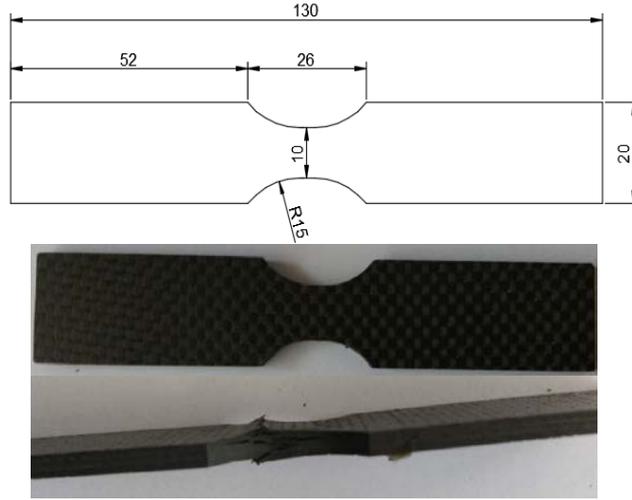


Figure 2: Tensile-compressive and compressive-compressive specimens and failure mode.

Fatigue life of braid composite laminate with three stress ratios are shown in the Fig. 3 using spot diagram. The straight lines in Fig. 3 are the S-N curves of braid composite laminate with different stress ratios, and the S-N curves can be expressed in the Eq. (11), (12) and (13).

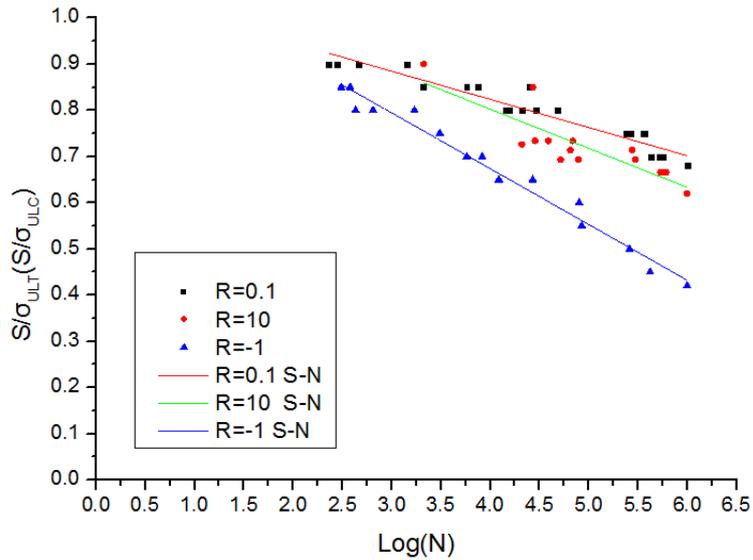


Figure 3: S-N curves with different stress ratios.

Tensile-Tensile S-N curve with stress ratio R=0.1.

$$S/\sigma_{UT} = 1.068 - 0.061 \times \log(N) \quad (11)$$

Compressive-Compressive S-N curve with stress ratio R=10.

$$S/\sigma_{UC} = 1.197 - 0.066 \times \log(N) \quad (12)$$

Tensile-Compressive S-N curve with stress ratio R=-1.

$$S/\sigma_{UC} = 1.156 - 0.122 \times \log(N) \quad (13)$$

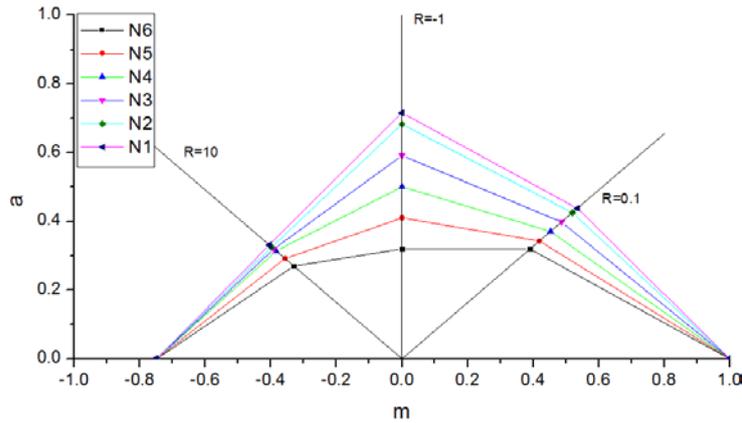


Figure 4: CLD of braid composite laminates.

By using the S-N curves with three stress ratios, the CLD (constant life of diagram) is established shown in Fig. 4. The Fig. 4 can be used to verify the master fatigue curve proposed in this paper for any stress amplitude and stress ratio.

#### 4 RESULTS AND DISSCUSSION

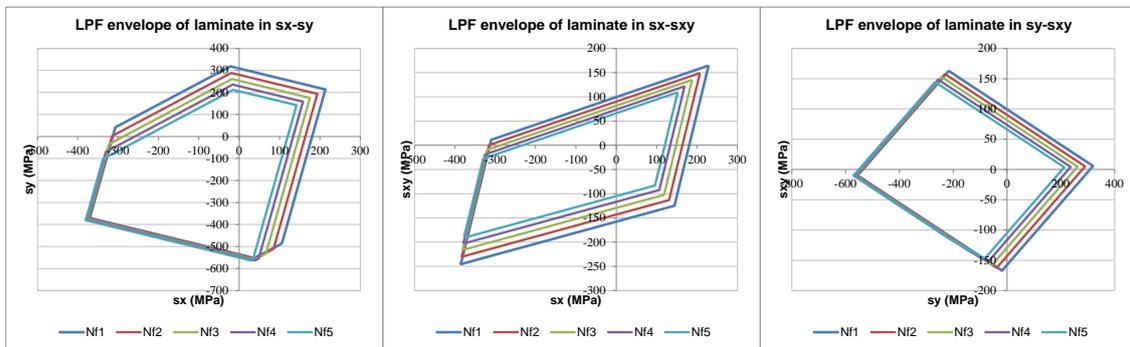


Figure 5: Failure envelope of laminate with stacking sequence  $[[ -75/15/60 ]_4 -75/15]$  under fatigue loading with stress ratio  $R=-1$  and maximum stress 113.43MPa.

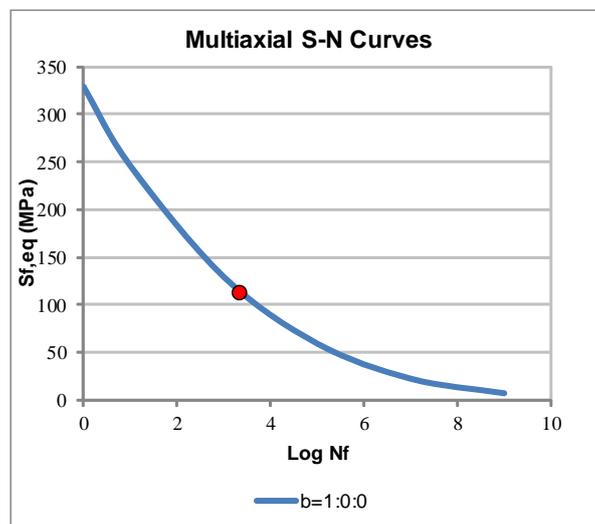


Figure 6: Fatigue life of composite laminate with stacking sequence  $[[ -75/15/60 ]_4 -75/15]$  under fatigue loading with stress ratio  $R=-1$  and maximum stress 113.43MPa.

To verify the master fatigue curve for the fatigue life prediction, unidirectional composite laminate [16] with ply sequence  $[-75/15/60]_4-75/15$  under fatigue loading is chosen for the comparison. The unidirectional laminate is under fatigue loading with stress ratio  $R=-1$  and maximum stress 113.43MPa. All the fatigue parameters can be founded in literature [12]. Figure 5 is fatigue failure envelope of laminate with  $Nf_1=100$  ,  $Nf_2=200$  ,  $Nf_3=400$  ,  $Nf_4=800$  ,  $Nf_5=1600$ . With the increase of cycle number, failure envelope of laminate becomes small. The progressive failure properties can be seen in the Fig. 5.

As shown in Fig. 6, when composite laminate with stacking sequence  $[-75/15/60]_4-75/15$  is under fatigue loading with stress ratio  $R=-1$  and maximum stress 113.43MPa, Fatigue cycle number of composite is 1601, and the numerical result is 1510. The computational accuracy is acceptable.

Property parameters		Experimental Results
$E_{11}$	[GPa]	526.15
$E_2$	[GPa]	420.43
$\nu_{12}$		0.3
$G_{12}$	[GPa]	2.7
$\sigma_{ULT}$	[MPa]	526.15
$\sigma_{ULC}$	[MPa]	420.43
$S_{12}$	[MPa]	72

Table 1: Properties of T700/Epoxy braid composite laminate.

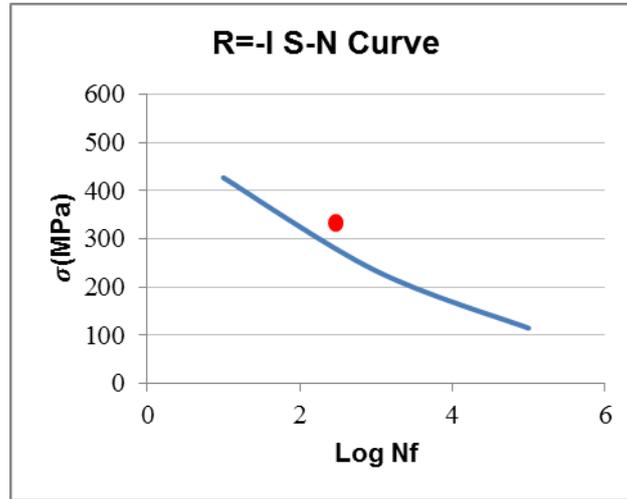


Figure 7: S-N curve of braid laminate under fatigue loading with stress ratio  $R=-1$  and maximum stress 357.36MPa.

After the verification of unidirectional laminate, fatigue life of braid composite laminate is predicted by master cure method. Since the 2D braid composite laminate is similar to the 0/90 ply sequence laminate, the properties of braid composite is shown in Table 1. When braid laminate is under fatigue loading with stress ratio  $R=-1$  and maximum stress 357.36MPa, fatigue life of laminate calculated by master fatigue curve is 252, and the experimental result is 102. Compared with the experimental result, although the deviation of numerical result is a little bigger, deviation of numerical result is still included in a scatter band of the factor of 3.

## 5 CONCLUSIONS

In this paper, master fatigue curve is proposed for the fatigue life prediction of braid composite laminate. CLD of braid composite laminate is obtained from the fatigue experiments. Numerical

results of unidirectional and braid composite laminate shows good agreements with the experimental results, and the numerical results show that the master fatigue is a good choice for the fatigue life prediction for the composite laminate. Due to the less fatigue experiments and stronger applicability, cost of fatigue life prediction could reduce significantly.

### ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Foundation of China (NO. 11402064), the Fundamental Research Fund for the Central Universities (Grant No. HIT. NSRIF. 201623). Part of this work was performed while the corresponding author was visiting the National University of Singapore and funded by the China Scholarship Council (CSC).

### REFERENCES

- [1] Z. Hashin and A. Rotem, A fatigue criterion for fibre reinforced composite materials. *Journal of Composite Materials*, **7**, 1973, pp. 448-464.
- [2] F. Ellyin and H. El-Kadi, A fatigue failure criterion for fiber reinforced composite laminate. *Composite Structures*. **15**, 1990, pp. 61-74.
- [3] K.L. Reifsnider and Z. Gao, A micromechanics model for composites under fatigue loading, *International Journal of Fatigue*, **13**, 1991, pp. 149-156.
- [4] Z. Fawaz and F. Ellyin, Fatigue failure model for fibre reinforced materials under general loading conditions. *Journal of Composite Materials*, **28**, 1994, pp.1432-1451.
- [5] M.H. Beheshty and B. Harris, A constant-life model of fatigue behavior for carbon-fibre composites: the effect of impact damage. *Composite Science and Technology*, **58**, 1998, pp. 9-18.
- [6] M.H. Beheshty, B. Harris and T. Adam, An empirical fatigue life model for high-performance fibre composites with and without impact damage. *Composites Part A: Applied Science and Manufacturing*, **30**, 1999, pp. 971-987.
- [7] M.H.R. Jen and C.H. Lee, Strength and life in thermoplastic composite laminates under static and fatigue loads Part I: Experimental. *International Journal of Fatigue*, **20**: 1998, pp. 605-615.
- [8] T.P. Philippidis and A.P. Vassilopoulos, Fatigue strength prediction under multiaxial stress. *Journal of Composite Materials*, **33**, 1999, pp.1578-1599.
- [9]. A. Plumtree and G.X. Cheng, A fatigue damage parameter for off-axis unidirectional fibre-reinforced composites. *International Journal of Fatigue*, **21**, 1999, pp. 849-856.
- [10]. I.P. Bond, Fatigue life prediction for GRP subjected to variable amplitude loading. *Composites Part A: Applied Science and Manufacturing*, **30**, 1999, pp. 961-970.
- [11]. E.A.S. Castillo, A.Fernandez-Canteli and A.S.Hadi, On fitting a fatigue model to data. *International Journal of Fatigue*, **21**, 1999. pp. 97-106.
- [12] X.S. Sun, A. Haris, V.B.C. Tan, T.E. Tay, S. Narasimalu and C.N. Della, A multi-axial fatigue model for fiber-reinforced composite laminates based on Puck's criterion. *Journal of Composite Materials*, **46(4)**, 2011, pp. 449-461.
- [13] Y. Miyano, M. Nakada, H. Cai, Formulation of long-term creep and fatigue strengths of polymer composites based on accelerated testing methodology. *Journal of Composite Materials*, **42**, 2008, pp. 1897-1919.
- [14] A. Puck, H. Schurmann, Failure analysis of FRP laminates by means of physically based phenomenological models. *Composites Science and Technology*, **62**, 2002, pp. 1633-1662.
- [15] Puck A, Kopp J and Knops M. Guidelines for the determination of the parameters in Puck's action plane strength criterion. *Composites Science and Technology*, **62**, 2002, pp. 371-378
- [16] T.P. Philippidis, A.E. Antoniou, T.T. Assimakopoulou, V.A. Passipoularidis, Fatigue tests on OB unidirectional & multidirectional fff-axis coupons: main test phase I', OPTIMAT BLADES, ENK6-CT2001-00552, OB\_TG2\_R030, [http://www.wmc.eu/public\\_docs/10350\\_000.pdf](http://www.wmc.eu/public_docs/10350_000.pdf).